

## Hydrodynamic Lubrication Analysis Of Slider Bearings Lubricated With Micropolar Fluids

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### *Abstract*

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*In this paper, a theoretical study of the effect of micropolar lubricants on the performance characteristics of wide inclined slider bearings is presented. The finite element method and Gauss Seidel iterative procedure have been used to simulate the modified Reynolds equation governing the micropolar lubricant flow in the bearing. The variations of pressure and load capacity are presented for instances of micropolar and bearing structural parameters. It has been observed that the micropolar parameters significantly influence the performance characteristics of the bearing.*

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**Keywords:** micropolar lubricant, finite element, slider bearing, Reynolds.

### **Nomenclature**

L	Length of bearing (m)
P	Pressure (N/m <sup>2</sup> )
N <sup>2</sup>	Coupling number
$\eta$	Dynamic oil viscosity ( Pa.s)
h	film thickness (m)
h <sub>1</sub>	Film thickness at x = 0 (m)
h <sub>0</sub>	Film thickness at x = L (m)
U	Velocity of slider (m/s)
A	Micropolar length

### **1.0 Introduction**

The study of micropolar fluids has received considerable attention due to applications in a number of processes that occur in industries such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, exotic lubricants, colloidal and suspension solution. In the study of all these problems the classical Navier-Stokes theory is inadequate. [7]. Micropolar fluids are a subset of the icromorphic fluid theory introduced in a pioneering paper by [4].

A number of investigators have used the micropolar fluid theory for the study of bearing problems. [12] studied the lubricating effectiveness of micropolar fluids in a dynamically loaded journal bearing. Results from the numerical analysis indicated that the effects of micropolar fluids on the performance of a dynamically loaded journal bearing depend on the size of material characteristic length and the coupling number. They concluded that, compared with Newtonian lubricants under a dynamic loading, the micropolar lubricants produce an obvious increase in the oil film pressure and oil film thickness. [1] carried out a theoretical study of a porous pivoted slider bearing lubricated with a micropolar fluid. The effect of micropolar fluid on bearing load is in agreement with those of [12]. However, they also indicated that the effect of porosity is to decrease the load capacity. [3] considered the three dimensional Reynolds equation for micropolar fluid lubricated bearings. [8] carried out a theoretical analysis of the effect of micropolar fluids on the lubrication characteristics of porous stepped inclined composite bearing. The generalized Reynolds type equation was derived for the most general porous bearing configuration (porous composite bearings) lubricated with micropolar fluid and closed form expressions obtained for the fluid

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film pressure, load carrying capacity, frictional force and coefficient of friction. It was observed that the micropolar fluid lubricants provide an increased load carrying capacity and decreased coefficient of friction as compared to the corresponding Newtonian case. [11], employed the method of calculus of variation to predict the characteristics of an externally pressurized bearing, using lubricant suspension, characterized as a micropolar fluid. They showed that the maximum load capacity increases as the step height ratio or the parameter characterizing the micro-structure of the suspension increases.

In this paper, the continuous Galerkin finite element model is used to simulate the modified Reynolds equation accounting for the use of micropolar lubricants in the clearance zone of an infinitely wide inclined slider bearing. Parametric studies are also carried using numerical experiments to show the effect of micropolar fluids on load capacity and pressure distribution of the infinitely wide inclined slider bearing.

### Reynolds Equation And Hydrodynamic Pressure

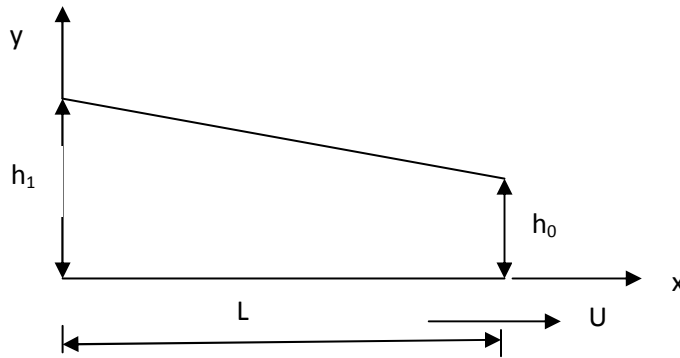


Figure 1: Physical geometry of inclined slider bearing

The geometry of the slider bearing under consideration is shown in Figure 1. The thickness of the micropolar oil film in the clearance zone is described by

$$h(x) = h_1 - \frac{(h_1 - h_0)}{L}x \quad (1)$$

The film thickness ratio is defined as

$$r = \frac{h_1}{h_0} \quad (2)$$

According to hydrodynamic lubrication theory, the pressure distribution in the clearance zone of a slider bearing can be described by the Reynolds equation. The fluid in the clearance zone is assumed to be laminar micropolar fluid.

