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A Comparative Study on Lognormal and Gamma Distributions for Life and Reliability of Kamag Machine

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Abstract

A comparative study on Lognormal and Gamma distributions for failure times, t, of kamag machine was undertaken to identify an appropriate distribution for the failure times, t, of the machine. The Gamma distribution of the random variable, failure times, t, has a smaller variance, $\hat{\sigma} = 24.335$ ***than the variance,*** $\hat{\sigma} = 86.490$ ***of the Lognormal. Hence, the Gamma distribution seems to be an appropriate distribution for the failure time, t, of kamag machine based on the data collected from a steel company.***

Keywords: kamag machine, reliability, failure, time, gamma, lognormal, distribution, variance, likelihood, parameter curve.

1.0 Introduction

According to [1] and [2], reliability is the probability that a device or system or machine will operate for a given period of time and under given conditions. It optimizes the life span distribution of systems. Life distribution is a function that assigns probabilities to failure times of a machine. In other words, it describes the length of life of a machine.

Gravrilov and Gavrilov [6] described the concept of failure as a very important phenomenon in the analysis of a machine's reliability. In other words, failure occurs when a machine deviates from the optimistically anticipated behaviour.

Colt and Dey [3] adopted maximum likelihood principles in the estimation of parameters of Gamma distribution. Barlov and Proschan [1] presented an ingenuous statistical model for life lengths of machines under dynamic loading.

Crowder et al [4], referred to the reliability of a machine as its ability to operate properly according to a specified standard.

Thomopoulos and Aroid [9] stated that a lognormally distributed random variable is the random variable where its logarithm is normally distributed. Thomopoulos and Long [10], discussed the bivariate lognormal distribution of structured reliability of machines. According to [5], a lognormal failure rate distribution should be expected for any machine.

In this work, we shall comparatively understudy the Lognormal and Gamma distributions for life and reliability of the Kamag Machine.

2.0 Lognormal Distribution

Consider a two parameter Lognormal distribution

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$$f_L = \begin{cases} \frac{1}{t\sigma\sqrt{2\pi}} \exp^{-\left[\frac{1}{2}\left(\frac{\log t-\mu}{\sigma}\right)^2\right]}, & t \geq 0 \\ 0, & elsewhere \end{cases} \tag{2.1}$$

where μ and σ are the shape and scale parameters of the distribution.

The probability that a transition to a state of failure will occur in a machine before or at the moment of operating time T [7] is

$F_L(t) \quad = P(T \leq t)$

$$= \int_0^t \frac{1}{\sigma x\sqrt{2\pi}} \exp^{-\left[\frac{1}{2}\left(\frac{\log t-\mu}{\sigma}\right)^2\right]} dx$$

$$= \Phi\left(\frac{\log t - \mu}{\sigma}\right) \tag{2.2}$$

is the standard lognormal distribution function.

Let $\underline{t} = (t_1, t_2, \cdots .t_N)$ be the failure times of a machine. The likelihood function for $\underline{t}$ is

$$L(\underline{t}; \mu, \sigma) = \prod_{i=1}^{N} f_L(\underline{t}; \mu, \sigma) \tag{2.3}$$

$$= \frac{\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^N \exp^{-\frac{1}{2}\left[\sum\left(\frac{\log t_i-\mu}{\sigma}\right)\right]}}{\prod_{i=1}^{N} t_i} \tag{2.4}$$

By the principle of maximum likelihood estimation, the parameters μ and σ are estimated from (2.4) as

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} \log t_i \tag{2.5}$$

and

$$\hat{\alpha} = \left[\frac{\sum_{i=1}^{N} \log t_i - N}{N}\right]^{\frac{1}{2}} \tag{2.6}$$

We obtain the k[th] moment of the lognormally distributed random variable, $\underline{t}$, about zero as

$$E(\underline{t}^k) = \int_{-\infty}^{\infty} \underline{t}^k f_L(\underline{t}) d\underline{t} \tag{2.7}$$

$$= e^{\mu k + \frac{k^2\sigma^2}{2}} \tag{2.8}$$

Hence, the variance of $\underline{t}$ is

$$Var_L(\underline{t}) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) \tag{2.9}$$

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3.0 Gamma Distribution

Consider a two parameter gamma distribution

$$f_g(t) = \begin{cases} \dfrac{t^{\alpha-1} e^{-\left(\frac{t}{\beta}\right)}}{\beta^{\alpha}\,\Gamma(\alpha)} \ , \ t \geq 0 \\ \\ 0 \ , \ elsewhere \end{cases} \tag{3.1}$$

where α and β are the shape and scale parameters of the distribution.

