Journal of the Nigerian Association of Mathematical Physics
***Volume* 17 (November, 2010), pp 103 - 108**

Numerical Modeling of a Rayleigh Step and Multi Step Slider Bearings using Finite Difference Method

Oladeinde[1] M.H and Akpobi[2] J.A

[1,2]Department of Production Engineering,
Faculty of Engineering, University of Benin.

Abstract

The oil film pressure in a step slider bearing is difficult to obtain because the discontinuous clearance leads to a discontinuous velocity profile. In this paper, the pressure distribution in a step slider bearing is obtained numerically by introducing a virtual clearance to satisfy the continuity of the velocity at the step. The governing equation is discretized using central difference method for each internal node in the flow volume and the resulting system of equations solved using Gauss Seidel scheme with a convergence criterion of 0.0001. The result obtained with the present method is in good agreement with analytical solution for the Rayleigh step slider. The method is also applied to a multi step slider bearing for which no analytical solution exists. The solution converges when the mesh is refined.

Nomenclature

h	film thickness (m)
$h_{i,}, h_{i+1,}, h_{i-1}$	nodal film thickness
p	Pressure (N/m^2)
$p_{\max}$	pressure at step land interface
$p_{i,} p_{i+1,} p_{i-1}$	Nodal pressures N/m^2)
U	slider velocity
μ	Viscosity of lubricant film
Δx	grid spacing

1.0 Introduction

Reynolds' equation plays an important role in analyzing the hydrodynamic operation of slider bearings. Though the analytical solution for the pressure distribution in a Rayleigh step slider bearing is available in the literature, the increasing use of numerical schemes for solving hydrodynamic lubrication problems has been observed. Additionally, to analyze some phenomena, we need to formulate some model problem correspondingly using appropriate numerical methods. There are a number of numerical models that can be used to construct an approximate solution. These include the finite element, finite difference, perturbation and boundary element methods. Perhaps, no other method of approximation has had a greater impact on the theory of hydrodynamic lubrication than the finite difference method. However, the use of the method in simulating step slider bearings is scanty in the literature. This situation is consequent upon the discontinuity occasioned by the discontinuity in film thickness.

The computation of the pressure distribution in a slider bearing without clearance discontinuity is fairly straightforward and has been considered by a number of researchers using different numerical schemes. [3] obtained the pressure field in an infinitely wide plane pad slider bearing. [2] used a finite volume method for numerical modeling of a Rayleigh step slider bearing. [4] applied the finite control volume method to simulate hydrodynamic lubrication problems.

Corresponding author: Tel (+2348039206421)

In this paper finite difference method is used to numerically model the pressure distribution in a Rayleigh step slider bearing by introducing a virtual clearance and gradient as suggested by [1] to cater for the discontinuity in film thickness. Having established the accuracy of the model, the method is extended to solve for the pressure distribution in a multi step slider bearing

Equation Of Motion

The oil film pressure in an infinitely wide Rayleigh step slider bearing shown in Figure 1 is obtained by solving Reynolds equation.

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right)=6\mu U\frac{dh}{dx} \tag{1}$$

