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Compilation of Instantaneous Source Functions for Varying Architecture of a Layered Reservoir with Mixed Boundaries and Horizontal Well Completion Part II: Normal and Inverted Letter 'P' Architecture

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Abstract

In this series, thirty-two(32) different models were identified for the normal and vertically inverted letter 'P' two-layered reservoir with a horizontal well in each layer. Relevant instantaneous source functions were also derived and tabulated for all the identified models.

1.0 Introduction

Horizontal or lateral wells may be drilled into a two-layered reservoir with either the normal or inverted latter 'P' architecture. The inversion be can vertically or horizontally. This architecture could have been discovered from studies which revealed that rather than one reservoir already completed there are actually two reservoirs in communication through an interface. On the other hand, one of two-layered reservoir may be required to serve as a means of fluid injection to enhance fluid mobility in the other layer. In some instances, fluids in the different layers may require separate production and testing and therefore different reservoir and wellbore treatment. Thus, correct procedures are needed for optimum performance prediction and characterization. Like in Part I, in this paper, instantaneous source functions for deriving pressure and their derivative distributions for a two-layered reservoir with horizontal or lateral well completions are derived for several cases of external fluid injection or patterns of influx into the layers. Table of Green's and source functions discussed in Ref 1 to 2 and as tabulated in the Appendix of Ref. 3 reproduced here, are utilized.

Normal and Inverted Letter 'P' Two-Layered Reservoir Description

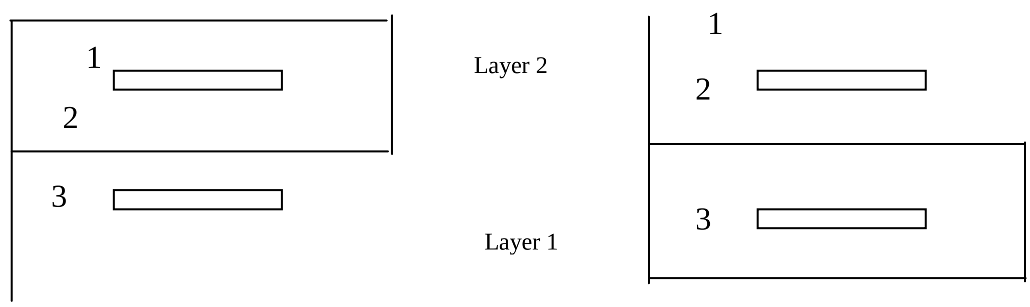
A two-layered reservoir with normal letter 'P' architecture is shown in Fig. 1a. A horizontal or lateral well is drilled into each layer. The top layer (Layer 2) has four external boundaries, which may be sealed or constant-pressured. The bottom layer (Layer 1) has only one external boundary, which may be sealed or constant-pressured, while it is infinite in all the other directions. Boundaries considered infinite are those that cannot be felt during a well test. The two layers are separated by a permeable interface. Therefore, the interface is an internal boundary and no fluid influx or injection is considered through.

Fig. 1b is an inverted letter 'P' architecture obtainable through vertical inversion of the normal letter 'P'. In this inverted form, the top layer corresponds to the bottom of the normal and bottom layer corresponds to the top.

Other forms of the inverted architectures shown in Fig. 2 are obtainable through horizontal inversion of the normal and the vertically inverted architecture. Therefore, in the architectures obtained through horizontal inversion, only the well positions are altered; the tops of wells in the parent architecture are bottoms of the horizontally inverted wells.

Derivation of Instantaneous Source Functions

Instantaneous source functions will be derived for the normal architecture and the vertically inverted architecture. Mastery of the derivations should assist in the derivation of instantaneous sources functions for all horizontally inverted architecture. However, a quick look at the architectures shows that for the purpose of well location consideration, all horizontally inverted architectures would have the same instantaneous source functions as their parent architectures. All the wells are considered as line sources. Mathematically, i.e., the y-boundaries are also ‘infinitely far away’.



