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Compilation of Instantaneous Source Functions for Varying Architecture of a Layered Reservoir with Mixed Boundaries and Horizontal Well Completion Part III: B-Shaped Architecture with Vertical Well in the Upper Layer

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Abstract

Two-layered reservoirs with a vertical well in the top layer and with letter 'B' architecture, is encountered very frequently in practice. Instantaneous source functions, which can be utilized in constructing pressure distributions for the purpose of fully characterizing such reservoirs, are derived in this paper.

Results show that sixty-four (64) different variations are possible, as a result of six (6) different external boundaries, which may be considered sealed or constant-pressured. With the interface also considered as a constant-pressure boundary, it is observed that the more the number of constant-pressure external boundaries, the fewer the source functions that could be written and vice-versa. Furthermore, only sources of the kind I(i) can be written for the vertical wells while sources of the kinds I(i), vi(i) and iii(i) are possible for only the x-axes of the horizontal well layer.

### Introduction

Part I and II of this series<sup>1-2</sup> derived instantaneous source functions for a two-layered reservoir with letter 'A' and letter 'S' architecture, respectively. In these architectures, horizontal wells were drilled in the layers. In this Part III, the frontiers of Part I and II are extended to letter 'B' architecture but with Layer 2 (top layer) completed with a vertical well. This configuration may result from later-in-life well re-entry with a horizontal well after initial completion with a vertical well in a reservoir initially thought to be singled-layered. It could also result from an oil recovery project, where a horizontal well is drilled to mobilize oil into an already existing vertical well or vice versa, under favourable technical and economic conditions.

Reference 3 derived instantaneous source functions for some selected two-layered reservoir with latter 'B' architecture but with both layers completed with a horizontal wells. Because this architecture is encountered more frequently in practice, this paper explores more of the possible configurations, especially with external constant-pressure applications.

Source and Green's functions in Refs.4 and 5 and Appendix of Ref. 1 are applied in this paper.

Letter 'B' Two- Layered Reservoir Geometry Description

Fig. 1 is a two-layered reservoir bounded on all six external boundaries and with a crossflow interface between its upper and lower boundaries, hence the letter 'B' architecture. The upper layer (Layer 2) is completed with a vertical well while the lower layer is completed with a horizontal well. A vertical well drains in 2D while a horizontal well drains in 3D. In both wells, flow can be governed chiefly by the nature of its reservoir boundaries,

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apart from well completion. An external constant-pressure boundary is capable of bringing flow in either well to ultimate, irreversible steady-state. Therefore, for prolong production of clean oil, well location becomes an important factor. Instantaneous sources and Green's functions will be written down for all possible well location in each layer, for the purpose of deriving the necessary pressure distribution.

#### **Results and Discussion**

Several well locations were considered for the wells in the layers in writing relevant instantaneous source functions. The functions showing the effects of critical boundaries, like constant-pressure and interface, are written down, where they exist. Another instantaneous source function that is not affected by boundary of any kind, i.e., the mandatory infinite-acting flow, is also written down for all models considered. Well location does not affect the number of flow directions in the wells but can strongly affect the number of flow periods in the wells especially horizontal wells. For this reason, both central and off-centered horizontal well locations are considered.



**Table 1** is a tabulation of results obtained. Sixty-four (64) different models emerged. With the interface assumed as a constant-pressure boundary, each layer has three external boundaries that can experience constant-pressure or be sealed.

All the instantaneous source functions for the vertical wells are of the kind I(i) on account of radial flow largely encountered in vertical wells. The constants,  $E_1$ , in all the suggested source functions are meant to fully characterize the vertical well flow transients at the inception of an external boundary, which may result in pseudosteady or steady-state flow, after the natural infinite-acting flow. Instantaneous source functions of the kind I(z = f) are suggested to account for interface effect in all the models. Source functions of the kind I(x=0) and I(x=x<sub>e</sub>) are suggested for constant-pressure boundaries along the x-axes, wherever they occur. I(x=0) means that the constant-pressure is at the starting edge of the layer, and I(x=x<sub>e</sub>) means that the constant-pressure is at the finishing end of the layer. However, some factors may prevent the prevalence of these kinds of external boundaries (involving constant-pressure) even if they are physically present.

