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Trajectory Coherent State And Coherent State For The Modified Caldirola-Kanai Oscillator

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Abstract

We construct the Trajectory Coherent State (TCS) and Coherent state (CS) in the framework of the Modified Caldirola Kanai Hamiltonian. We also evaluate the minimum uncertainty relation.

Keywords:Coherent state,modified caldirola-Kanai oscillator,uncertainty relation.

1.0 Introduction

The damped Harmonic Oscillator plays a central role in the quantum theory of lasers and masers [1-2] and its coherent state are used in describing different fields of theoretical physics [3]. Different theoretical methods to construct quantum states and techniques to measure them have been developed in quantum optics and atomic physics [4]. Glauber [5] had reviewed in his paper that the coherent states could be successfully used for problem of quantum optics. However, Glauber's coherent state is more useful tools to find quantum states, especially of an oscillator that displace its initial wave packet and exhibit a classical property [6].

In recent times, different authors have constructed the coherent states for particles with various potentials. Yeon and Um [7] have constructed the coherent states for the damped Harmonic Oscillator with a time-dependent frequency and coherent states for the damped Harmonic Oscillator [8]. Hartley and Ray [9] have evaluated and obtained the coherent states for time-dependent harmonic oscillator base on the Invariant method of Lewis-Riesenfield theory [10-11]. Nieto [12] have recently found the wave functions of the displaced and squeezed number states for the static oscillator and [13] have constructed the coherent states for particles in a general potential.

Different authors have also evaluated coherent states of damped Harmonic Oscillators in the frame of Caldirola-Kanai Hamilton [14, 16] and the path Integral of the damped harmonic oscillator of the modified Caldirola-Kanai oscillator [15]. The approximate solution of the Schrödinger wave equation for particle in general potentials whose space co-ordinate and momentum quantum mechanical expectation values were exact solutions of the corresponding classical Hamiltonian equations have been constructed by [14]. The State describing this process was called Trajectory – Coherent State (TCS) and the consequence of their construction is base on the complex WKB methods [17-19].

In this paper, we follow the approach [14, 17-19] and construct the Trajectory-Coherent State (TCS) of a damped Harmonic Oscillator (DHO) in the frame of modified Caldirola-Kanai Hamiltonian [15].

The organization of the paper is as follows:

In section II, we review the Integral of motion of the modified Caldirola-Kanai Oscillator. In Section, III, we construct the trajectory coherent state of the modified Caldirola-Kanai Hamiltonian.

Section IV focuses on the expectation values and the uncertainty relations while section V gives a brief conclusion.

2.0 Integral of motion of modified Calirola-Kanai Oscillator.

We consider the quantum damped oscillator in the frame of Caldirola-Kanai model [7, 8, 11, 16], using Integral of motion methods. The Hamiltonian of the modified Caldirola-Kanai Oscillator is [16],

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$$\hat{H}(t)=\frac{1}{2m}e^{-Sin\gamma t}\,\hat{p}^2+\frac{1}{2}mw^2(t)e^{Sin\gamma t}\hat{q}^2 \qquad (2.1)$$

where m is the mass of the oscillator, γ is the damping coefficient, $\hat{q}$ and $\hat{p}$ are the co-ordinate and momentum operators, and w (t) is time-dependent frequency of the oscillator. The Hamiltonian equations describe a classical damped motion whose classical co-ordinate q(t) and momentum p(t) takes the form.

$$\dot{q}=pe^{-Sin\beta\gamma t}$$
$$\dot{p}=-w^2(t)e^{Sin\beta\gamma t}q, \qquad (2.2)$$

and the classical equation of motion becomes

$$\ddot{q}(t)+\beta\gamma Cos\gamma t\dot{q}(t)+w^2(t)q(t)=0 \qquad (2.3)$$

Using Equation (2.1), we write the Lagrangian and the mechanical energy as [9, 16]

$$L=e^{Sin\beta\gamma t}\left[\frac{1}{2}m\dot{q}^2-\frac{1}{2}mw^2q^2\right],$$
$$E=\frac{1}{2m}e^{-Sin\beta\gamma t}p^2+\frac{1}{2}mw^2q^2e^{Sin\beta\gamma t} \qquad (2.4)$$

The Invariant method due to Lewis and Risenfield can be used to find the exact quantum states of the time-dependent modified Caldirola-Kanai Hamiltonian.