The equation governing the pressure in the slider gap is given in dimensional form as

$$\frac{\delta}{\delta x} \left[ \frac{h^3}{\eta} \phi(A, N, h) \frac{\delta P}{\delta x} \right] + \frac{\delta}{\delta y} \left[ \frac{h^3}{\eta} \phi(A, N, h) \frac{\delta P}{\delta y} \right] = 6U \frac{dh}{dx} \quad (3)$$

In equation (3),  $N^2$  is the coupling number,  $P$  is the hydrodynamic pressure,  $A$  is the characteristic length of the micropolar fluid and  $h$  is the gap height given by equation (1)

The function  $\phi(A, N, h)$  is given by [10]

$$\phi(A, N, h) = 1 + 12 \frac{A^2}{h^2} - 6 \frac{NA}{h} \coth \left( \frac{Nh}{2A} \right) \quad (4)$$

For a Newtonian fluid, equation (4) has a value equal to unity, thereby changing equation (3) into the Newtonian fluid case. For the infinitely wide bearing under consideration in the present paper, the second term in equation (3) is considered to be negligible in relation to the first. The governing equation reduces to

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$$\frac{d}{dx} \left[ \frac{h^3}{\eta} \phi(A, N, h) \frac{dP}{dx} \right] = 6U \frac{dh}{dx} \quad (5)$$

The boundary conditions are specified as the pressure at the ends of the bearing.

$$P = 0 \text{ at } x = 0, \quad P = 0 \text{ at } x = L \quad (6)$$

### Description Of Numerical Scheme

To formulate a finite element model of equation (1), we begin with the weak formulation by multiplying it by a test function  $v$ , integrating both sides and we obtain

$$\int v \frac{d}{dx} \left[ \frac{h^3}{\eta} \phi(A, N, h) \frac{dP}{dx} \right] d\Omega = \int v 6U \frac{dh}{dx} d\Omega \quad (7)$$

Simplifying equation (7) by integrating the first term by parts, we obtain

$$v \left[ \frac{h^3}{\eta} \phi(A, N, h) \frac{dP}{dx} \right] - \int \frac{h^3}{\eta} \phi(A, N, h) \frac{dP}{dx} \frac{dv}{dx} d\Omega = \int v 6U \frac{dh}{dx} d\Omega \quad (8)$$

In equation (8), the region  $\Omega$  is defined by  $0 \leq x \leq L$ . We define equation (9) as a finite sum of the product of nodal pressure and basis function over an element from which

$$P = \sum_{i=1}^n N_i(x) p_i \quad (9)$$

$$\frac{dP}{dx} = p_i \sum_{i=1}^n \frac{dN_i}{dx} \quad (10)$$

$N_i(x)$  are linearly independent basis functions and  $p_i$  are unknown nodal pressures which are yet to be determined. The weak formulation given by equation (8) is a variational statement of equation (5) and the test functions  $v(x)$  are chosen to be identical to the basis functions  $N_i(x)$ . The basis functions  $N_i(x)$  in equation (9) are shown in equation (11). The bearing domain is divided into three node quadratic elements with the pressure within each element approximated using equation (9). The nodal pressures are obtained by computing elemental stiffness equations using equation (8) to equation (11), assembling the respective elemental stiffness equations by enforcing continuity of the nodal degree of freedom, imposing the boundary conditions in equation (6) and using an iterative method to solve the resulting system of equations.

$$N_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} \quad N_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} \quad N_3(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} \quad (11)$$