Given the bounded nature of the lateral boundaries of Layer 1, the horizontal wells in this layer are presumed to experience any of the three kinds of source strengths in the x-axes unless any boundary there is a constant-pressure boundary. These are (1) early time infinite-acting source strength for central well location, (2) an infinite slab source

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strength in an infinite slab reservoir, and (3) a possible infinite plane source in infinite slab reservoir, owing to offcentered well location. These are source numbers II(i), vi(i), and iii(i), respectively. It is observed that the more the number of external boundaries with constant-pressure, the fewer the number of source functions that can be written down. For example, Models 1 to 7 have more instantaneous source functions than Models 18, 19, 51, etc. this is so because once one constant-pressure boundary is felt, the existence of all other boundaries, whether sealing or another constant-pressure, will not be felt under the same transient regime, if the external fluids have comparatively larger compressibilities than the produced fluids.

All the z-axes source functions at late times are infinite plane sources in infinite slab reservoirs, with an upper constant-pressure interface an lower boundary which may be sealed or constant-pressured. However, at early times, even the z-axes sources are considered to be of the kind I(i) in all models for Layer 1.

In all the models, the y-axes contribute infinite plane sources in infinite reservoirs; that is, the horizontal wells are considered as line sources.

| S  | Model      | Instantaneous Sou   | urce Function  |   |   |
|----|------------|---|--|---|---|
| /N | Diagram    |   |  |   |   |
|    |            | x-axis  | z-axis   | Layer 1   | Layer 2   |
| 1. |            | Both layers<br>sealed at both ends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $E_1 vii(x).I(y).v(z)$ $E_1 II(x).I(y).v(z)$ $E_1 iii(x).I(y).I(z)$                                       | $\begin{array}{c} E_2I(x).I(z)\\ E_2I(x).I(y)\\ E_2I(y).I(z) \end{array}$                             |
| 2. |            | Both layers<br>sealed at both ends  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>sealed bottom, interface<br>at top | $ \begin{array}{c} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array} $      | $\begin{array}{c} E_2I(x).I(z=0)\\ E_2I(x).I(y)\\ E_2I(x).I(z=f)\end{array}$                          |
| 3. |            | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>ends | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $ \begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array} $      | $\begin{array}{c} E_{2}I(x=0).I(z=f) \\ E_{2}I(x).I(z) \\ E_{2}I(x).I(z=f) \end{array}$               |
| 4. | → <u> </u> | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>bottom, sealed top    | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $ \begin{array}{l} E_1 x(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \end{array} $         | $\begin{array}{c} E_2I(x).I(z)\\ E_2I(x).I(y)\\ E_2I(y).I(z) \end{array}$                             |
| 5. |            | Top layer,<br>constant-pressure at<br>top, sealed bottom.<br>Bottom layer, sealed<br>ends | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $ \begin{array}{c} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array} $      | $\begin{array}{c} E_2I(x=x_e).I(z=0) \\ E_2I(x).I(y) \\ E_2I(x).I(z=f) \\ E_2I(x=0).I(y) \end{array}$ |
| 6. |            | Top layer, sealed<br>ends. Bottom layer<br>constant-pressure at<br>top, sealed bottom     | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $  \overline{E_1 ix(x).I(y).v(z)} \\  \overline{E_1 II(x).I(y).I(z)} \\  \overline{E_1 II(x).I(y).v(z)} $ |   |

