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On The Flow of Maxwell Fluid Between Two Walls Induced By A Constantly Accelerating Plate

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Abstract

The flow of a Maxwell fluid between two side walls induced by a constant accelerating plate is revisited. In the present investigation, we employed asymptotic technique by assuming small and large relaxation times λ. We proved the uniqueness of our solution based on some simplifying assumption; the result shows that λ has much influence on the velocity field. A comparison of large and small relaxation times λ shows that the velocity is higher when the relaxation time is small. Moreover the flow is reversed when the relaxation time is large, that is 1/λ is small.

Keywords: - Asymptotic techniques, Maxwell fluid, constantly accelerating plates and velocity fields.

1.0 Introduction

In this paper, we revisit the work in [1], in which the Authers established exact solutions corresponding to the unsteady flow of a Maxwell fluid induced by a constantly accelerating plate between two side walls perpendicular to the plate. In their work, solutions were obtained by means of the Fourier transforms. However, in the present study we are interested in the asymptotic behaviour of the relaxation time (λ) that is when λ is small and the behaviour when λ is large under same condition of Maxwell fluid between two side walls induced by a constantly accelerating plates. Literature is rich for viscoelastic fluid flow [3, 4] where the simplest subclass of the rate type fluids take into consideration the stress relaxation effects and the flow between two walls of a plate that is suddenly moved.

The paper is organized as follows: in section one of the paper we give a brief introduction and in section two the model is formulated while section three is concerned with the detailed method of solution, in section four, results are presented and discussed while section five concludes the paper.

2.0 Governing Equations

Following [1], we examine the flow between two walls at z = 0 and z = d

$$V = V(y,z,t) = (u(y,z,t),0,0) \tag{2.1}$$

Assume that the stress S is a function of y, z and t, then (2.1) can be written as

$$S = S(y,z,t) \tag{2.2}$$

Hence,

$$(1+\lambda\partial_t)\tau_1 = \mu\partial_y u$$
$$(1+\lambda\partial_t)\tau_2 = \mu\partial_z u \tag{2.3}$$

For shear stress, $\tau_1 = S_{xy}$ and $\tau_2 = S_{xz}$

In the absence of body forces, the balance of linear momentum reduces to

$$\partial_y\tau_1 + \partial_z\tau_2 - \partial_x P = \rho\partial_t u,$$
$$\partial_y P = \partial_z P = 0 \tag{2.4}$$

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Eliminating τ_1 and τ_2 from equations (2.3) and (2.4), then we have:

$$\tau_1 = \frac{\mu\partial_y u}{(1+\lambda\partial_t u)} \quad \text{and} \quad \tau_2 = \frac{\mu\partial_z u}{(1+\lambda\partial_t u)} \tag{2.5}$$

and (2.5) becomes

$$\frac{\mu\partial_y^2 u}{(1+\lambda\partial_t u)} + \frac{\mu\partial_z^2 u}{(1+\lambda\partial_t u)} - \partial_x P = \rho\partial_t u, \tag{2.6}$$

$$\partial_y P = \partial_z P = 0$$

Simplifying this, we obtain

$$\mu\partial_y^2 u + \mu\partial_z^2 u - (1+\lambda\partial_t u)\partial_x P = (1+\lambda\partial_t)\rho\partial_t u \tag{2.7}$$

that is:

$$\frac{\partial u}{\partial t} + \lambda\frac{\partial^2 u}{\partial t^2} = \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{\partial P}{\partial x}\left(1+\lambda\frac{\partial u}{\partial t}\right) \tag{2.8}$$

We assume that the flow along the z co-ordinate is negligible, and then equation (2.8) reduces to

$$\frac{\partial u}{\partial t} + \lambda\frac{\partial^2 u}{\partial t^2} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2.9}$$

with the initial and boundary conditions,

$$u(y,0) = \frac{\partial u}{\partial t}(y,0) = 0;\ y>0,\ u(0,t) = At; t>0 \ \text{ and } \ u(d,t) = 0 \tag{2.10}$$

when $\frac{\partial P}{\partial x} = 0$

3.0 Method of Solution

For small λ: To obtain our result, we use asymptotic expansion of the form:

$$u = u_0 + \lambda u_1 + \lambda^2 u_2 + \lambda^3 u_3 + ... \tag{3.1}$$

Substituting equation (3.1) in equation (2.9) and equating in favour of powers of λ, we obtain

$$\lambda^0: \quad \frac{\partial u_0}{\partial t} = \nu\left(\frac{\partial^2 u_0}{\partial y^2}\right), \quad u_0(y,0) = \frac{\partial u_0}{\partial t}(y,0) = 0; y>0, \quad u_0(0,t) = At, u_0(d,t) = 0 \tag{3.2}$$

