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Collisional Effect On Magnetosonic Solitons In A Dusty Plasma Slab

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Abstract

An analytical investigation of collisional effect on magnetosonic solitons in a dusty plasma slab is presented. We have derived and presented solutions of nonlinear magetohydrodynamic equations for a warm dusty magnetoplasma. It is observed that, our work could be considered a general case for magnetosonic solutions in a dusty plasma slab. While the neglection of the collisional effect reduces our results to exactly what is obtained by Marklund et al, 2007.

Keywords: Magnetosonic Solitons, dusty plasma.

1.0 Introduction

Fast and slow magnetosonic modes are the fundamental modes of magnetized plasma from the magnetohydrodynamic (MHD) view. The fast magnetosonic waves propagate perpendicular to the ambient magnetic field. Earth's bow shock wave is one of the examples of fast magnetosonic waves that is formed due to the interaction between the solar wind and the obstacle of the Earth's magnetic field.

Various number of literature abound on nonlinear magnetohydrodhynamic waves, magnetosonic solitary waves as well as magnetosonic shock waves in different kind of plasma [1, 2, 3, 4, 5, 6, 7, 8, 9]. Dusty plasma physics studies the properties of heavier charged dust in the presence of traditional electrons and ions. These mixtures occur in heliospheric and astrophysical plasmas, and in laboratory and technological applications [10, 11, 12, 13, 14].

In this report, we have extended the work of [2] by studying the effect of collision on fast magnetosonic solutions (FMS) in a dusty plasma. In section II, we have described the model and the basic equations governing the model. Section III contains the analysis of collisional effect on fast magetosonic solitons in a dusty plasma. While discussion and conclusion, are presented in section IV.

2.0 Basic Equations

The dusty plasma we are studying consists of three components, i.e, extremely massive dust particles that are considered to be immobile, inertialess electron and ion. The nonlinear dynamics of FMS waves in a dusty magnetoplasma is described by the following set of equations:

$$0 = -n_e e\hat{E} - \nabla\hat{P}_e - n_e \frac{e\hat{V}_e}{c} \times \hat{B} - \nu_{en} n_e m_e \hat{V}_e \tag{2.1}$$

$$\frac{\partial n_i}{\partial t} + \hat{\nabla}.\left(n_i \hat{V}_i\right) = 0 \tag{2.2}$$

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$$m_i n_i\left(\frac{\partial}{\partial t}+\hat{V}_i.\hat{\nabla}\right)\ \hat{V}_i = n_i e\hat{E}-\nabla\hat{P}_i + n_i\frac{e\hat{V}_i}{c}\times\hat{B}-v_{in}n_i m_i\hat{V}_i$$

(2.3)

where n_e, n_i are the electron and ion number density respectively, e is the electronic charge, $\hat{E}$ is wave electric field, $\hat{B}$ is the sum of the ambient and wave magnetic fields, $P_e = n_e T_e$ and $P_i = n_i T_i$ are the electron and ion pressure respectively, $T_e(T_i)$ is the electron (ion) electron (ion) temperature, $V_e(V_i)$ is the electron (ion) fluid velocity, m_i is the ion mass, and C is the speed of light in vacuum.

The ampere's law and Faradeny's law used to supplement equations (1-3) are defined respectively as:

$$\nabla\times\hat{B}=\frac{4\pi e}{c}\left(n_i\hat{V}_i - n_e\hat{V}_e\right)$$

(2.4)

$$\frac{\alpha\hat{B}}{\partial t}=-C\nabla\times\hat{E},$$

(2.5)

While quasi-neutrality condition is given as

$$n_i = n_e + z_d n_d$$

(2.6)

Where n_d is the number density of the dust and z_d the number of electrons on each dust grain.

3.0 Derivation Of Nonlinear Magnetohydrodynamic Equations With Collisional Effect

Eliminating $\hat{E}$ from equations (2.1) and (2.3) and making use of Ampere's law, we obtain

$$\left(\frac{\partial}{\partial t}+\hat{V}_i.\hat{\nabla}\right)\hat{V}_i = \frac{-1}{m_i}\left[\frac{T_e}{n_i - z_d n_d}+\frac{T_i}{n_i}\right]\nabla n_i$$
$$-\frac{e z_d n_d}{m_i c\left(n_i - z_d n_d\right)}\hat{V}_i\times\hat{B}-v_{in}v_i+\frac{\hat{\nabla}\times\hat{B}\times\hat{B}}{4\pi m_i(n_i - z_d n_d)}$$

(3.1)

With the use of inertialess electron momentum equation; Ampere's law equation and Faraday's law equation, we obtain magnetic induction as:

$$\frac{\partial\hat{B}}{\partial t}=\hat{\nabla}\times\left[\frac{n_i}{(n_i - z_d n_d)}\hat{V}_i\times\hat{B}-\frac{C}{4\pi e\left(n_i - z_d n_d\right)}\left(\nabla\times\hat{B}\right)\times\hat{B}\right]$$

(3.2)