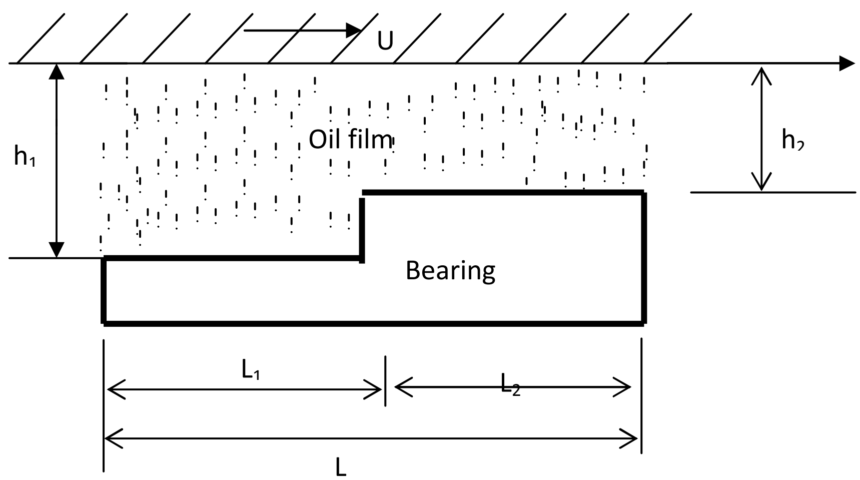

Figure 1: Infinitely wide Rayleigh Step slider bearing

The boundary conditions are a specification of the pressures at the inlet and outlet of the step bearing as shown in equation (2). The oil film pressure is obtained by solving equations (1) and (2). In equation (2), L is the length of the bearing

$$P=0 \quad \text{at} \quad x=0,L \tag{2}$$

DISCRETIZATION OF GOVERNING EQUATION

Differentiating the left hand side of equation (1) leads to

$$3h^2\frac{dh}{dx}\frac{dp}{dx}+h^3\frac{d^2p}{dx^2}=6\mu U\frac{dh}{dx} \tag{3}$$

Considering the flow of lubricant through three consecutive sections namely, $i-1$, i and $i+1$, respectively, equation (3) can be rewritten for the cross section through i as

$$3h_i^2\frac{dh_i}{dx}\left(\frac{p_{i+1}-p_{i-1}}{2\Delta x}\right)+h_i^3\left(\frac{p_{i-1}-2p_i+p_{i+1}}{\Delta x^2}\right)=6\mu U\frac{dh_i}{dx} \tag{4}$$

Equation (4) can be simplified further without much mathematical rigor to obtain

$$p_i=\left(\frac{1}{2}+\frac{3\Delta x}{4h_i}\frac{dh_i}{dx}\right)p_{i+1}+\left(\frac{1}{2}-\frac{3\Delta x}{4h_i}\frac{dh_i}{dx}\right)p_{i-1}-\frac{3\mu U\Delta x^2}{h_i^3}\frac{dh_i}{dx} \tag{5}$$

Equation (5) is the pressure relation between three consecutive nodes in the flow field.

For a slider bearing without step(s) equation (5) can be written for all internal nodes in the flow field and the resulting system of equations solved using Gauss Seidel method. However, the discontinuity in the clearance

Corresponding author: Tel (+2348039206421)

occasioned by the step in the bearing complicates the definition of the clearance h_i and its gradient at the step. To overcome this challenge, a virtual clearance and gradient at the step, as suggested by [1], are used. For a grid point i at the step, the clearances at neighboring nodes h_{i-1} and h_{i+1} are used to define virtual clearance by

$$h_i = \sqrt{\frac{h_{i+1}^3 + h_{i+1}h_{i-1} + h_{i-1}^3}{3}} \tag{6}$$

The gradient at the step is given by

$$\frac{dh_i}{dx} = \frac{2h_i}{3\Delta x}\left(\frac{h_{i+1}^3 - h_{i-1}^3}{h_{i+1}^3 + h_{i-1}^3}\right) \tag{7}$$

In solving equations (1) and (2) over the entire bearing domain including the step and land, we use equation (5) and note that when the internal node for which equation (5) is to be written is not at a step, equation (5) reduces to

$$p_i = \frac{1}{2}p_{i+1} + \frac{1}{2}p_{i-1} \tag{8}$$

If the node under consideration is at a step, then equations (6) and (7) are substituted into equation (5).

