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On The Structure of The Inverse of a Linear Constant Multivariable System

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Abstract

The paper establishes criteria for invertibility of a large class of linear constant multivariable systems with feedback loops which can be characterized using an interconnection of scalar blocks where the input-output relation for each block is described by a strictly proper rational transfer function or by a linear constant multivariable system described by an interconnection of scalar blocks. It is shown that the use of this representation has certain advantages in the design of multivariable feedback systems. typical examples were considered to indicate the corresponding application.

Keywords: Stability Functions, multivariable systems, scalar blocks, interconnected electric power systems.

1.0 Introduction

The invertibility of dynamical systems is a decoupling problem [1, 2] and is often addressed in conjunction with synthesis of state or output feedback law [3, 4]. The general problem was considered by Silverman [5] on the existence and construction of inverse systems by methods similar to those of Obinabo and Ikpeazu [6].
Inversion of dynamic systems [7, 8] arises naturally in the time solution of deterministic systems, for example, in the whitening filter [9], which is the inverse of a dynamic model of a random process, or in the determination of pursuer and evader strategies [10] in a stochastic game. In these applications, the fundamental requirement is to make the input functions control the outputs independently, in other words, a single input is made to affect only a single output. This is a decoupling problem [1, 2] generally addressed in conjunction with synthesis of state or output feedback law [3, 4]. For systems with square transfer function matrices the necessary and sufficient condition for existence of state feedback laws for decoupling are equivalent to the conditions for system inversion. A process of finding a state feedback law for this purpose is given by considering the system described by the relations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1.1}$$

where $\underline{u}$ is the m-dimensional input vector, y is the r-dimensional output vector and $\underline{x}$ is the n-dimensional state vector. The matrices A, B, C and D have dimensions compatible with $\underline{x}$, y and $\underline{u}$. By taking Laplace transforms of (1.1) it can shown that

$$y(s) = \left(C(sI-A)^{-1}\ B + D\right)\underline{u}\ (s) = Q(s)\underline{u}(s) \tag{1.2}$$

where

$$Q(s) = \left(C(sI-A)^{-1}\ B + D\right) \tag{1.3}$$

$Q(s)$ is a rational transfer function matrix of dimension ($r \times m$). various methods are available for computing $Q(s)$ and they involve the computation of the characteristic matrix $(sI - A)^{-1}$ using an algorithm and then using

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equation (1.3) to compute $Q(s)$ by first determining $\det\,(sI - A)$, which gives the common denominator of each entry of $Q(s)$, while the numerator of each entry is obtained in terms of the Markov parameters of the system.

Equation (1.3) may be rewritten in the form

$$Q(s) = C(sI - A)^{-1}\ B + D$$

$$= D + C\ (s) = C(sI + s^{-2}\ A + s^{\ -3} A^2 A^2 + \cdots)B$$

$$= D + CBs_{-1} + CABs^{-2} + CA^2\, Bs^{-3}\ + \cdots \quad (1.4)$$

$$= D + y_o s^{\ -1} + s\ y_1 s^{-2} + \cdots \quad (1.5)$$

where

$$y_1 = CA^i B, +\ y_1 s^{-2}\ \ + \cdots \quad (1.6)$$

are the so-called parameters of the system.
The purpose of this paper was to determine the transfer function of a system from its Markov parameters using an algorithm for determining whether a system is invertible. The algorithm incorporates a very simple criterion for determining the invertibility. The method was applied to continuous-time systems described in state-space form, by a rational matrix transfer function or by a set of linear constant differential equation.