a **b**

Nomenclature
1 Layer 1 Horizontal well
2 Interface
3 Layer 2 Horizontal well

Fig. 1: Normal and Inverted Letter ‘P’ Two-Layered Reservoir Architectures

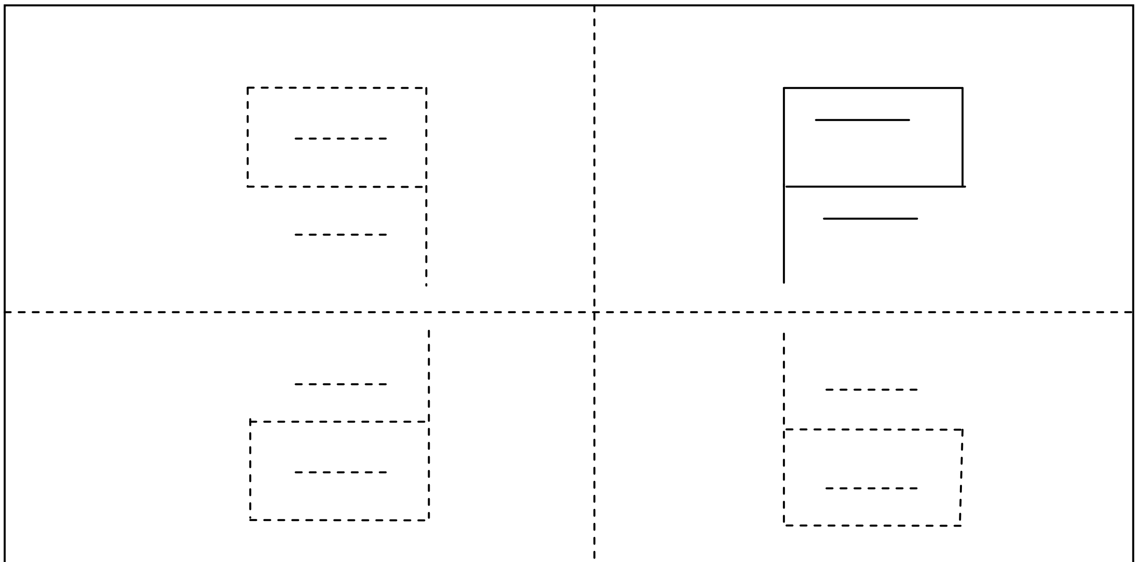


Fig. 2: Mirror Inversions of Letter ‘P’ Architecture (Not to scale)

Results and Discussion

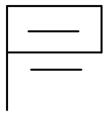
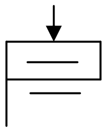
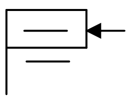
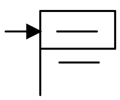
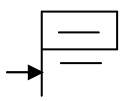
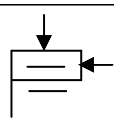
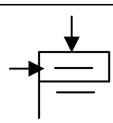
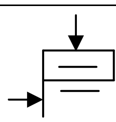
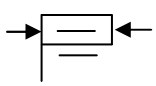
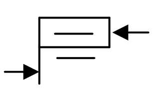
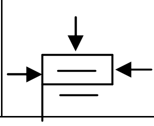
No external energy (influx or injection) is considered for all ‘infinitely far away’ boundaries and the interface. Each of the five external boundaries varied as sealed or constant-pressured or a combination in such a way that each resulting model is unique and not duplicated. Because no external energy is imposed on an ‘infinitely far away’ boundary, source function of the kind II(x), assuming central well location along the x-axes are written down for all the layers with ‘infinitely far away’ boundaries. However, off-centered well locations, where the wells behave as infinite plane sources in infinite slab reservoirs, source functions of the kind iii(x) are suggested. In practice, the function iii(x) is more common in layers where there are fewer or no external influx or injection oe where transients are propagated rapidly away from a constant-pressure boundary. In all the models, instantaneous source functions ii(x).I(y). I(z) is

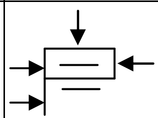
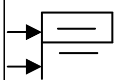
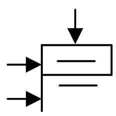
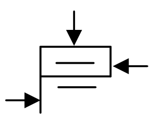
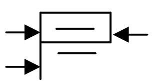
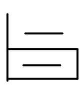
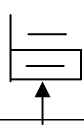
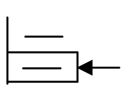
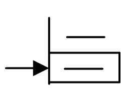
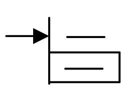
suggested to denote early radial flow, mandatory in any reservoir flow, no matter the nature of the external boundaries.