### **Table 1: Description of Relevant Source Functions**

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| 7.  |           | Both layers<br>sealed at both ends   | Top layer, sealed<br>top, interface bottom.<br>Bottom layer, constant-<br>pressure at bottom,<br>interface at top                  | $\begin{array}{c} E_{1} \\ vii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).I(z) \\ E_{1}II(x).I(y).I(z) \end{array}$   | $\begin{array}{c} E_2 I(x).I(z) \\ E_2 I(x).I(y) \\ E_2 I(y).I(z) \end{array}$                                    |
|-----|-----------|--|--|--|---|
|     | _         |  |  |  |   |
| 8.  |           | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>ends  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>sealed bottom, interface<br>at top               | $\begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1II(x).I(y).v(z) \\ E_1iii(x).I(y).I(z) \end{array}$   | $\begin{array}{l} E_2 I(x=\!0).I(z=\!0) \\ E_2 I(x=\!x_e).I(z=\!f) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z) \end{array}$ |
| 9.  |           | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer, sealed<br>ends | Top layer, constant-<br>pressure top, interface at<br>bottom. Bottom layer,<br>sealed bottom, interface<br>at top                  | $ \begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array} $   | $\begin{array}{c} E_2I(x=x_e).I(z=0) \\ E_2I(x).I(z=f) \\ E_2I(x).I(y) \\ E_2I(x).I(z) \end{array}$               |
| 10. |           | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>bottom, sealed top     | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>sealed bottom, interface<br>at top               | $\begin{array}{l} E_{1}x(x).I(y).v(z) \\ E_{1}II(x).I(y).v(z) \\ E_{1}II(x).I(y).I(z) \end{array}$   | $\begin{array}{c} E_{2}I(x).I(z=0) \\ E_{2}I(x).I(y) \\ E_{2}I(x).I(z=f) \end{array}$                             |
| 11. |           | Top layer, sealed<br>ends. Bottom layer,<br>sealed bottom,<br>constant-pressure at<br>top  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>sealed bottom, interface<br>at top               | $\begin{split} E_1 i x(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{split}$   | $\begin{array}{l} E_2 I(x).I(z=0) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z=f) \end{array}$                                |
| 12. |           | Both layers<br>sealed at both ends   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>constant-pressure at<br>bottom, interface at top | $ \begin{array}{l} E_1 \operatorname{vii}(x).I(y).\operatorname{iv}(z) \\ E_1 \operatorname{iii}(x).I(y).\operatorname{iv}(z) \\ E_1 \operatorname{iii}(x).I(y).I(z) \\ E_1 \operatorname{II}(x).I(y).I(z) \end{array} $ | $\begin{array}{c} E_{2}I(x).I(z=0) \\ E_{2}I(x).I(y) \\ E_{2}I(x).I(z=f) \end{array}$                             |
| 13. |           | Top and bottom<br>layers, constant-<br>pressure at bottoms,<br>sealed tops                 | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                              | $\begin{array}{c} E_{1}x(x).I(y).v(z) \\ E_{1}II(x).I(y).v(z) \\ E_{1}II(x).I(y).I(z) \end{array}$   | $\begin{array}{c} E_2 I(x=0).I(y) \\ E_2 I(x).I(z=f) \\ E_2 I(x).I(y) \end{array}$                                |
| 14. | ★         | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed ends              | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                              | $ \begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array} $   | $\begin{array}{c} E_2I(x=0).I(y) \\ E_2I(x=x_e).I(y) \\ E_2I(x).I(z=f) \\ E_2I(x).I(z=0) \end{array}$             |
| 15. | ▶ <u></u> | Toplayer,constant-pressureatbottom,sealedbottomlayer,sealedbottom,constant-pressureattop   | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                              | $E_1 ix(x).I(\overline{y}).v(z)$ $E_1 II(x).I(y).I(z)$ $E_1 II(x).I(y).v(z)$   | $\begin{array}{c} E_{2}I(x=0).I(y) \\ E_{2}I(x).I(z=f) \\ E_{2}I(x).I(z=f) \end{array}$                           |
| 16. |           | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>ends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                              | $\begin{array}{c} E_{1} vii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).I(z) \\ E_{1}II(x).I(y).I(z) \end{array}$  | $\begin{array}{c} E_{2}I(x=0).I(y)\\ E_{2}I(x).I(z=f)\\ E_{2}I(x).I(z=f) \end{array}$                             |

