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A Variable Thermal Conductivity Flow of A Micropolar Fluid Over A Stretching Surface in A Non-Darcian Porous Medium

Adepoju O.A[1] *And Ayeni R.O*[2]
[1,2]**Department of Pure and Applied Mathematics,**
Ladoke Akintola University of Technology,Ogbomoso, Nigeria.

Abstract

We revisited the paper of Mahmoud et al, on the hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation.We show that even when the thermal conductivity depends linearly or quadratically on temperature the problem still has a unique solution.

1.0 Introduction

The boundary layer flow of a micropolar fluid past a semi-infinite plate has been studied by [10] whereas a similarity solution for boundary layer flow near stagnation point was presented by [6]. The boundary layer flow of micropolar fluid past a semi-infinite plate was studied by [1], taking into account the gyration vector normal to the xy-plane and micro inertia effects. [9] studied hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation. Flow and heat transfer of a micropolar fluid past a continuously moving plate were studied by [12]. By drawing the continuous strips through a quiescent electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. [8] studied micropolar flow over a porous stretching sheer with strong suction or injection. [4] investigated thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. [11] discussed the effect of thermal radiation on MHD asymmetric flow of an electrically conducting fluid past a semi-infinite plate.

All the above studies were confined to a fluid with constant viscosity. However, it is known that this physical property may change significantly with temperature. [3] analyzed a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. [7] studied the influence of fluid property variation on the boundary layers of a stretching surface. [2] discussed the effect of radiation on free convection flow of a fluid with variable viscosity from a porous vertical plate. In this work, we present a variable thermal conductivity flow of a micropolar fluid over a stretching surface in a non-Darcian porous medium.

2. Mathematical Formulation

Consider a steady, two-dimensional laminar flow of an incompressible, electrically conducting micropolar fluid over a continuously moving stretching surface embedded in a non-Darcian porous medium which issues from a thin slit. The x-axis is taken along the stretching surface in the direction of the motion and y-axis is perpendicular to it. We assume that the velocity is proportional to its distance from the slit. A uniform magnetic field B_0 is imposed along y-axis. Then under the usual boundary layer approximations, the flow and heat transfer of a micropolar fluid in porous medium with non-Darcian effects included are governed by the following equations;

The equation of momentum is given by

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+k_1\frac{\partial N}{\partial y}-\frac{\upsilon\varphi u}{k}-c\varphi u^2-\frac{\sigma B_O^2}{\rho}u \qquad (2.1)$$

The continuity equation is

[1]Corresponding author: E-mail okay_4uall@yahoo.com , Tel. +2348035618740

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(2.2)

The angular momentum equation is

$$G_1\frac{\partial^2 N}{\partial y^2}-2N-\frac{\partial u}{\partial y}=0$$

(2.3)

And finally the energy equation is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_P}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_P}\frac{\partial k}{\partial y}\frac{\partial T}{\partial y}+\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}$$

(2.4)

with the following boundary conditions

$$y=0:u=ax,\quad v=0,\quad T=T_w,\quad N=0,$$
$$y\to\infty:u\to 0,\quad T\to T_\infty,\quad N\to 0,$$

(2.5)

Where $\upsilon=(\mu+S)/\rho$ is the apparent kinematic viscosity, μ is the coefficient of dynamic viscosity, S is a constant characteristic of fluid, N is the microrotation component, $k_1=S/\rho\,(>0)$ is the coupling constant, $G_1\,(>0)$ is the microrotation constant, ρ is the fluid density, u and v are the velocity components along x and y directions respectively, φ is the porosity, k is the permeability of the porous medium, c is Forchheimer's inertia coefficient, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid far away from the plate, T_w is the temperature of the plate, K is the thermal conductivity, C_p is the specific heat at constant pressure, σ is the electrical conductivity, B_0 is an external magnetic field, and q_r is the radiative heat flux.