One can introduce a pair of linear invariant operators [10], linear in both co-ordinate and momentum for the solution ε(t) of Eq. (2.5),

$$\hat{a}(t)=\frac{i}{\sqrt{\hbar}}\left[\varepsilon^*(t)\hat{p}-m(t)\varepsilon^*(t)\hat{q}\right] \;,$$
$$\hat{a}^+(t)=-\frac{i}{\sqrt{\hbar}}\left[\varepsilon(t)\hat{p}-m(t)\varepsilon(t)\hat{q}\right], \qquad (2.5)$$

where ε(t) is the solution of Eq.(2.3) and the m(t) is the time-dependent mass defined in Eq. (2.1) as $m\ (t)=me^{Sin\gamma t}$.

The solution of the classical equation of motion ε(t) must satisfy the Wronskian condition [16].

$$m\frac{d}{d\beta}e^{Sin\beta\gamma t}\left[\varepsilon(t)\dot{\varepsilon}^*(t)-\varepsilon^*(t)\dot{\varepsilon}(t)\right]=i, \qquad (2.6)$$

which make the time-dependent Commutation and creation operators of Eq. (2.5) to satisfy the standard boson commutation relation at equal time,

$$\left[\hat{a}(t),\hat{a}^+(t)\right]=1 \qquad (2.7)$$

The eigenfunctions of the operators in Eq. (2.5) is given by [11, 15].

$$\psi_n(q,t)=\left(\frac{1}{2^n n!\sqrt{2\pi\hbar\varepsilon^*\varepsilon}}\right)^{1/2}\left(\frac{\varepsilon}{\sqrt{\varepsilon^*\varepsilon}}\right)^{n+1/2}H_n\left(\frac{q}{\sqrt{2\hbar\varepsilon^*\varepsilon}}\right)$$
$$\times\exp\left[\frac{ime^{Sin\gamma t}\dot{\varepsilon}^*(t)}{2\hbar\varepsilon^*}q^2\right]; \qquad (2.8)$$

and these are the exact quantum state of the Schrödinger equation [15]

$$i\hbar\frac{d}{dt}\psi(q,t)=H_{ck}\psi(q,t), \qquad (2.9)$$

where H_{ck} is the Hamiltonian of the modified Caldirola-Kanai oscillator of Eq.(2.1).

The Cauchy problem arising from Eq. (2.9) is defined as [14, 19]

$$\left|0\right\rangle_{t=0}=\psi_0(q,t,h)\big|_{t=0}$$
$$=N\exp\left\{\frac{i}{\hbar}\left(p_0(q-q_0)+\frac{b}{2}(q-q_0)^2\right\} \qquad (2.10)$$

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where $q_0 = q(t)\big|t=0$ and $p_0 = p(t)\big|t=0$.

The Trajectory Coherent State (TCS) wave function is the solution of the WKB type [14, 16, 19]

$$\left|0\right\rangle = \psi_0(q,t,\hbar) = N\psi(t)e^{i/\hbar S(q,t)}, \tag{2.11}$$

where the normalization constant N, is given by

$$N = \left(\frac{\operatorname{Im} b}{\pi\hbar}\right)^{1/4} \tag{2.12}$$

and

$$\psi(t) = \left(Z(t)\right)^{-\frac{1}{2}} \tag{2.13}$$

where Z (t) is the classic trajectory.