| 17. | → <u>+</u><br>→ <u>+</u><br>→ ← | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed bottom,<br>constant-pressure at<br>top           | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>sealed bottom, interface<br>at top                | $ \begin{array}{l} E_1 ix(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $   | $\begin{array}{c} E_2 I(x=0).I(y) \\ E_2 I(x=x_e).I(z=0) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z=f) \end{array}$     |
|-----|---------------------------------|---|---|--|---|
| 18. | →<br>_<br>+<br>-<br>+           | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed ends   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>constant-pressure at<br>bottom, interface at top  | $ \begin{array}{l} E_1 \operatorname{vii}(x).I(y).iv(z) \\ E_1 \operatorname{iii}(x).I(y).iv(z) \\ E_1 \operatorname{iii}(x).I(y).I(z) \\ E_1 \operatorname{II}(x).I(y).I(z) \end{array} $ | $\begin{array}{l} E_2I(x=0).I(y) \\ E_2I(x=x_e).I(z=0) \\ E_2I(x).I(y) \\ E_2I(x).I(z=f) \end{array}$         |
| 19. | →<br>→<br>→                     | Toplayer,constant-pressureatbothends.Bottomlayer,constant-pressureatbottom,sealedtop                                      | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{split} E_1 x(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \end{split}$   | $\begin{array}{l} E_2I(x=\!0).I(y) \\ E_2I(x=\!x_e).I(z=\!0) \\ E_2I(x).I(y) \\ E_2I(x).I(z=\!f) \end{array}$ |
| 20. | ►<br>                           | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>both ends          | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $\begin{split} E_1 viii(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_1 viii(x).I(y).I(z) \end{split}$  | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y)\\ E_2I(x).I(z=f) \end{array}$                             |
| 21. | →<br>→<br>↑                     | Toplayer,constant-pressureatbottom,sealedbottomlayer,constant-pressureatbothends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $\begin{split} & E_1 viii(x).I(y).iv(z) \\ & E_1 II(x).I(y).iv(z) \\ & E_1 viii(x).I(y).I(z) \end{split}$  | $\begin{array}{l} E_2 I(x{=}0).I(z{=}f) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z{=}f) \end{array}$                    |
| 2.  |                                 | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>both ends   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 viii(x).I(y).iv(z)\\ E_1II(x).I(y).iv(z)\\ E_1 viii(x).I(y).I(z) \end{array}$  | $\begin{array}{l} E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z=f) \end{array}$                          |
| 23. | →<br>—                          | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>bottom, sealed top | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{split} E_1 x(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \end{split}$   | $\begin{array}{l} E_2I(x=x_e).I(z=0)\\ E_2I(x).I(y)\\ E_2I(x).I(z=f) \end{array}$                             |
| 24. | →<br><br><br>↓                  | Toplayer,constant-pressurebottom, sealed top.Bottom layer, sealedbottom, constant-pressure at top                         | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{split} & E_1 ix(x).I(y).v(z) \\ & E_1 II(x).I(y).I(z) \\ & E_1 II(x).I(y).v(z) \end{split}$  | $\begin{array}{c} E_2I(x=0).I(z=0) \\ E_2I(x).I(y) \\ E_2I(x).I(z=f) \end{array}$                             |
| 25. | ► <u>+</u> +                    | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed ends   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 iii(x).I(y).I(z) \end{array}$   | $\begin{array}{l} E_2I(x=0).I(z=0) \\ E_2I(x).I(y) \\ E_2I(x=x_e).I(z=f) \\ E_2I(x=x_e).I(z=0) \end{array}$   |