ANALYTICAL SOLUTION

Reynolds equation governing the hydrodynamic pressure given by equation (1) can be written as

$$\frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx} - \frac{Uh}{2}\right) = 0 \tag{9}$$

The integration of equation (9) over each flow region in Figure 1 gives

$$\frac{h_1^3}{12\mu}\frac{dp_1}{dx_1} - \frac{Uh_1}{2} = -q_x \qquad \frac{h_2^3}{12\mu}\frac{dp_2}{dx_2} - \frac{Uh_2}{2} = -q_x \tag{10}$$

In equation (10), q_x is known as the volumetric flow rate which is constant and equal in the two flow domains. For the step and land regions, the boundary conditions are as given by

$$x_1 = 0, \quad p_1(0) = 0 \quad \text{and} \quad x_1 = L_1, \quad p_1(0) = p_{\max} \tag{11a}$$

$$x_2 = 0, \quad p_1(0) = p_{\max} \quad \text{and} \quad x_2 = L_2, \quad p_2(0) = 0 \tag{11b}$$

The step pressure can be determined by equating the flow rates in the step and land regions given in equation (10), noting that the pressure gradients in the step and land regions are given by equations (12a) and (12b). The pressure at the step shown in equation (13) is obtained.

$$\frac{dp_1}{dx_1} = \frac{p_{\max}}{L_1} \tag{12a}$$

$$\frac{dp_2}{dx_2} = -\frac{p_{\max}}{L_2} \tag{12b}$$

`

$$p_{\max} = \frac{6\mu UL_2}{h_2^2}P_{step}(\alpha,\beta) \tag{13}$$

In equation (13) $P_{step}(\alpha,\beta)$ is defined by equation (14), where α is defined as the film thickness step to land ratio and β is defined as the step to land length ratio.

$$P_{step} = \frac{\alpha - 1}{1 + \alpha^3\beta} \tag{14}$$

NUMERICAL EXAMPLE 1

Consider a Rayleigh step bearing with the following data. Compute its pressure distribution and load capacity

Corresponding author: Tel (+2348039206421)

$$h_1 = 50\mu m, \quad h_2 = 25\mu m, \quad L_1 = 50mm, \quad L_2 = 50mm \quad U = 10m/s, \quad \mu = 0.1Ns/m^2,$$

The flow domain is divided into a mesh of 9 nodes with node spacing Δx being equal to $12.5mm$. Writing equation (5) for all nodes in the flow domain, noting that equation (5) for a node not at a step can be written as equation (8) the system of equations shown below is obtained.

$$p_1 = \frac{1}{2}p_0 + \frac{1}{2}p_2, \quad p_2 = \frac{1}{2}p_1 + \frac{1}{2}p_3, \quad p_3 = \frac{1}{2}p_4 + \frac{1}{2}p_2, \quad P_4 = \frac{1}{9}P_5 + \frac{8}{9}P_3 - 1.333x10^7$$

$$p_5 = \frac{1}{2}p_6 + \frac{1}{2}p_4, \quad p_6 = \frac{1}{2}p_7 + \frac{1}{2}p_5, \quad p_7 = \frac{1}{2}p_8 + \frac{1}{2}p_6$$

Imposing the boundary conditions $p_0 = p_8 = 0$ on the system of equations above and representing the resulting condensed system in matrix form, equation (13) is obtained.

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -8 & 9 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 11.997x10^7 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (15)$$

Equation (15) exhibits diagonal dominance and was solved using Gauss Seidel method to obtain the solution below.

$$P^T = \left(1.333x10^7 \quad 2.666x10^7 \quad 3.999x10^7 \quad 5.332x10^7 \quad 3.999x10^7 \quad 2.666x10^7 \quad 1.333x10^7\right)$$

The solution obtained using analytical method is compared with the numerical solution in
Table 1

Table 1: Comparison of results using analytical and present method

Node Position	0.0125	0.0250	0.0375	0.0500	0.0625	0.0750	0.0875
Analytical Method	$1.333x10^7$	$2.666x10^7$	$4.000x10^7$	$5.333x10^7$	$4.000x10^7$	$2.666x10^7$	$1.333x10^7$
Present Method	$1.333x10^7$	$2.666x10^7$	$3.999x10^7$	$5.332x10^7$	$3.999x10^7$	$2.666x10^7$	$1.333x10^7$

Corresponding author: Tel (+2348039206421)

NUMERICAL EXAMPLE 2
Compute the pressure distribution in the multi- step slider bearing shown in Figure 2