The probability that a transition to a state of failure will occur in a machine before or at the moment of operating time T [8] is

$$F_g(t) = P(T \leq t) \tag{3.2}$$

$$= \int_0^t \frac{x^{\alpha-1} e^{-x}}{\Gamma(\alpha)} dx \tag{3.3}$$

$$= \frac{\Gamma_t(\alpha)}{\Gamma(\alpha)} \tag{3.4}$$

where

$$\Gamma_t(\alpha) = \int_0^t x^{\alpha-1} e^{-x} dx \tag{3.5}$$

Let $\underline{t} = (t_1, t_2, \ldots t_N)$ be the failure times of a machine. The likelihood function of $\underline{t}$ is

$$L(\underline{t}; \alpha, \beta) = \prod_{i=1}^{N} f_g(\underline{t}; \alpha, \beta) \tag{3.6}$$

$$= \beta^{-\alpha N} \left[\Gamma(\alpha)\right]^{-N} \left[\prod_{i=1}^{N} t_i^{\alpha-1} e^{-\frac{1}{\beta}\sum_{i=1}^{N} t_i} \right] \tag{3.7}$$

By principles of maximum likelihood estimation, we estimate the parameters α and β for the observation $\underline{t} = (t_1, t_2, \ldots t_N)$ as

$$\hat{\beta} = \frac{1}{\hat{\alpha}} \sum_{i=1}^{N} t_i \tag{3.8}$$

and

$$\hat{\alpha} = \frac{3 - s + \sqrt{(s-3)^2 + 24s}}{12s} \tag{3.9}$$

where

$$s = \log\left[\frac{1}{N}\sum_{i=1}^{N} t_i\right] - \frac{1}{N}\sum_{i=1}^{N} \log t_i$$

(3.10)

We obtain the k^{th} moment of gamma distribution random variable, $\underline{t}$, about the origin as

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$$E(\underline{t}^k) = \int_{-\infty}^{\infty} \underline{t}^k f_g(\underline{t}) d\underline{t} \quad (3.11)$$

$$= \frac{\beta^k \Gamma(\alpha + k)}{\Gamma(\alpha)} \quad (3.12)$$

Hence, the variance of t is

$$Var_g(\underline{t}) = \alpha\beta^2 \quad (3.13)$$

4.0 Data Collection

Data on the failure times of kamag machine were collected from a steel company at Warri for the period of 3 years (2006 – 2008). Kamag Machine is used to carry scrap of about 20 tons weight from scrap dump (bucket) to furnace (melting shop) for melting.

Averagely, the data is presented in Table 1:

Table 1: Failure times of Kamag Machine

Months	**Failure times in minutes**	**Failure times in days**
January	5526	3.8375
February	9680	6.6916
March	6589	4.5757
April	1230	0.8542
May	4089	2.8396
June	12634	8.7736
July	5544	3.8500
August	10780	7.4861
September	21830	15.1597
October	9202	6.3903
November	10064	6.9889
December	178	0.1236

5.0 Analysis of Data and Results

5.1 Analysis of Data

Consequently, the parameters of Lognormal and Gamma distributions for failure times, t, in days of kamag machine are obtained from Table 2.

Table 2: Determination of the parameters: $\hat{\mu}$, $\hat{\sigma}, \hat{\alpha}$, $\hat{\beta}$:

N	**t**	**Logt**	**logt** – $\hat{\mu}$	**(logt** – $\mu)^2$
1	3.8575	1.3448	0.0478	0.0019
2	6.6916	1.9009	0.5999	0.3599
3	4.5757	1.5208	0.2198	0.0483
4	0.8542	– 0.15576	– 1.4586	2.1275
5	2.8396	1.0437	– 0.2573	0.0662
6	8.7736	2.1717	0.8707	0.7581
7	3.8500	1.3481	0.0471	0.0022
8	7.4861	2.0130	– 0.7120	0.5069
9	15.1597	2.7186	1.4176	2.0096
10	6.3903	1.8548	0.5538	0.3070

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11	6.9889	1.9442	0.6432	0.4137
12	0.1236	– 2.0907	– 3.3917	11.5036

$\hat{\mu}$ = 1.3010, $\hat{\sigma}$ = 1.2283, $\hat{\alpha}$ = 1.3029, $\hat{\beta}$ = 4.3218

The lognormal and gamma cumulative distributions of $\underline{t}$ at respective times t_i (days) are shown in Table 3:

Table 3: Lognormal[$F_L(\underline{t})$] and Gamma[$F_g(\underline{t})$] cumulative distributions

$\underline{t}$	$F_L(\underline{t})$	$F_g(\underline{t})$
3.8375	0.5160	0.9820
6.6916	0.6879	0.9990
4.5757	0.5714	0.9930
0.8542	0.1170	0.6320
2.8396	0.4168	0.9500
8.7736	0.7611	–
3.8500	0.5160	0.9820
7.4861	0.7190	0.9990
15.1597	0.8749	–
6.3903	0.6736	0.9980
6.9889	0.6985	0.9990
0.1236	0.0026	–

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Graph of $F_L(\underline{t})$ and $F_g(\underline{t})$ against time $\underline{t}$ is shown in Fig. 1.