| 26. | → <u></u>                                | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>both ends                                    | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $ \begin{array}{c} E_1 viii(x).I(y).v(z) \\ E_1 viii(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $  | $\begin{array}{c} E_2 I(x).I(z{=}0) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z{=}f) \end{array}$                    |
|-----|--|--|---|---|---|
| 27. | <ul> <li>↓ ↓ ↓</li> <li>↓ ↓ ↓</li> </ul> | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, constant-<br>pressure at bottom,<br>sealed top | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $\begin{array}{c} E_{1}x(x).I(y).v(z) \\ E_{1}II(x).I(y).v(z) \\ E_{1}II(x).I(y).I(z) \end{array}$  | $\begin{array}{c} E_2I(x=0).I(z=f) \\ E_2I(x).I(y) \\ E_2I(x=x_e).I(y) \\ E_2I(x=x_e).I(z=f) \end{array}$ |
| 28. | ★  | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed bottom,<br>constant-pressure at<br>top  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $ \begin{split} & E_1 ix(x).I(y).v(z) \\ & E_1 II(x).I(y).I(z) \\ & E_1 II(x).I(y).v(z) \end{split} $   | $\begin{array}{l} E_2I(x=0).I(z=f) \\ E_2I(x).I(y) \\ E_2I(x=x_e).I(y) \\ E_2I(x=x_e).I(z=f) \end{array}$ |
| 29. | → <u> </u>                               | Both layers,<br>constant-pressure at<br>bottom.  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{array}{l} E_1 x(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \end{array}$   | $\begin{array}{l} E_2I(x=0).I(z=f) \\ E_2I(x).I(y) \\ E_2I(x).I(z=0) \\ E_2I(x).I(z=f) \end{array}$       |
| 30. | *<br>*<br>*                              | Toplayer,constant-pressureatbottom,sealedbottomlayer,constant-pressureatbothends                                 | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $ \begin{array}{c} E_1 viii(x).I(y).v(z) \\ E_1 viii(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $  | $\begin{array}{c} E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \\ E_2 I(x).I(z=f) \end{array}$                      |
| 31. | *  | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>both ends | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $\begin{array}{l} E_1 viii(x).I(y).v(z) \\ E_1 viii(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array}$  | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$  |
| 32. |  | Both layers,<br>constant-pressure at<br>tops, sealed bottoms   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $ \begin{array}{l} E_1 ix(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $  | $\begin{array}{c} E_2 I(x=x_e).I(z=0) \\ E_2 I(x=x_e).I(z=f) \\ E_2 I(x).I(y) \end{array}$                |
| 33. |  | Top layer, sealed<br>ends. Bottom layer,<br>sealed bottom,<br>constant-pressure at<br>top                        | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $ \begin{array}{l} E_1 ix(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i ix(x).I(y).I(z) \end{array} $  | $\begin{array}{l} E_2I(x).I(z=0) \\ E_2I(x).I(z=f) \\ E_2I(x).I(y) \end{array}$                           |
| 34. |  | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer, sealed<br>ends                       | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $ \begin{array}{c} E_1 \operatorname{vii}(x).I(y).iv(z) \\ E_1 \operatorname{iii}(x).I(y).iv(z) \\ E_1 \operatorname{iii}(x).I(y).I(z) \\ E_1 \operatorname{III}(x).I(y).I(z) \end{array} $ | $\begin{array}{c} E_2 I(x\!=\!\!x_e).I(z\!=\!\!0) \\ E_2 I(x).I(z\!=\!\!f) \\ E_2 I(x).I(y) \end{array}$  |