$$\lambda^1: \quad \frac{\partial u_1}{\partial t} + \frac{\partial^2 u_0}{\partial t^2} = \nu\left(\frac{\partial^2 u_1}{\partial y^2}\right), \quad u_1(y,0) = 0, u_1(0,.t) = 0, t.>0, \quad u_1(d,t) = 0 \tag{3.3}$$

the zeroth order solution of equation (3.2) gives

$$u_0(y,t) = At\left(1 - \frac{erf\left(\frac{y}{2\sqrt{\nu t}}\right)}{erf\left(\frac{d}{2\sqrt{\nu t}}\right)}\right) \tag{3.4}$$

and equation (3.3) gives (for more, see[2])

$$u(y,t) = \sum_{n=1}^{\infty}\left(\int_0^t \exp^{-\nu\lambda_n(t-\tau)} f_n(\tau)d\tau\right) Sin\frac{n\pi}{d}y + \sum_{n=1}^{\infty} b_n \exp^{-\nu\lambda_n t}\ Sin\frac{n\pi}{d}y \tag{3.5}$$

This gives the solution of the problem.

Again for large λ: We use the expansion of the form

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$$u = u_0 + \frac{1}{\lambda}u_1 + \frac{1}{\lambda^2}u_2 + \frac{1}{\lambda^3}u_3 + ... \quad (3.6)$$

inserting equation (3.6) into equation (2.9), we obtain this

$$\lambda^0 : \quad \frac{\partial^2 u_0}{\partial t^2} = 0, \quad u_0(y,0) = \frac{\partial u_0}{\partial t}(y,0) = 0; y > 0, \quad u_0(0,t) = At, \quad u_0(d,t) = 0 \quad (3.7)$$

$$\lambda^{-1} : \quad \frac{\partial u_0}{\partial t} + \frac{\partial^2 u_1}{\partial t^2} = \nu \frac{\partial^2 u_0}{\partial y^2}$$
$$u_1(y,0) = 0, \; u_1(d,t) = 0 \quad (3.8)$$
$$u_1(0,t) = 0, t > 0$$

$$\lambda^{-2} : \quad \frac{\partial u_1}{\partial t} + \frac{\partial^2 u_2}{\partial t^2} = \nu \frac{\partial^2 u_1}{\partial y^2}$$
$$u_2(y,0) = 0 \quad (3.9)$$
$$u_2(d,t) = 0, \quad u_2(0,t) = 0, t > 0$$

$$\lambda^{-3} : \quad \frac{\partial u_2}{\partial t} + \frac{\partial^2 u_3}{\partial t^2} = \nu \frac{\partial^2 u_2}{\partial y^2}$$
$$u_3(y,0) = 0 \quad (3.10)$$
$$u_3(d,t) = 0, \quad u_3(0,t) = 0, t > 0$$

and so on.
Solving equations (3.7 – 3.10) one by one with our

$$k(y) = A\left(1 - \frac{y^2}{d^2}\right),$$

we obtained,

$$u_0 = At\left(1 - \frac{y^2}{d^2}\right), \quad u_1 = \frac{At^2}{2}\left(\frac{y^2}{d^2} - 1\right), \quad u_2 = \frac{At^3}{6}\left(1 - \frac{y^2}{d^2}\right), \quad u_3 = \frac{At^4}{4!}\left(\frac{y^2}{d^2} - 1\right)$$

and we insert u_0, u_1, u_2, u_3 ... into equation (3.6) to obtain,

$$u = At\left[1 - \frac{y^2}{d^2}\right] + \frac{1}{\lambda}\frac{At^2}{2}\left[\frac{y^2}{d^2} - 1\right] + \frac{1}{\lambda^2}\frac{At^3}{3!}\left[1 - \frac{y^2}{d^2}\right] + \frac{1}{\lambda^3}\frac{At^4}{4!}\left[\frac{y^2}{d^2} - 1\right] + ... \quad (3.11)$$

which can be written in this form $u(y,t) = A\lambda\left(\frac{y^2}{d^2} - 1\right)\sum_{n=0}^{\infty}\left(\frac{1}{(n+1)!}\left(\frac{-t}{\lambda}\right)^{n+1}\right)$ (3.12)

and more conveniently in Taylor series expansion for the exponential function which converges to this form:

$$u(y,t) = A\lambda\left(\frac{y^2}{d^2} - 1\right)\left(e^{-\frac{t}{\lambda}} - 1\right) \quad (3.13)$$

4.0 Discussion Of Results

Figure 4.1 shows the graph of u against y for t equals 5s, 10s and 15s respectively when relaxation time λ is small which are okay as the boundary conditions $u_0(0, t) = At$ and $u_0(d, t) = 0$ are satisfied and it clearly shows that the flow is unsteady and that the velocity has a maximum at y = 0 and increases as time also increases.
Figure 4.2 shows the graph of u against t for 1/λ equals 0.1, 0.105, 0.11, and 0.12 respectively and it shows that the flow is reversed when the relaxation time is large or termed as back flow.
Figure 4.3 shows the graph of u against y for t equals 5s, 10s and 15s respectively when relaxation time is large which satisfies all the boundary conditions and also shows that the velocity has a maximum at y = 0 and increases as time increases as well.