Using the following transformations:

$$\eta=\left(\frac{a}{\upsilon}\right)^{1/2}y,\ \varphi=(a\upsilon)^{1/2}xf(\eta),\ N=\left(\frac{a^3}{\upsilon}\right)^{1/2}xg(\eta),$$

(2.6)

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\qquad u=\frac{\partial\varphi}{\partial y},\qquad v=-\frac{\partial\varphi}{\partial x},$$

$$q_r=\left(-\frac{4\sigma_0}{3k_0}\right)\frac{\partial T^4}{\partial y}$$

(2.7)

where σ_0 is the Stefan-Boltzmann constant and k_0 is the mean absorption coefficient.

Using the above transformations we have as thus,

$$v=-\frac{\partial\varphi}{\partial x},\qquad u=\frac{\partial\varphi}{\partial y},\ u=axf'(\eta),$$

$$\therefore\frac{\partial^2 u}{\partial y^2}=\frac{a^2}{\upsilon}xf'''(\eta),\quad \frac{\partial u}{\partial x}=af'(\eta),\quad \frac{\partial N}{\partial y}=\frac{a^2}{\upsilon}xg'(\eta),\quad \frac{\partial^2 N}{\partial y^2}=\frac{a^{\frac{5}{2}}}{\upsilon^{\frac{3}{2}}}xg''(\eta)$$

Thus equation (2.1) above becomes,

[1]Corresponding author: E-mail okay_4uall@yahoo.com , Tel. +2348035618740

$$f'''+f''f+Lg'-\left(D_a^{-1}+R\right)f'-(1+\lambda)f'^2=0$$

(2.8)

Equation (2.3) reduces to, $Gg''-\left(2g+f''\right)=0$

(2.9)

With k being constant equation (2.4) takes the form

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}$$

With $k=k_0\left(\alpha\theta+1\right)$ equation (2.4) reduces to, $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_0\alpha}{\rho C_P}\frac{\partial \theta}{\partial y}\frac{\partial T}{\partial y}+\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}$

(2.10)

and eventually becomes $3F\theta''+\frac{3F}{(\alpha\theta+1)}\theta'^2+3FP_r f\theta'+4P_r\left\{r\theta''(r\theta+1)^3+3(r\theta+1)^2r^2\theta'^2\right\}=0$

(2.11)

With $k=k_0\left(\alpha\theta^2+1\right)$ and $\theta=\theta(\eta)$,

equation (2.4) reduces to, $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{2\theta k_0\alpha}{\rho C_P}\frac{\partial \theta}{\partial y}\frac{\partial T}{\partial y}+\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}$

(2.12)

$T=\left(T_w-T_\infty\right)\theta+T_\infty$

(2.12) becomes:

$$3F\theta''+\frac{6\alpha F}{\alpha\theta^2+1}\theta\theta'^2+3FP_r f\theta'+4P_r\left\{r\theta''(r\theta+1)^3+3(r\theta+1)^2r^2\theta'^2\right\}=0 \qquad (2.13)$$

where $L=k_1/\upsilon$ denotes the coupling constant parameter, $D_a^{-1}=\varphi\upsilon/ka$ denotes the inverse Darcy number, $R=\left(\sigma_0B_0^2\right)/\rho a$ denotes the magnetic parameter, $\lambda=c\varphi x$ denotes the inertia coefficient parameter $G=G_1a/\upsilon$ denotes the microrotation parameter, $P_r=\left(\upsilon\rho C_p\right)/k$ denotes the Prandtl number $F=\left(\rho C_pk_0\upsilon\right)/\left(4\sigma_0T_\infty^4\right)$ denotes the radiation parameter, $r=\left(T_w-T_\infty\right)/T_\infty$ is the relative difference between the temperature of the surface and the temperature far away from the surface.

The corresponding boundary conditions are:

$$f(0)=0,\quad f'(0)=1,\quad \theta(0)=1,\quad g(0)=0,\quad f'(\infty)=0,\quad \theta(\infty)=0,\quad g(\infty)=0$$

3.0 Existence And Uniqueness

Theorem 1: Problem (2.11) subject to initial conditions $\theta(0)=1$, $\theta'(0)=-k$ $(k>0)$ has a unique solution in $D=\{(\theta,\theta'(\eta),\eta),0\le\theta\le1,-k\le\theta'(\eta)\le0,\eta\ge0\}$

Theorem 2: Problem (2.13), subject to initial conditions $\theta(0)=1,\theta'(0)=-k$ $(k>0)$ has a unique solution in $D=\{(\theta,\theta'(\eta),\eta),0\le\theta\le1,-k\le\theta'(\eta)\le0,\eta\ge0\}$.