| 35. |             | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>bottom, sealed top                                 | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{c} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array}$  | $\begin{array}{l} E_{2}I(x).I(z=0) \\ E_{2}I(x).I(z=f) \\ E_{2}I(x).I(y) \end{array}$                            |
|-----|-------------|--|---|---|--|
| 36. | ►<br>       | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>ends                              | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{c} E_{1} vii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).iv(z) \\ E_{1}iii(x).I(y).I(z) \\ E_{1}II(x).I(y).I(z) \end{array}$   | $\begin{array}{l} E_2 I(x=0).I(z=f) \\ E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \end{array}$                           |
| 37. | → <u></u>   | Toplayer,constant-pressureattop,sealedbottom.Bottomlayer,constant-pressureatbottom,sealedtop                           | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $ \begin{array}{l} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array} $  | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 38. |             | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>bottom, constant-<br>pressure top | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $ \begin{array}{l} E_1 ix(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i ix(x).I(y).I(z) \end{array} $  | $\begin{array}{l} E_2I(x=0).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 39. |             | Both layers,<br>constant-pressure<br>bottoms, sealed tops  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $\begin{array}{c} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array}$  | $\begin{array}{l} E_2I(x=0).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 40. |             | Both layers,<br>constant-pressure<br>tops, sealed bottoms  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $\begin{array}{l} E_1 ix(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i ix(x).I(y).I(z) \end{array}$  | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 41. |             | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>both ends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $ \begin{split} & E_1 viii(x).I(y).iv(z) \\ & E_1 II(x).I(y).iv(z) \\ & E_i viii(x).I(y).I(z) \end{split} $   | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 42. |             | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed ends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $ \begin{array}{l} E_1 \operatorname{vii}(x).I(y).\operatorname{iv}(z) \\ E_1 \operatorname{iii}(x).I(y).\operatorname{iv}(z) \\ E_1 \operatorname{iii}(x).I(y).I(z) \\ E_1 \operatorname{III}(x).I(y).I(z) \end{array} $ | $\begin{array}{l} E_2 I(x\!=\!\!x_e).I(z\!=\!\!f) \\ E_2 I(x\!=\!\!0).I(z\!=\!\!f) \\ E_2 I(x).I(y) \end{array}$ |
| 43. | → <u></u> + | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, sealed ends  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $\begin{array}{c} E_1 vii(x).I(y).v(z) \\ E_1II(x).I(y).v(z) \\ E_1iii(x).I(y).I(z) \end{array}$  | $\begin{array}{c} E_2 I(x\!=\!x_e).I(z\!=\!f) \\ E_2 I(x\!=\!0).I(z\!=\!f) \\ E_2 I(x).I(y) \end{array}$         |
| 44. |             | Top layer, sealed<br>ends. Bottom layer,<br>constant-pressure at<br>bottom, sealed top                                 | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{array}{c} E_1 x(x).I(y).v(z)\\ E_1 II(x).I(y).v(z)\\ E_1 II(x).I(y).I(z) \end{array}$   | $\begin{array}{c} E_2 I(x).I(z{=}0) \\ E_2 I(x).I(z{=}f) \\ E_2 I(x).I(y) \end{array}$                           |