### Theoretical Model

The slider bearing under consideration here is narrowing in linear style similar to that of [9]. The upper bearing surface is kept stationary as in the study of [5]. For the present analysis, the velocity of the lower surface is chosen in the range 2.55 – 10.21m/s. The bearing length used in the analysis is selected between the choice of [5] which equals 1.25mm and Ai et al [1998] which equals 14.5mm. The coupling number ( $N^2$ ) used for the simulation falls within the range  $0 \leq N^2 \leq 0.9$ , where  $N^2 = 0$  represents the Newtonian case. The micropolar length ( $A$ ) is within the range  $0 \leq A \leq 100$ , where the lubricant exhibits increasing Newtonian behaviour as  $A$  approaches 0. The lubricant has a dynamic viscosity equal 0.38304 Pa.s

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## RESULTS AND DISCUSSION

### Validation of Finite Element results

A suitable assessment of solution accuracy is crucial to establish confidence in a numerical model. Consequently, the accuracy of the solution obtained using the present method is first examined. To achieve this end, simulations were carried out using  $N^2 = 0.5$ ,  $A = 20$ ,  $h_1 = 1\text{mm}$ ,  $h_0 = 0.5\text{mm}$ ,  $L = 10\text{mm}$  and  $\mu = 0.38304\text{ Pa}\cdot\text{s}$  on eight meshes with successive refinement. A necessary condition for the reliability of finite element result is that the nodal degree of freedom which is the pressure in our case obtained by solving equation (1) with the boundary conditions become mesh independent as the size of the element approaches zero. Table 1 shows the pressure obtained at selected points as the element length is halved progressively.

**Table 1: Finite element solution at selected points using different mesh densities**

Distance	P (N/m <sup>2</sup> ) 10 elements	P (N/m <sup>2</sup> ) 20 elements	P (N/m <sup>2</sup> ) 40 elements	P (N/m <sup>2</sup> ) 80 elements	P (N/m <sup>2</sup> ) 160 elements	P (N/m <sup>2</sup> ) 320 elements	P (N/m <sup>2</sup> ) 640 elements	P (N/m <sup>2</sup> ) 1280 elements
0.0001	152.3575	152.6803	152.7616	152.7820	152.7870	152.7883	152.7886	152.7886
0.0002	301.7432	302.4169	302.5866	302.6293	302.6396	302.6423	302.6430	302.6431
0.0003	443.9243	444.9734	445.2373	445.3037	445.3198	445.3241	445.3251	445.3259
0.0004	572.6767	574.0666	574.4282	574.5194	574.5417	574.5475	574.5488	574.5491
0.0005	678.5086	680.3346	680.7937	680.9094	680.9376	680.9449	680.9467	680.9472
0.0006	747.5343	748.7038	750.2490	750.3864	750.4204	750.4289	750.4311	750.4316
0.0007	758.3366	760.7297	761.3310	761.4828	761.5204	761.5296	761.5321	761.5327
0.0008	677.8106	680.1584	680.7491	680.8984	680.9353	680.9441	680.9467	680.9473
0.0009	453.5072	455.2539	455.6924	455.8042	455.8320	455.8387	455.8402	455.8407

The solution of equation (5) under the boundary conditions stated in equation (6) for different meshes is shown in Table 1. The table shows the converging trend of the results which have been obtained through numerical experimentation. The solution obtained by the method is shown to be stable and convergent.

### Pressure Distribution

The effect of coupling number ( $N^2$ ) on pressure distribution in an inclined slider bearing of length ( $L$ ) equal 10mm, inlet film thickness ( $h_1$ ) equal 0.001mm, outlet film thickness ( $h_0$ ) equal 0.0005, Micropolar length ( $A$ ) equal 100 and slider velocity ( $U$ ) equal 5m/s is shown in Figure 2. The plot shows that pressure build up is augmented with increase in coupling number. Numerical computation shows that increasing the coupling number from 0.3 to 0.5 results in an increase in nodal pressure by a factor approximately 1.40. The effect of slider velocity on the pressure in the lubricant film is shown in Figure 3. Increase in velocity has a similar effect on the pressure distribution as increase in coupling number. A 67% increase in coupling number (0.3 to 0.5) causes an increase in nodal pressure by a factor of 1.4. A similar percentage increase in velocity of the slider from 5m/s to 8.35m/s results in increase in nodal pressure by a factor of 1.67. This suggests a linear relationship between increase in pressure and nodal pressure for an inclined slider bearing lubricated by a Micropolar fluid. The effect of film thickness ratio defined as the ratio of the inlet and outlet film thickness on the pressure distribution is shown in Figure 4. The plot shows that nodal pressure is increased as the film thickness ratio increases. This is as a result of the increased wedge effect in the bearing. In particular, an increase in the film thickness ratio by a factor equal to 2 increases the maximum pressure in the bearing by a factor equal to 3.55 with the position of the maximum pressure moving downstream towards the outlet of the bearing as film thickness ratio increases.