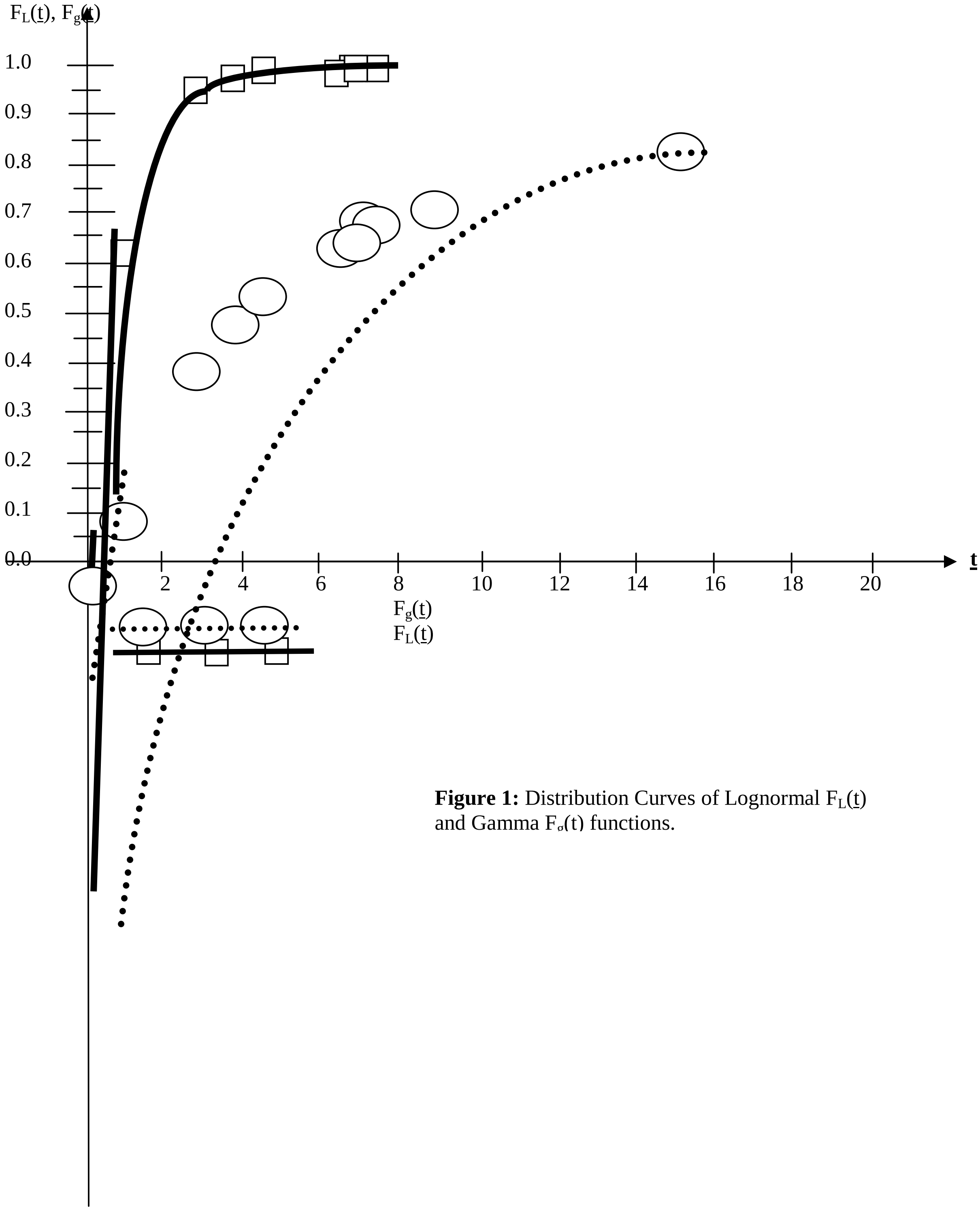

Figure 1: Distribution Curves of Lognormal $F_L(\underline{t})$ and Gamma $F_g(t)$ functions.

The variance of failure times in days of Kamag Machine for Lognormal and Gamma distributions, are $Var_L(\underline{t}) = 86.490$, $Var_g(\underline{t}) = 24.335$ respectively.

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5.2 Results

(i) The variance of gamma distributed random variable t is smaller than that of lognormal distribution. Statistically, the distribution with smaller variance is more preferred.

(ii) The $F_g(\underline{t})$ curve move up parabollically, indicating that the kamag machine is failing with respect to time, and there is need to take decision at this point in replacing failed parts to improve efficiency.

But the $F_L(\underline{t})$ curve at a point takes a straight line shape showing that the kamag machine failing with respect to time at that point is constant. Hence, it puts the engineers at alert to always do random checks on the machine to forestall any breakdown.

6.0 Conclusion

Gamma distribution for life and reliability of Kamag machine has minimal variance when compared to Lognormal distribution.

Evidently, gamma distribution is a better distribution than Lognormal, distribution, for life and reliability of kamag machine.

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