| 45. | ↓<br>↓<br>↓          | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer, sealed<br>ends                       | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{array}{l} E_1vii(x).I(y).v(z)\\ E_1II(x).I(y).v(z)\\ E_1iii(x).I(y).I(z) \end{array}$                     | $\begin{array}{l} E_2 I(x\!=\!x_e).I(z\!=\!0) \\ E_2 I(x).I(z\!=\!f) \\ E_2 I(x).I(y) \end{array}$  |
|-----|----------------------|--|---|---|---|
| 46. | → <u> </u>           | Top layer,<br>constant-pressure at<br>bottom, sealed top.<br>Bottom layer, sealed<br>ends                        | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{array}{l} E_1 vii(x).I(y).v(z) \\ E_1II(x).I(y).v(z) \\ E_1iii(x).I(y).I(z) \end{array}$                  | $\begin{array}{l} E_2 I(x=0).I(z=f) \\ E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \end{array}$  |
| 47. |                      | Both layers,<br>constant-pressure at<br>tops, sealed bottoms   | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $ \begin{split} & E_1 ix(x).I(y).v(z) \\ & E_1 II(x).I(y).I(z) \\ & E_1 II(x).I(y).v(z) \end{split} $             | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$  |
| 48. | *                    | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure,<br>sealed top  | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $\begin{split} & E_1 x(x).I(y).v(z) \\ & E_1 II(x).I(y).v(z) \\ & E_1 II(x).I(y).I(z) \end{split}$                | $\begin{array}{l} E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$  |
| 49. | <b>★</b><br><b>→</b> | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>both ends | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{split} & E_1 viii(x).I(y).v(z) \\ & E_1 viii(x).I(y).I(z) \\ & E_1 II(x).I(y).v(z) \end{split}$           | $\begin{array}{l} E_2 I(x=x_e).I(z=f) \\ E_2 I(x=x_e).I(z=0) \\ E_2 I(x).I(y) \end{array}$  |
| 50. |                      | Top layer, constant-<br>pressure at both ends.<br>Bottom layer,<br>constant-pressure at<br>bottom, sealed top    | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                               | $\begin{split} E_1 x(x).I(y).v(z) \\ E_1 II(x).I(y).v(z) \\ E_1 II(x).I(y).I(z) \end{split}$                      | $\begin{array}{l} E_2 I(x\!=\!\!0).I(z\!=\!\!f) \\ E_2 I(x\!=\!\!x_e).I(z\!=\!\!f) \\ E_2 I(x).I(y) \end{array}$                                |
| 51. |                      | Both layers,<br>constant-pressure at<br>both ends  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 viii(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i viii(x).I(y).I(z) \end{array}$            | $\begin{array}{l} E_2I(x=\!0).I(z=\!f) \\ E_2I(x=\!x_e).I(z=\!0) \\ E_2I(x=\!x_e).I(z=\!f) \\ E_2I(x=\!0).I(z=\!0) \\ E_2I(x).I(y) \end{array}$ |
| 52. | → <u> </u> ←         | Both layers,<br>constant-pressure at<br>both ends  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom                | $\begin{split} & E_1 viii(x).I(y).v(z) \\ & E_1 viii(x).I(y).I(z) \\ & E_1 II(x).I(y).v(z) \end{split}$           | $\begin{array}{l} E_2I(x=\!0).I(z=\!f) \\ E_2I(x=\!x_e).I(z=\!0) \\ E_2I(x=\!x_e).I(z=\!f) \\ E_2I(x=\!0).I(z=\!0) \\ E_2I(x).I(y) \end{array}$ |
| 53. |                      | Toplayer,constant-pressureatbottom,sealedbottomlayer,constant-pressureatbothends                                 | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 viii(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i viii(x).I(y).I(z) \end{array}$            | $\begin{array}{l} E_2I(x=\!0).I(z=\!f) \\ E_2I(x=\!0).I(z=\!0) \\ E_2I(x).I(y) \end{array}$   |
| 54. |                      | Toplayer,constant-pressureatbothends.Bottomlayer,sealedbottom,constant-pressureat                                | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $ \begin{split} & E_1 i x(x). I(y). i v(z) \\ & E_1 II(x). I(y). i v(z) \\ & E_i i x(x). I(y). I(z) \end{split} $ | $\begin{array}{c} E_2I(x=0).I(z=f) \\ E_2I(x=x_e).I(z=0) \\ E_2I(x=x_e).I(z=f) \\ E_2I(x=0).I(z=0) \\ E_2I(x).I(y) \end{array}$                 |