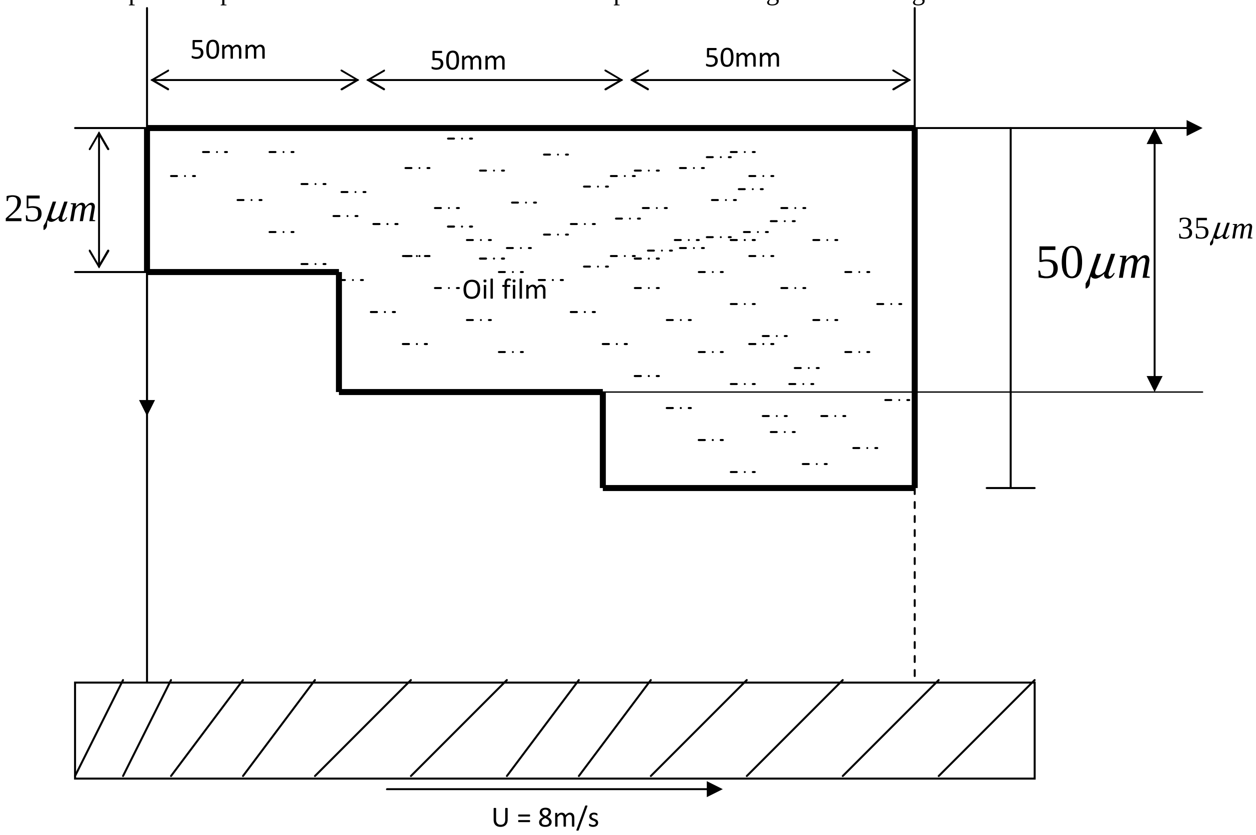

Fig. 2: infinitely wide Rayleigh multi step slider bearing

The flow domain of the multi step slider bearing is divided into a mesh of 7 nodes with $\Delta x = 0.025m$. Writing equation (5) for all internal nodes at a step of the multi step slider and equation (8) for all internal nodes not located at a step, and subsequently imposing boundary conditions (equation (2)) namely specification of the pressures at the inlet and outlet of the multi step slider bearing, the following system of equations cast in matrix format results

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -0.745 & 1 & -0.255 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -0.733 & 1 & -0.267 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 16083395.383 \\ 0 \\ 30769230.76 \\ 0 \end{pmatrix}$$

Solving the forgoing equation, the following solution is obtained

$$p_1 = 2.94305\text{x}10^7 \quad p_2 = 5.88611\text{x}10^7 \quad p_3 = 8.17724\text{x}10^7 \quad p_4 = 1.04684\text{x}10^8$$
$$p_5 = 5.23418\text{x}10^7$$

In order to verify the convergence and reliability of the forgoing numerical solution for the multi step slider in the absence of analytical solution, the mesh is refined further into 13 nodes and the finite difference equation written for each node in the flow domain once again. The following system of equations is obtained.