Using the same argument as [2] for one dimensional FMS wave; we have $\hat{\nabla}=\hat{X}\frac{\partial}{\partial x}, \hat{V}_i = u(x)\hat{X}+\omega(x)\hat{Y}$ and $\hat{B}=B(x)z,$ where $\hat{X}\left(\hat{Y},\hat{Z}\right)$ are the unit vector along the x(y,z) axis in Cartesian coordinates. The MHD equations reduces to the following expressions with the index i dropped:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=0$$

(3.3)

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$$\left(\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}\right)u=\frac{-1}{m}\left(\frac{T_e}{(n-z_d n_d)}+\frac{T}{n}\right)\frac{\partial n}{\partial x}$$

$$-\frac{1}{4\pi m(n-z_d n_d)}B\frac{\partial B}{\partial x}-\frac{e z_d n_d}{m(n-z_d n_d)}\frac{\omega}{c}B-v_{in}u$$

(3.4)

$$\left(\frac{\partial n}{\partial t}+u\frac{\partial}{\partial x}\right)\omega=\frac{e z_d n_d}{m(n-z_d n_d)}\frac{u}{c}B-v_{in}\omega$$

(3.5)

also

$$\frac{\partial B}{\partial t}=-\frac{\partial}{\partial x}\left(\frac{n}{n-z_d n_d}UB\right)$$

(3.6)

From stationary solutions as $\frac{\partial}{\partial t}=0,$ the integration of continuity equation gives the velocity component u according to the following expression

$$u(x)=\frac{u_0 n_0}{n(x)},$$

(3.7)

where u_0 is some constant of integration. The velocity component ω from equation (3.5) can be written as

$$\frac{d\omega}{dx}=\frac{e}{m}\frac{z_d n_d B}{c(n-z_d n_d)}-v_{in}$$

(3.8)

above equation can be written in energy form as:

$$\frac{d\omega}{dx}\left(\frac{m\omega^2}{2}\right)=\frac{e}{c}\frac{z_d n_d B}{(n-z_d n_d)}\omega-mv_{in}\frac{\omega^2}{u}$$

(3.9)

The momentum equation along x-direction according to equation (3.4) is given by the following expression

$$\frac{d}{dx}\left(\frac{mu^2}{2}\right)=-T_e\frac{d}{dx}\ln(n-z_d n_d)-T\frac{d}{dx}(\ln n)$$

$$-\frac{1}{(n-z_d n_d)}\frac{d}{dx}\left(\frac{B^2}{8\pi}\right)-\frac{e}{c}\frac{z_d n_d B\omega}{(n-z_d n_d)}-v_{in}u$$

(3.10)

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Adding equations (3.9) and (3.10) above gives

$$\frac{d}{dx}\left[\frac{m(u^2+\omega^2)}{2}+T_e\ln n_e+T\ln n\right]+n_e^{-1}\frac{d}{dx}\left(\frac{B^2}{8\pi}\right)+\left(u+\frac{m\omega^2}{u}\right)v_{in}=0. \tag{3.11}$$

We are also going to consider the fact that, magnetic field is frozen in [2] as

$$\frac{B}{B_0}=\frac{n_e}{n_0}, \tag{3.12}$$

Substituting the values of equation (3.7) and (3.12) into equation (3.8), then integrating reduces it to:

$$\omega(x)=\int_{-\infty}^{x} n_d(x')dx'-v_{in}\omega\int_{-\infty}^{x} n(x')\,dx' \tag{3.13}$$

Making use of the following normalized variables:

$$n_d\to\frac{z_d n_d}{n_0},n\to\frac{n}{n_0},u\to\frac{u}{u_0},\omega\to\frac{\omega}{u_0},V_T\equiv\left(\frac{T}{m}\right)^{\frac{1}{2}}\to\frac{V_T}{u_0},$$

$V_A\equiv\left(\frac{B_0^2}{4\pi mn_0}\right)^{\frac{1}{2}}\to\frac{V_A}{u_0},T_e\to\frac{T_e}{T}$ and $x\to\frac{x\omega_c}{u_0}$, where $\omega_c=\frac{eB_0}{mc}$ is the ion cyclotron frequency in equation (3.11) and integrating gives

$$\frac{1}{2n^2}+\frac{\omega^2}{2}+V_T^2\left[T_e\ln(n-n_d)+\ln n\right]+V_A^2 n+v_{in}\left(\frac{u}{m}+\frac{\omega^2}{u}\right)x=W_{0z}, \tag{3.14}$$

where W_{0z} is a constant.

4.0 Discussion and Conclusion

Our investigation of solutions for nonlinear magnetohydrodynamic equations for a warm dusty magnetoplasma reduces exactly to what had been obtained by [2], when the collissional effect is neglected in our analytical results with respect to our equations (3.8 – 3.11) and (3.13 – 3.14). As can be observed in our equations (3.8), (3.9), (3.10) and (3.13), the collisional effect does not really add much to the result hence its neglection by [2]. While for equations (3.11) and (3.14), the collisional effect has some positive contribution to the results.

In summary, we could consider our results to represent a general case, which reduces to the special case of [2] when the collisional effect is neglected. The result presented in this report should be useful in understanding the collisional effect on magnetosonic solitons in a dusty plasma.

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