|     | top   |   |  |   |
|-----|---|---|--|---|
|     |   |   |  |   |
| 55. | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>both ends          | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{split} & E_1 viii(x).I(y).iv(z) \\ & E_1 II(x).I(y).iv(z) \\ & E_i viii(x).I(y).I(z) \end{split}$    | $\begin{array}{l} E_2I(x=x_e).I(z=0)\\ E_2I(x=x_e).I(z=f)\\ E_2I(x).I(y) \end{array}$   |
| 56. | Top layer,<br>constant-pressure at<br>both ends. Bottom<br>layer, constant-<br>pressure at bottom,<br>sealed top          | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array}$             | $\begin{array}{l} E_2I(x=0).I(z=f) \\ E_2I(x=x_e).I(z=0) \\ E_2I(x=x_e).I(z=f) \\ E_2I(x=0).I(z=0) \\ E_2I(x).I(y) \end{array}$ |
| 57. | Both layers, both<br>ends at constant-<br>pressures   | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, sealed bottom                                  | $\begin{array}{c} E_1 viii(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i viii(x).I(y).I(z) \end{array}$       | $\begin{array}{l} E_{2}I(x=0).I(z=f) \\ E_{2}I(x=x_{e}).I(z=f) \\ E_{2}I(x).I(y) \end{array}$                                   |
| 58. | Both layers,<br>constant-pressures at<br>bottom, sealed tops  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array}$             | $\begin{array}{l} E_2I(x=\!0).I(z=\!f) \\ E_2I(x=\!0).I(z=\!0) \\ E_2I(x).I(y) \end{array}$                                     |
| 59. | Both layers,<br>constant-pressures at<br>tops, sealed bottoms   | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $ \begin{array}{l} E_1 ix(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i ix(x).I(y).I(z) \end{array} $         | $\begin{array}{l} E_2 I(x=x_e).I(z=0) \\ E_2 I(x=x_e).I(z=f) \\ E_2 I(x).I(y) \end{array}$                                      |
| 60. | Top layer, sealed<br>top, constant-pressure<br>at bottom. Bottom<br>layer, sealed bottom,<br>constant-pressure at<br>top  | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 i x(x). I(y). i v(z) \\ E_1 II(x). I(y). i v(z) \\ E_i i x(x). I(y). I(z) \end{array}$ | $\begin{array}{l} E_2 I(x=0).I(z=f) \\ E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \end{array}$  |
| 61. | Top layer, sealed<br>bottom, constant-<br>pressure at top.<br>Bottom layer,<br>constant-pressure at<br>bottom, sealed top | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, constant-<br>pressure at bottom | $\begin{array}{l} E_1 x(x).I(y).iv(z) \\ E_1 II(x).I(y).iv(z) \\ E_i x(x).I(y).I(z) \end{array}$             | $\begin{array}{c} E_2 I(x=x_e).I(z=0) \\ E_2 I(x=x_e).I(z=f) \\ E_2 I(x).I(y) \end{array}$                                      |
| 62. | Top layer,<br>constant-pressures at<br>both ends. Bottom<br>layer, sealed bottom,<br>constant-pressure at<br>top          | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, interface<br>at top, constant-pressure<br>at bottom                 | $\frac{E_1 i x(x).I(y).iv(z)}{E_1 II(x).I(y).iv(z)}$ $\frac{E_1 i x(x).I(y).I(z)}{E_1 i x(x).I(y).I(z)}$     | $\begin{array}{l} E_2I(x=0).I(z=f) \\ E_2I(x=x_e).I(z=f) \\ E_2I(x).I(y) \end{array}$   |

| 63. |                | Toplayer,constant-pressureatbottom,sealedbottomlayer,constant-pressureatbothends | Top layer, constant-<br>pressure at top, interface<br>at bottom. Bottom layer,<br>interface at top, sealed<br>bottom | $ \begin{array}{l} E_1 viii(x).I(y).v(z) \\ E_1 viii(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $ | $\begin{array}{c} E_2 I(x=0).I(z=f) \\ E_2 I(x=0).I(z=0) \\ E_2 I(x).I(y) \end{array}$   |
|-----|----------------|--|--|--|--|
| 64. | → <u></u><br>→ | Both layers,<br>constant-pressures at<br>both ends                               | Top layer, sealed<br>top, interface at bottom.<br>Bottom layer, sealed at<br>bottom, interface at top                | $ \begin{array}{l} E_1 viii(x).I(y).v(z) \\ E_1 viii(x).I(y).I(z) \\ E_1 II(x).I(y).v(z) \end{array} $ | $\begin{array}{c} E_2 I(x=0).I(z=f) \\ E_2 I(x=x_e).I(z=f) \\ E_2 I(x).I(y) \end{array}$ |

# Conclusion

Instantaneous source functions for a two-layered reservoir with letter 'B' architecture and with a vertical well in the top layer have been tabulated. The external boundaries were varied with sealing and constant-pressure boundaries. The interface was considered as permeable and therefore behaves as a constant-pressure internal boundary. Compilation of all the possible instantaneous source functions shows that:

- 1. Sixty-four (64) different models are possible
- 2. There are six external boundaries defining the two-layered reservoir, which may be sealed or constantpressured
- 3. The vertical wells present source functions of the kind I(i) while those of horizontal wells are of the kinds II(i), vi(i) and iii(i) along the x-axes only
- 4. The more the number of external boundaries with constant-pressure, the fewer the source functions that can be written down, and vice versa

## Nomenclature

- E<sub>1</sub> Late time flow boundary effects constant for Layer 1
- $E_2$  Late time flow boundary effects constant for Layer 2
- (i) Axial flow directions x, y, or z
- i Distances in x, y, or z directions, ft

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