2.0 The Invertibility Problem

To obtain the dynamic response giving the change in frequency as a function of time for a step change in load, we must obtain the Laplace inverse of (7). Here, the necessary and sufficient conditions for existence of the inverse [5, 11, 12] are shown to be equivalent to the conditions of state feedback laws. A procedure of finding a state feedback law is established here for this purpose. First, we consider the general operational representation

$$y(t) = \frac{b_o p^m + b_1 p^{m-1} + \ldots + b_{m-1} p + b_m}{p^n + a_1 p^{n-1} + \ldots + a_{n-1} p + a_n}\ f(t) = \frac{M(p)}{L(p)}\ f(t), \qquad n \geq m \quad (2.1)$$

Laplace transforming will then give the transformed output response

$$Y(s) = \frac{M(s)F(s) + D(s)}{L(s)} \quad (2.2)$$

where $D(s)$ is a polynomial of degree $n-1$ related to initial conditions. The function $G(s) = \frac{M(s)}{L(s)}$ represents the system transfer function relating the transformed input-output functions. With zero initial conditions we obtain

$$Y(s) = \frac{M(s)F(s)}{L(s)} = G(s)F(s) = \frac{A(s)}{B(s)} \quad (2.3)$$

where $A(s)$ and $B(s)$ are rational polynomials in the complex frequency variable s. The time response of the linear system including the effects of the initial conditions is given by the inverse transform as follows:

$$x(t) = L^{-1}\left(\frac{A(s)}{B(s)}\right) + L^{-1}\left(\frac{D(s)}{L(s)}\right) \quad (2.4)$$

based on standard inverse form obtained by a partial fraction expansion. The partial fractions required for obtaining a time solution can, for the case of distinct roots, be obtained by the method of residues.

Example:

Now we consider the system (1) where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^m$ and A, B, C and D are real constant matrices of appropriate size. The system is invertible, that is,

$$\det\left[D + C(sI - A)^{-1} B\right] \neq 0, \quad (2.5)$$

if and only if, there exists the matrices $F \in R^{m\times n}$ and $G \in R^{m\times m}$ such that the system together with the state feedback law

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$$u(t) = Fx(t) + Gv(t) \tag{2.6}$$

results in a closed-loop transfer function matrix relating the new input $v(t)$ and the output $y(t)$ which has a nonsingular form, implying that the system is decoupled. For systems with square transfer function matrices the necessary and sufficient condition for existence of state feedback laws for decoupling are equivalent to the conditions for system inversion.

We now consider a representation of a linear constant multi-machine electric power system with the transfer function matrix [13].

$$Q(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{2}{s+2} \\ \frac{-1}{(s+1)(s+2)} & \frac{1}{s+2} \end{bmatrix} \tag{2.7}$$

By expanding (16) about $s = \infty$ we obtain: $Q(s) = Q_o + Q_1 s^{-1} + Q_2 s^{-2} + \ldots$ (2.8)

and letting

$$(Q(s))^{-1} = s(\bar{Q}(s))^{-1} \tag{2.9}$$

where $Q_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $Q_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $Q_2 = \begin{bmatrix} -1 & -2 \\ -1 & -2 \end{bmatrix}$, $Q_3 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $Q_4 = \begin{bmatrix} -1 & -2 \\ -7 & -8 \end{bmatrix}$, $Q_2 = \begin{bmatrix} 36 & -36 \\ -18 & 18 \end{bmatrix}$

… etc.

This result was expressed as: $(Q(s))^{-1} = E_0 s + D(s)$ (1.10)

The state space representation of $D(s)$ was obtained using the algorithm outlined in [2, 3, 5].

Conclusion

This study has shown that for a given multivariable system, the expression describing the dynamics can be reduced to a transfer function with the order of the numerator not exceeding that of the denominator. This was possible only when the entry of $D(s)$ was in reduced form. By making appropriate substitutions the individual entries of $D(s)$ was obtained using (2.4) for computing the inverse of the system. With the system transfer function $Q(s)$ given as in (3) and assuming that the system has an equal number of inputs and outputs, i.e., $r = m$, it is clear that the inverse is given by $Q(s)^{-1}$, and that it exists as long as $\det(Q(s))$ is not identically zero.

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