The action S (q, t) in Eq. (2.11) is given as

$$S[q,t] = \int_0^t \{\dot{q}(t)p(t) - H(q(t), p(t), t)\}dt + p(t)(q - q(t))$$
$$+\frac{1}{2}\frac{W(t)}{Z(t)}(q-q(t))^2, \tag{2.14}$$

Where *Z (t)* and *W (t)* are the classical trajectories in addition to *q(t)* and *p(t)*

3.0 Classical Trajectory of MCK Oscillator

The Hamiltonian of Eq.(2.1) is obtained from Eq. (2.4) as [15].

$$H = \frac{\partial L}{\partial \dot{q}}\dot{q} - L(p,q,t) \tag{3.1}$$

where H is t he classical Hamiltonian of the system and the classical trajectory is defined as

$$\dot{q}(t) = \frac{\partial H(x,p,t)}{\partial p}, \ \dot{p}(t) = -\frac{\partial H(q,p,t)}{\partial x}. \tag{3.2}$$

In addition to the classical trajectory Eq. (3.2), we define an additional classical trajectory *Z(t)* and w*(t)* as

$$Z(t) = \frac{H_{pp}\dot{W}(t) + H_{xp}\dot{Z}(t)}{H_{xp}H_{px} - H_{pp}H_{xx}} = \frac{\partial q(t)}{\partial p_0} \tag{3.3}$$

$$W(t) = \frac{H_{qq}\dot{Z}(t) + H_{px}\dot{W}(t)}{H_{qq}H_{pp} - H_{pq}H_{qp}} = \frac{\partial p(t)}{\partial q} \tag{3.4}$$

Equations (3.4 – 3.3) are subjected to the following conditions:

$$Z(0) = 1, \left(\frac{\partial q}{\partial p}\right)_0 = \frac{1}{b}$$
$$W(0) = b + \frac{\sigma(0)}{\Omega(0)} \tag{3.5}$$

Similarly, the classical Hamiltonian *Hqq*, *Hpp*, *Hpq* and *Hqp* are given as

$$H_{qq}(t) = \frac{d^2}{dq^2}H(q,p,t),$$
$$H_{pp}(t) = \frac{\partial^2}{\partial p^2}H(q,p,t),$$
$$H_{pq}(t) = \frac{\partial}{\partial p}\frac{\partial}{\partial q}H(q,p,t),$$
$$H_{qp}(t) = \frac{\partial}{\partial q}\frac{\partial}{\partial p}H(q,p,t) \tag{3.6}$$

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respectively.

The quantity b in Eq. (3.5) is a complex number obeying the boundary conditions $\operatorname{Im} b > 0$ [14, 16; 19]. The solution to the classical equation of motion, Eq. (2.3) is

$$q(t) = e^{-\beta\gamma t \, casp\gamma t}\left[Ae^{i\Omega t} + Be^{-i\Omega t}\right] \tag{3.7}$$

Using Eqs (3.2 – 3.7), we obtain the following differential equations

$$q(t) = \frac{1}{2m\Omega} e^{-\beta\gamma t Cos\beta\gamma t}\left[2P_0 Sin\Omega t + 2m\Omega Cos\Omega t + mq_0 Sin\Omega t\sigma(t)\ \right], \tag{3.8}$$

$$p(t) = \frac{1}{-4\Omega} e^{\beta\gamma t Cos\beta\gamma t}\left[\left(2\sigma(t) - 4\Omega Cos\Omega t\right)p_0 \ + m\left(\sigma^2(t) + 4w^2\right)q_0 Sin\Omega t\ \right], \tag{3.9}$$

$$Z(t) = \frac{\partial q(t)}{\partial p} = \frac{1}{2m\Omega} e^{-\beta\gamma t Cos\beta\gamma t}\left[2Sin\Omega t + 2m\Omega Cos\Omega t + \frac{m}{b}\sigma(t) Sin\Omega t\ \right], \tag{3.10}$$

$$W(t) = \frac{\partial p(t)}{\partial q} = \frac{1}{-4\Omega} e^{\beta\gamma t Cos\gamma t}\left[2\sigma(t) - 4\Omega Cos\Omega t\ \ b + m\left(\sigma^2(t) + 4w^2\right) Sin\Omega t\ \right],$$

(3.11)

where $\sigma(t) = -\beta\gamma\cos\beta\gamma t + \beta^2\gamma^2 t\sin\beta\gamma t$ and $\Omega(t) = \left(w^2(t) - \tfrac{1}{4}\gamma^2\right)^{1/2}$ and Eqs. (3.10 - 3.11) becomes $Z(0) = 1$

$$W(0) = b - \frac{\alpha}{\sqrt{1 - \frac{1}{4}\frac{\alpha^2}{2}}} \tag{3.12}$$

where $\alpha = \dfrac{\gamma}{2w(0)}$ and w(0) reduces to the result of Ref. [16, 19] when $\alpha \to 0$.