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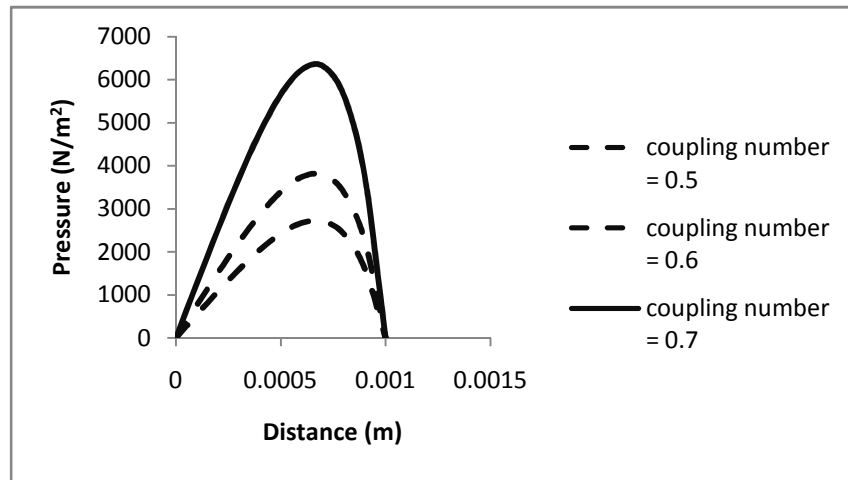


Figure 2: Pressure against distance along the bearing for different coupling numbers and  $A = 40$ ,  $h_1 = 0.005$ ,  $h_0 = 0.0005$ ,  $U = 5\text{m/s}$

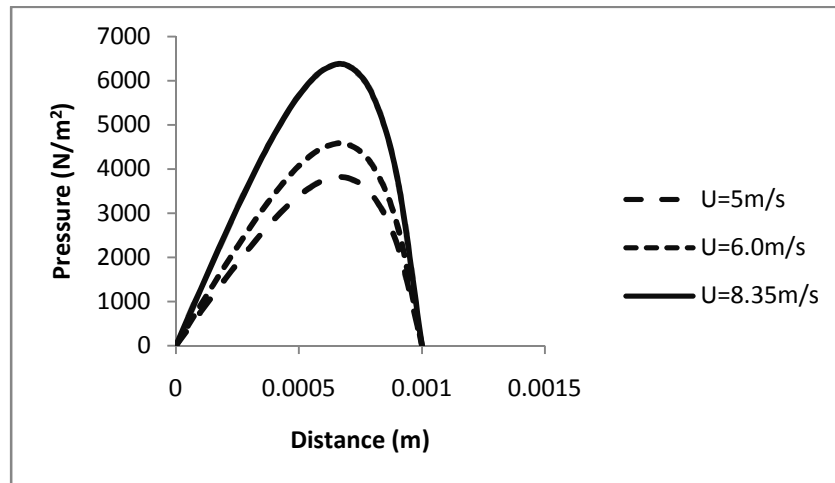


Figure 3: Pressure against distance for different slider velocities and  $N^2 = 0.5$ ,  $A = 100$ ,  $h_1 = 0.005$ ,  $h_0 = 0.0005$ .