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5.0 Conclusion

We have studied the flow of Maxwell fluid between two walls induced by a constantly accelerating plate and our results generally showed that for both small and large relaxation times λ and 1/λ respectively, the flow increases with time in the channel.
However, of interest is the back flow behaviour for axi – symmetrical case when relaxation time is large, that is, 1/λ which was not considered in the study in [1].
We used asymptotic technique to find the nature of the Maxwell flow. The solutions exist and they are unique under the two cases of relaxation times λ. Thus, in a flow between parallel plates, the boundary conditions are important.

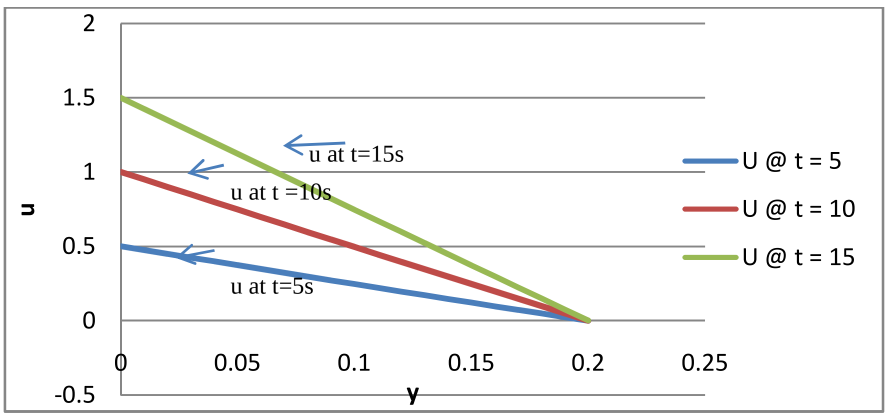

Figure 4.1: The graph of u against y for t equals 5s, 10s and 15s

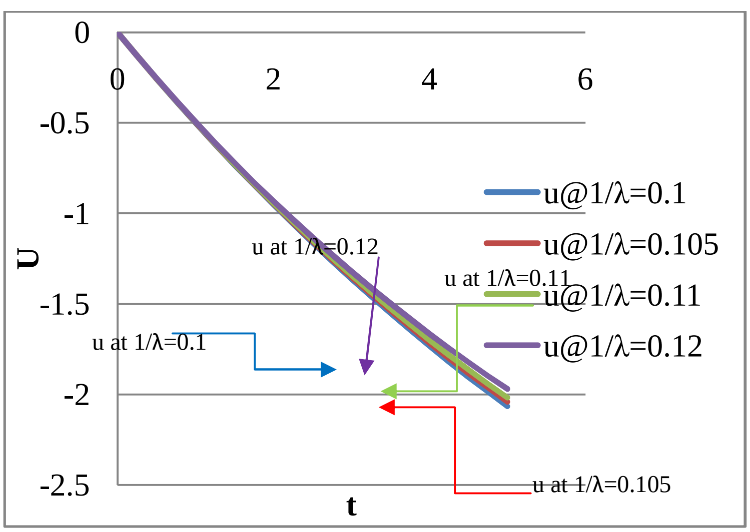

Figure 4.2: The graph of u against t for 1/ λ equals 0.1, 0.105, 0.11and 0.12

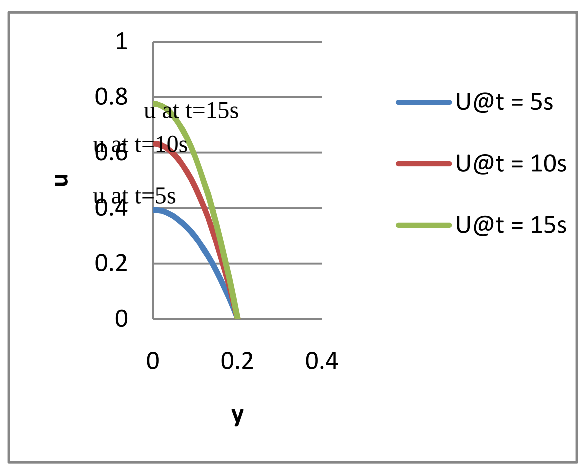

Figure 4.3: The graph of u against y for t equals 5s, 10s and 15s.

6.0 REFERENCES

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