Corresponding author: Tel (+2348039206421)

$$
\begin{pmatrix}
2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.745 & 1 & -0.255 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.733 & 1 & -0.267 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2
\end{pmatrix}
\begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \\ p_{10} \\ p_{11} \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 8.042\text{x}10^6 \\ 0 \\ 0 \\ 0 \\ 1.538\text{x}10^7 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

Solving the system of equation above using Gauss Seidel method the following solution is obtained.

$p_1 = 1.47142\text{x}10^7 \quad p_2 = 2.94284\text{x}10^7 \quad p_3 = 4.41426\text{x}10^7 \quad p_4 = 5.88568\text{x}10^7$
$p_5 = 7.03081\text{x}10^7 \quad p_6 = 8.17594\text{x}10^7 \quad p_7 = 9.32107\text{x}10^7 \quad p_8 = 1.04662\text{x}10^8$
$p_9 = 7.84965\text{x}10^7 \quad p_{10} = 5.23310\text{x}10^7 \quad p_{11} = 2.61655\text{x}10^7$

Table 2 Solution obtained using present method for multi step slider using two different meshes.

Node position	0.025	0.050	0.075	0.100	0.125
7 node Solution	2.94284x10	$5.88568\text{x}10^7$	$8.17594\text{x}10^7$	$1.04662\text{x}10^8$	$5.23310\text{x}10^7$
13 node Solution	2.94305x10	$5.88611\text{x}10^7$	$8.17724\text{x}10^7$	$1.04684\text{x}10^8$	$5.23418\text{x}10^7$

DISCUSSION OF RESULTS

The result obtained using the present method for the Rayleigh step bearing is shown in Table 1 with the analytical solution. Point wise comparison of the two independent solutions shows a close agreement. Refinement of the mesh only increases the computational cost and time but not the accuracy of the result. The multi step slider bearing in numerical example 2 has no analytical solution. Hence, the convergence characteristic of the solution method was investigated by mesh refinement. Table 2 shows the nodal pressure at selected points using a mesh of 7 and 13 nodes respectively. The result shows clear convergence and improvement in the solution as the number of nodes is increased. It has been shown that the present method produces stable and convergent results and can be used to further predict the performance characteristics of the step slider bearing.

Conclusion

A finite difference method which successfully handles the discontinuity in clearance at the step of Ryaleigh step and multi step bearing has been developed and applied to solve Reynolds equation. The method has been used to solve for the pressure distribution in the bearing configurations. The method has been demonstrated to produce admirable close solution to the analytical solution where they exist and it is also convergent with mesh refinement.

Corresponding author: Tel (+2348039206421)

References

[1] Ogata H., (2005), Thermodynamic Lubrication Analysis Method of Step bearings, IHI Engineering Review, vol 38, no 1, pp 6 – 10

[2] Hong Y., Chen .C. Wang J and Li. X (2003), Numerical modeling of a Newtonian and a Birgham fluid in a Rayleigh step bearing, Transactions of the ASME, vol 125, pp 206 -210

[3] Oladeinde M.H and Akpobi J.A (2008), Finite element simulation of performance characteristics of infinitely wide plane pad slider bearing, Advances in Materials and System Technologies, pp 637 – 642

[4] Oladeinde M.N and Akpobi J.A (2010), Control volume method applied to Hydrodynamic lubrication Problem, to appear in the International Journal of Engineering Research, vol 4, June 2010.

Corresponding author: Tel (+2348039206421)