Remark: For the proof we need the following ;

Let

$$\begin{aligned}
x_1' &= f_1(x_1,\dots x_n,t), & x_1(t_0)&=x_{10}\\
x_2' &= f_2(x_1,\dots x_n,t), & x_2(t_0)&=x_{20}\\
&\vdots\\
x_n' &= f_n(x_1,\dots x_n,t), & t_n(t_0)&=x_{n0}
\end{aligned}$$

[1]Corresponding author: E-mail okay_4uall@yahoo.com , Tel. +2348035618740

$$f(x,t) = \begin{pmatrix} f_1(x_1,...x_n,t \\ f_2(x_1,...x_n,t) \\ . \\ . \\ f_n(x_1,...x_n,t) \end{pmatrix}, \quad \underline{X} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}, \quad \underline{X}_0 = \begin{pmatrix} x_{10} \\ x_{20} \\ . \\ . \\ x_{n0} \end{pmatrix}$$

That is $\underline{x}^1 = f\left(\underline{x},t\right), \underline{x}\left(t_0\right) = \underline{x}_0$ (3.1)

<u>Theorem:</u> [5]

Let D denote the region

$$\left|\underline{x}-\underline{x}_0\right| \le b, \ \left|t-t_0\right| \le a.$$

If the partial derivatives $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, ...n$ are continuous in D, then (3.1) has a unique solution.

We are now in a position to prove theorem 1

Let $x_1 = \eta \quad, x_2 = \theta \quad, x_3 = \theta'$

Then (2.11) becomes

$$x_1' = f_1\left(x_1,x_2,x_3\right) = 1, x_1\left(0\right) = 0 \quad, x_2' = f_2\left(x_1,x_2,x_3\right) = x_3, x_2\left(0\right) = 1$$

$$x_3' = f_3\left(x_1,x_2,x_3\right)$$

$$= -\left\{\frac{3Fx_3^2}{\alpha x_2+1} 3FP_r f\left(x_1\right)x_3 + 4P_r\left(3\left(rx_2+1\right)^2 r^2 x_3^2\right)\right\} \Big/ \left(3F + 4P_r r\left(rx_2+\right)^3\right),$$

$$x_3\left(0\right) = -k$$

Now $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, 3$ are continuous. Hence by [5], theorem (1) holds.

Proof of Theorem 2

Let $x_1 = \eta, \quad x_2 = \theta, \quad x_3 = \theta'$

Then (2.13) becomes

$$x_1' = f_1\left(x_1,x_2,x_3\right) = 1, x_1\left(0\right) = 0$$

$$x_2' = f_2\left(x_1,x_2,x_3\right) = x_3, x_2\left(0\right) = 1$$

$$x_3' = f_3\left(x_1,x_2,x_3\right) \quad = -\left\{\frac{6\alpha Fx_2x_3^2}{\alpha x_2^2+1} + 3FP_r f\left(x_1\right)x_3 + 12P_r\left(r^2x_3^2\left(rx_2+1\right)\right)\right\} \Big/ \left(3F + 4P_r r\left(rx_2+1\right)^3\right)$$

Now $\frac{\partial f_i}{\partial x_j}, i, j = 1, 2, 3$ are continuous, hence by [5], the problem has a unique solution.

[1]Corresponding author: E-mail okay_4uall@yahoo.com , Tel. +2348035618740

Conclusion

The resulting equation governing the flow of a micropolar fluid over a stretching surface in a non Darcian porous medium with variable thermal conductivity were shown to have a unique solution, even when the thermal conductivity is linear and also when it is quadratic.

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[1]Corresponding author: E-mail okay_4uall@yahoo.com , Tel. +2348035618740