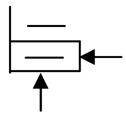
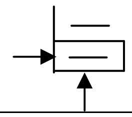
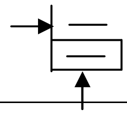
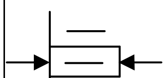
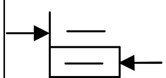
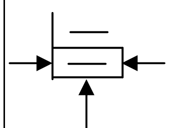
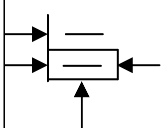
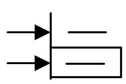
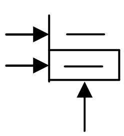
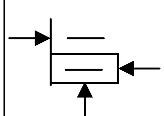
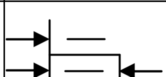
Because the interface is assumed as a constant-pressure boundary, multiplicative constants E_1 and E_2 are considered for Layer 1 and 2, respectively, for the purpose of determining full characterization of the interface.

Results obtained are tabulated in Table 1 for thirty-two (32) different models of both the normal and vertically inverted letter ‘P’ architectures. Models 1 to 16 are for the normal architecture while Models 17 to 32 are for vertically inverted architectures. In both normal and inverted architectures, the mandatory infinitely far away boundaries along the z-axes are generally represented by source function of the kind I(z). with only one (1) boundary that may be sealed or constant-pressured, in normal Layer 1 and vertically inverted Layer 2, source functions like vi(x) and vii(x) are considered, respectively, for constant-pressure effect at the bottom of the well and no sealing boundary off-centered well location effect along the x-axes. This is deliberately done even though there are no visible pairs of parallel boundaries apart from the single boundary which may be sealed or constant-pressured. This action is in vie of artificial no-flow boundaries, which are set up if the transients are felt by the single natural boundary. If the transients are not stopped by the physical single natural boundary, the well transients would behave as if the reservoir is infinite in the direction opposite to the natural boundary. If the single natural boundary is constant-pressured and is felt first by transients, infinite behavior would be brought to an end under the prevailing transient regime.

Instantaneous source functions, like viii(x).I(y).viii(z) are considered for completely constant-pressured pair of external boundaries, while vii(x).I(y).I(z), vii(x).I(y). v(z), vii(x).I(y). v(z) and vi(x).I(y).v(z), are considered for partially sealed and partially constant-pressured external boundaries. These instantaneous source functions are, however, used to describe final flow periods.

S/N	Model Diagram	Instantaneous Source Function			
		x-axis	**z-axis**	**Layer 1**	**Layer 2**
1.		Top layer, sealed ends. Bottom layer, sealed bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z) E_2iii(x).I(y).vi(z)
2.		Top layer, sealed ends. Bottom layer, sealed bottom, infinite top	Top layer, constant-pressure at top, interface at bottom. Bottom layer, interface at top, infinite bottom	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2vii(x).I(y).iv(z) E_2II(x).I(y).I(z)
3.		Top layer, sealed bottom, constant-pressure at top Bottom layer, sealed bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2v(x).I(y).vi(z) E_2II(x).I(y).I(z)
4.		Top layer, constant-pressure at bottom, sealed top. Bottom layer, sealed bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2vi(x).I(y).vi(z) E_2II(x).I(y).I(z)
5.		Top layer, sealed ends. Bottom layer, constant-pressure at bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z) E_2iii(x).I(y).vi(z)
6.		Top layer, sealed bottom, constant-pressure at top. Bottom layer, sealed bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2v(x).I(y).viii(z) E_2II(x).I(y).I(z)
7.		Top layer, constant-pressure at bottom, sealed top. Bottom layer, sealed bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2vi(x).I(y).vi(z) E_2II(x).I(y).I(z)
8		Top layer, sealed ends. Bottom layer, constant-pressure at bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2vii(x).I(y).iv(z) E_2II(x).I(y).I(z)
9.		Top layer, constant-pressure at both ends. Bottom layer, sealed bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2viii(x).I(y).vi(z) E_2II(x).I(y).I(z)
10.		Top layer, sealed bottom, constant-pressure at top. Bottom layer, constant-pressure at bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2v(x).I(y).vi(z) E_2II(x).I(y).I(z)
11.		Top layer, constant-pressure at both ends. Bottom layer, sealed bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vii(x).I(y).I(z) E_1iii(x).I(y).I(z) E_1II(x).I(y).v(z)	E_2viii(x).I(y).vi(z) E_2II(x).I(y).I(z)