The annihilation operator *a(t)* and creation operator $\hat{a}^+(t)$ in terms of the trajectories z(t) and w(t) are given by [14, 19].

$$\hat{a}(t) = \left(2\hbar \operatorname{Im} b\right)^{-\frac{1}{2}}\left\{z(t)\left(\hat{p} - p(t)\right) - w(t)\left(\hat{x} - x(t)\right)\right\},$$

$$\hat{a}^+(t) = \left(2\hbar \operatorname{Im} b\right)^{-\frac{1}{2}}\left\{z^*(t)\left(\hat{p} - p(t)\right) - w^*(t)\left(\hat{x} - x(t)\right)\right\} \tag{3.13}$$

and they satisfies the commutation relation

$$\left[a, a^+\right] = 1 \tag{3.14}$$

The eigenfunction of the trajectory coherent state and its coherent counterpart are defined as [14, 16].

$$\left|n\right\rangle = \left(n!\right)^{-\frac{1}{2}}\left(\hat{a}^+\right)^n\left|0\right\rangle, \tag{3.15}$$

$$\left|\alpha\right\rangle = e^{\left(\alpha\hat{a}^+ - \alpha^* a\right)}\left|0\right\rangle, \tag{3.16}$$

respectively.

4.0 Expectation Values and the Uncertainty Relations

The co-ordinate and momentum operators in terms of the four classical trajectories q(t), p(t), z(t) and w(t) are expressed as

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$$\hat{q}(t) = q(t) - \frac{i}{\left(\frac{2\,\mathrm{Im}\,b}{\hbar}\right)^{\frac{1}{2}}}\left\{z(t)\hat{a}^{+}(t) - z^{*}(t)\hat{a}(t)\right\},$$

$$\hat{p}(t) = p(t) - \frac{i}{\left(\frac{2\,\mathrm{Im}\,b}{\hbar}\right)^{\frac{1}{2}}}\left\{w(t)\hat{a}^{+}(t) - w^{*}(t)\hat{a}(t)\right\}, \tag{4.1}$$

The expectation values and the uncertainty relations for the trajectory and coherent state are defined by the following relations:

$$\langle \hat{q} \rangle_{Tcs} = q(t), \qquad \langle \hat{p} \rangle_{Tcs} = p(t),$$

$$\langle \hat{q} \rangle_{cs} = q(t) - \frac{i}{\left(\frac{2\,\mathrm{Im}\,b}{\hbar}\right)^{\frac{1}{2}}}\left\{\alpha^{*} w(t) - \alpha w^{*}(t)\right\},$$

$$\langle \hat{q}^2 \rangle_{Tcs} = q^2(t) + \frac{\hbar}{\mathrm{Im}\,b}\left(n + \frac{1}{2}\right)|z(t)|^2$$

$$\langle p^2 \rangle_{Tcs} = p^2(t) + \frac{\hbar}{\mathrm{Im}\,b}\left(n + \frac{1}{2}\right)|w(t)|^2 \tag{4.2}$$

using Eqs. (3.8 – 4.2), we obtain the uncertainty in position for trajectory coherent state (TCS) and coherent state (CS) as

$$\left(\Delta \hat{q}^2\right)_{TCS} = \langle q^2(t) \rangle_{TCS} - \langle q(t) \rangle_{TCS}^2$$

$$= \frac{\hbar\left(n + \frac{1}{2}\right)}{\mathrm{Im}\,b}|z(t)|^2 = e^{-2\gamma t \cos \beta t}\,\frac{\hbar\left(n + \frac{1}{2}\right)}{\mathrm{Im}\,b}\left[1 + \frac{1}{\alpha^2}\right.$$

$$\left.\left\{1 + \frac{\sigma(t)}{\theta} + \left(\frac{\sigma(t)}{2\theta}\right)^2 - \alpha^2\right\}\sin^2 \Omega t + \frac{1}{\alpha}\left(1 + \frac{\sigma(t)}{\theta}\right)\sin 2\Omega t\right], \tag{4.3}$$

where Imb = μmΩ and the coherent state (CS) is obtained from Eq. (4.3) when n→0, and we obtain.