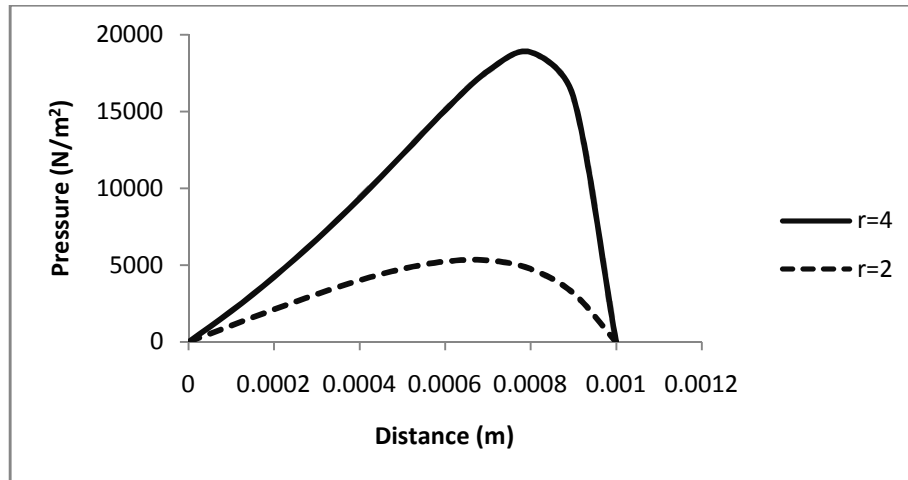


Figure 4: Pressure against distance for different film thickness ratios

### Load Capacity

The effect of coupling number on load capacity ( $W$ ) for two different film thickness ratios is shown in Figure 5. The plot illustrates that in general, the bearing load is enhanced as the coupling number increases. Figure 5 also shows that bearing load is increased as the film thickness ratio of the bearing increases. In particular, increase in the film thickness ratio by a factor equal to 2 increases the point wise pressure along the bearing by a factor of 3.04. From the standpoint of bearing performance, a higher coupling number brings about improved performance for a slider bearing lubricated with Micropolar lubricant for a given Micropolar length. Numerical computations demonstrate that there is no significant increase in the load capacity for higher Micropolar length greater than 10 for a given coupling number.

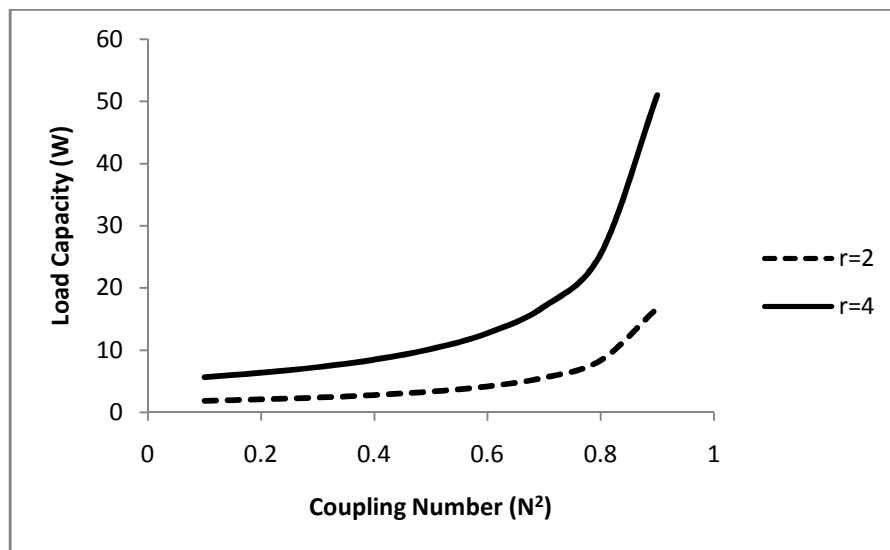


Figure 5: Plot of load capacity ( $W$ ) against coupling number ( $N^2$ ) for Micropolar length ( $A$ ) equal 10

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## Conclusion

In this paper, the effect of Micropolar lubricants on the pressure and load capacity of an inclined slider bearing is presented. Numerical computations has shown that bearing pressure and load is improved when a Micropolar lubricant is used compared to the Newtonian case. This finding is consistent with those of Wang and Zhu (2006) and Agrawal and Bhat (1980) who independently reported increase in pressure and load for journal bearings lubricated with micropolar fluids It has also been demonstrated that for a given coupling number, the load capacity increases with aspect ratio which reinforces the finding of Naduvinaman and Marali (2008) who reported a similar trend with increase in profile parameter for dynamically loaded slider bearing lubricated with micropolar lubricants. This finding is attributable to the increased wedge effect which result on the bearing. The potential of the finite element method in simulating hydrodynamic lubrication problems has also been demonstrated.

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