12.		Top layer, constant-pressure at both ends. Bottom layer, constant-pressure at bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2viii(x).I(y).vi(z) E_2II(x).I(y).I(z)
13.		Top layer, constant-pressure at bottom, sealed top. Bottom layer, constant-pressure at bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2vi(x).I(y).vi(z) E_2II(x).I(y).I(z)
14.		Top layer, constant-pressure at bottom, sealed top. Bottom layer, constant-pressure at bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2vi(x).I(y).vi(z) E_2II(x).I(y).I(z)
15.		Top layer, sealed bottom, constant-pressure at top. Bottom layer, constant-pressure at bottom, infinite top	Top layer, constant-pressure at top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2v(x).I(y).iv(z) E_2II(x).I(y).I(z)
16.		Top layer, constant-pressure at both ends. Bottom layer, constant-pressure at bottom, infinite top	Top layer, sealed top, interface bottom. Bottom layer, infinite bottom, interface top	E_1II(x).I(y).v(z) E_1vi(x).I(y).I(z) E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2viii(x).I(y).vi(z) E_2II(x).I(y).I(z)
17.		Top layer, sealed bottom, infinite top. Bottom layer, sealed ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1vii(x).I(y).v(z) E_1iii(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
18.		Top layer, sealed bottom, infinite top. Bottom layer, sealed ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1vii(x).I(y).viii(z) E_1iii(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
19.		Top layer, sealed bottom, infinite top. Bottom layer, sealed bottom, constant-pressure at top	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1v(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
20.		Top layer, sealed bottom, infinite top. Bottom layer, constant-pressure at bottom, sealed top	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
21.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, sealed ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1vii(x).I(y).v(z) E_1iii(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)

22.		Top layer, sealed bottom, infinite top. Bottom layer, sealed bottom, constant-pressure at top	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1v(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
23.		Top layer, sealed bottom, infinite top. Bottom layer, constant-pressure at bottom, sealed top	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1vi(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
24.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, sealed ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1vii(x).I(y).viii(z) E_1iii(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
25.		Top layer, sealed bottom, infinite top. Bottom layer, constant-pressure at both ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1viii(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
26.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, sealed bottom, constant-pressure at top	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1v(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
27.		Top layer, sealed bottom, infinite top. Bottom layer, constant-pressure at both ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1viii(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z)
28.		Top layer, sealed bottom, infinite top. Bottom layer, constant-pressure at both ends	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1viii(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
29.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, constant-pressure at bottom, sealed top	Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1vi(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
30.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, constant-pressure at bottom, sealed top	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1vi(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
31.		Top layer, constant-pressure at bottom, infinite top. Bottom layer, sealed bottom, constant-pressure at top	Top layer, infinite top, interface bottom. Bottom layer, interface top, constant-pressure bottom	E_1v(x).I(y).viii(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)
32.			Top layer, infinite top, interface bottom. Bottom layer, interface top, sealed bottom	E_1viii(x).I(y).v(z) E_1II(x).I(y).I(z)	E_2II(x).I(y).vi(z) E_2vii(x).I(y).I(z) E_2iii(x).I(y).I(z) E_2vii(x).I(y).vi(z) E_2II(x).I(y).I(z)

Conclusions

Derivation of relevant instantaneous source functions for a layered reservoir with letter 'p' architecture shows that:

(1) The letter 'P' normal or inverted architecture is characterized by two mandatory infinitely far away boundaries; i.e., in the x and z-axes.
(2) Vertical inversion turns top of architecture bottom and bottom top.
(3) Horizontal inversion turns top of horizontal well bottom and bottom top.
(4) Thirty-two (32) different models are possible for both the normal and vertically inverted letter 'P' architectures.
(5) For the only boundary that can be sealed or constant-pressured, source functions of the kind vi(x) and vii(x) characterize instantaneous source functions along the x-axes.
(6) In the letter 'P' architecture, whether normal or inverted, one layer must be made up of four (4) boundaries that may be sealed or constant-pressured, while the other layer must consist of one (1) boundary that may be sealed or constant-pressured.
(7) For centrally located wells, the most prevalent instantaneous source function is II(x). I(y). I(z).

Nomenclature

E_1 Late time flow boundary effects constant for Layer 1
E_2 Late time flow boundary effects constant for Layer 2
(i) Axial flow directions x, y, or z
i Distances in x, y, or z directions, ft

References

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[3] Adewole, E.S.: "Compilation of Instantaneous Source Functions for Varying Architecture of a Layered Reservoir with Mixed Boundaries and Horizontal Well Completion," Part I: e-Shaped Architecture, paper submitted to *Nigeria Association of Mathematical Physics*, 2010.