$$\left(\Delta \hat{q}^2\right)_{CS} = \frac{\hbar/2}{\mathrm{Im}\,b}e^{-2\beta\gamma t \cos \beta\gamma t}\left[1 + \frac{1}{\alpha^2}\left\{1 + \frac{\sigma(t)}{\theta} + \left(\frac{\sigma(t)}{2\theta}\right)^2 - \alpha^2 \sin^2 \Omega t + \frac{1}{\alpha}\left(1 + \frac{\sigma(t)}{2\theta}\right)\sin 2\Omega t\right]\right. \tag{4.4}$$

Similarly, we obtain the uncertainty in momentum for the trajectory coherent state (TCS) and coherent state (CS) as

$$\langle \Delta \hat{p} \rangle_{TCS} = \langle P^2 \rangle_{TCS} - \langle p \rangle_{TCS}^2 = \frac{\left(n + \frac{1}{2}\right)\hbar}{\mathrm{Im}\,b}|w(t)|^2 = \left(n + \tfrac{1}{2}\right)\hbar\,\mathrm{Im}\,b\left[1 + \frac{1}{\theta^2}\left\{\frac{1}{4}\left(\frac{\sigma^2(t)}{\Omega}\right)^2\right.\right.$$

$$\left.\left. + \frac{1}{4}\frac{\sigma^3(t)}{\Omega^2} + \sigma(t) + \frac{1}{16}\frac{\sigma^4(t)}{\Omega^2} + \frac{1}{4}\sigma^2(t) + \frac{\theta^2}{\mu^2} - \theta^2\right\}\sin^2 \Omega t - \left(\frac{1}{2} + \theta\Omega\right)\sin 2\Omega t\right], \tag{4.5}$$

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$$\langle\Delta\hat{p}\rangle_{CS} = \frac{\hbar}{2\,\mathrm{Im}\,b}\left[1+\frac{1}{\theta^2}\left\{\frac{1}{4}\left(\frac{\sigma^2(t)}{\Omega}\right)^2 + \frac{1}{4}\frac{\sigma^3(t)}{\Omega^2}+\sigma(t)+\frac{1}{16}\frac{\sigma^4(t)}{\Omega^2}+\frac{1}{4}\sigma^2(t)\right.\right.$$

$$\left.\left.+\frac{\theta^2}{\mu^2}-\theta^2\right\}\sin^2\Omega t-\left(\frac{1}{2}+\theta\Omega\right)\sin 2\Omega t\right], \quad (4.6)$$

where θ = μΩ and α = mΩ in Eqs. (4.3 – 4.6).

The uncertainty relation is obtained for the trajectory coherent state (TCS) and coherent state (CS) from Eqs. (4.3 – 4.6) as

$$\langle\Delta p\Delta q\rangle_{TCS} = \hbar\left(n+\frac{1}{2}\right)\left[1+\frac{1}{\alpha^2}\left\{1+\frac{\sigma(t)}{\theta}+\left(\frac{\sigma(t)}{2\theta}\right)^2-\alpha^2\right\}\sin^2\Omega t+\frac{1}{\alpha}\left(1+\frac{\sigma(t)}{2\theta}\right)\sin 2\Omega t\right]^{\frac{1}{2}}$$

$$\times\left[1+\frac{1}{\theta^2}\left\{\frac{1}{4}+\left(\frac{\sigma^2(t)}{\Omega}\right)^2+\frac{1}{4}\frac{\sigma^3(t)}{\Omega^2}+\sigma(t)+\frac{1}{16}\frac{\sigma^4(t)}{\Omega^2}+\frac{1}{4}\sigma^2(t)+\frac{\theta^2}{\mu^2}-\theta^2\right\}\sin^2\Omega t-\left(\frac{1}{2}+\theta\Omega\right)\sin 2\Omega t\right]^{\frac{1}{2}},$$

(4.7)

or we simplify Eq. (4.7) in the form

$$\langle\Delta p\Delta q\rangle_{TCS} = \hbar\left(n+\frac{1}{2}\right)[1+\Lambda(t)]^{\frac{1}{2}}, \quad (4.8)$$

where

$$\Lambda(t) = \left[\frac{1}{\theta^2}\left\{\frac{1}{4}\left(\frac{\sigma^2(t)}{\Omega}\right)^2+\frac{1}{4}\frac{\sigma^3(t)}{\Omega^2}+\sigma(t)+\frac{1}{16}\frac{\sigma^4(t)}{\Omega^2}+\frac{\sigma^2(t)}{4}+\frac{\theta^2}{\mu^2}-\theta^2\right\}\right.$$

$$\left.+\frac{1}{\alpha^2}\left\{1+\frac{\sigma(t)}{\theta}+\left(\frac{\sigma(t)}{2\theta}\right)^2-\alpha^2\right\}\right]\sin^2\Omega t+\left[\frac{1}{\alpha}\left(1+\frac{\sigma(t)}{2\theta}\right)-\left(\frac{1}{2}+\theta\Omega\right)\right]\sin 2\Omega t$$

$$+\left[\frac{1}{\alpha\theta^2}\left\{\frac{1}{4}\left(\frac{\sigma^2(t)}{\Omega}\right)^2+\frac{\sigma^3(t)}{4\Omega^2}+\sigma(t)+\frac{1}{16}\frac{\sigma^4(t)}{\Omega^2}+\frac{\sigma^2(t)}{4}+\frac{\theta^2}{\mu^2}-\theta^2\right\}\right.$$

$$\left.-\frac{1}{\alpha^2}\left(\frac{1}{2}+\theta\Omega\right)\left\{1+\frac{\sigma(t)}{\theta}+\left(\frac{\sigma(t)}{2\theta}\right)^2-\alpha^2\right\}\right]\sin 2\Omega t\sin^2\Omega t-\frac{1}{\alpha}\left(1+\frac{\sigma(t)}{2\theta}\right)$$

$$\left(\frac{1}{2}+\theta\Omega\right)\sin^2 2\Omega t+\frac{1}{\alpha^2\theta^2}\left\{1+\frac{\sigma(t)}{\theta}+\left(\frac{\sigma(t)}{2\theta}\right)^2-\alpha^2\right\}\ \left\{\frac{1}{4}\left(\frac{\sigma^2(t)}{\Omega}\right)^2+\frac{\sigma^3(t)}{4\Omega^2}\right.$$

$$\left.+\sigma(t)+\frac{\sigma^4(t)}{16\Omega^2}+\frac{\sigma^2(t)}{4}+\frac{\theta^2}{\mu^2}-\theta^2\right\}\sin^4\Omega t$$

and

$$\langle\Delta p\Delta q\rangle_{CS} = \frac{\hbar}{2}[(1+\Lambda(t))]^{\frac{1}{2}}, \quad (4.9)$$

Equations (4.8) and (4.9) are the minimum uncertainty relation for the trajectory coherent state (TCS) and coherent state (CS) respectively.

V CONCLUSION

We have constructed the trajectory coherent state (TCS) and coherent state (CS) using the modified Caldirola-Kanai Hamiltonian. These states obey the minimum uncertainty relation, Eqs. (37), and (40). When Λ(t)

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$\rightarrow 0$, we obtain the uncertainty relation of the simple Harmonic oscillator. We conclude that the trajectory coherent state (TCS) and coherent state (CS) for the damped harmonic oscillator with time-dependent frequency described by the modified Caldirola-Kanai Hamiltonian constructed above satisfy the basic properties of trajectory coherent state (TCS) and coherent states.

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