Journal of the Nigerian Association of Mathematical Physics
***Volume* 16 (May, 2010), pp 521 - 530**
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Empirical Analysis of Priority on a FCFS Queue Discipline In Nigerian Banking System

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Abstract

Queues are virtually unavoidable phenomenon in Nigerian banking system. While banks stride to meet customers services' satisfaction, customers who do not go immediately into service must wait in line (if any). The queuing discipline in banks has been first-come-first-served (FCFS). What happen to the waiting time distribution when priority is encapsulated into a FCFS discipline is the interest of this paper. Data from a typical leading bank in Nigeria are analyzed using chi-squared technique on the assumption of Poisson arrival and exponential service times. The need for improved service control measures on banks' peak periods is emphasized.

Keywords: Queuing, Waiting Time, Priority, Poisson, Exponential, Chi-Squared.

1.0 Introduction

Queuing theory is concerned primarily with congestion caused by stochastic effects, that is, more arrivals than expected, or longer service times than usual. The pertinent aspects are: the probabilistic arrival pattern of the customers; the probability law of the time taken by a server to serve a customer; the number of servers present at the service facility; the size of the waiting line if only a limited number of customers may wait; and, the queue discipline (which is the rule by which a customer is selected to be served) (see [7]).

Globally, one is interested in how the queue length tends to rise and fall as time passes, while locally, the waiting times of individual customers are of interest (see also [3]).

In Nigerian banking system, the most practicable queuing system is the FCFS with inherent priority for a multi-sewer facilities. In many situations, the mean service rate or the mean arrival rate may not remain constant. As the queue increases, empirical evidence shows that servers tend to increase the rate of service, and some customers may balk and return another time or day [2]. Thus, some special customers of the bank also have their special treats depending on their priority class despite long queue. When this happened, the types of probability distributions for waiting times are often the subject of interest. The intent of this paper is to verify, empirically, whether or not, priority on a FCFS queue discipline, will have any significant effect on a FCFS waiting time distribution. Here, customers are treated as discrete while the corresponding queue size (i.e number of customers in queue) is integral valued. The basic process is: customers requiring service are generated between 8.00 am and 3.00pm over a period of five days of the bank's peak period; the customers enter the queuing system and join a queue for teller (deposit and withdrawal section of the service facility); at certain times, a member of the queue is selected for service by some rule known as the queue discipline; the required service is performed for the customer by the service mechanism, after which the customer leaves the system.

The hypotheses are alternatives noted over time for a queuing system with S>1 servers. Whereas, a Chi-squared test would be used in collapsing the hypotheses. The hypotheses are:

(i) The arrival time of this bank does not follow a Poisson distribution.

(ii) The service time does not follow exponential distribution.

(iii) The waiting time distribution is not exponential

The following notations will clarify issues:

λ = average number of customers per unit time

μ = average number of service completion per unit time

ρ = the utilization of traffic intensity factor

$1/\lambda$ = expected interarrival time

$1/\mu$ = expected service time

When λ_n (i.e the mean arrival rate of new customer when n customers are in system) is a constant for all n, it is denoted by λ, while $\mu_n = S\mu$ when $n \geq 1$.

$\rho = \lambda/s\mu$ (i.e. The utilization factor for the service facility)

1.0 The FCFS Queuing System:

Here, we assume that customers join the end of the queue and begin service in the order of their arrival. Let W(t) be the waiting time in the system if there are n customers in the system at time t, then

$$W(t) = n \overset{\lim}{\rightarrow} \infty \sum_{i=1}^{n} \rho(W_i \leq t)/n \quad , \quad t \geq o \tag{2.1}$$

Where for every t, (2.1) is the proportion of customers who have waiting time $\leq$t.

Let Y_n be the proportion of arrivals who find n other customers in the system on arrival, (n =0, 1, …), and let X_t denote a random variable with $[Y_n\}$distribution.

Then, X_t is the number of customers in the system found by an arrival.

Suppose W be a FCFS waiting time of a typical customer with distribution defined by (2.1), then,

$$W = \sum_{i=1}^{X_t+1} s_i \tag{2.2}$$

That is, for a i-server FCFS queue, a customer's waiting time is "the sum of the service time of those customers found in queue + the remaining service time of the customer (if any) found in service + the service time of the arriving customer". Since service time is exponential, the remaining service time is ~ exp (μ). Then (2.2) is the sum of X_t + 1 iid exponential random variables.

Again, since arrivals are Poisson, then

$$\{Y_n\} = \{P_n\} \tag{2.3}$$

That is, for each n, the proportion of arrivals that find n customers in system is equal to the proportion of time there are n customers in the system. Note:

$$P_n(t) = P\{X(t) = n\} \tag{2.4}$$

Suppose the interarrival or service time is represented with a random variable T, then, T will have an exponential distribution with parameter λ if its probability density function is

$$f_T(t) = \begin{cases} \lambda\lambda^{-\lambda} & for \qquad t \geq 0 \\ 0 & , for \quad t < 0 \end{cases} \tag{2.5}$$

And the cumulative probabilities are given as

$$\begin{aligned} P\{T \leq t\ \} &= i - \lambda^{-\lambda t} \\ P\{T > t\ \} &= \lambda^{-\lambda t} \end{aligned} \quad \text{, for } t \geq 0 \tag{2.6}$$

The expected value and variance of T are given respectively as

$$E(T) = 1/\lambda\ , \text{ and}$$

$$\text{Var (T)} = \frac{1}{\lambda^2} \qquad (2.7)$$

Suppose that the time between consecutive occurrences of some particular kind of arrivals or service completions by a continuously busy server has an exponential distribution with parameter λ, then there exist a relationship between Poisson and exponential distribution. (see [4]). Thus, we have:

$$P\{X(t) = n\ \} = \frac{(\lambda t)^n \lambda^{-\lambda t}}{n!}, t \geq 0, n = 0,1,2,........ \qquad (2.8)$$

When n = 0,

$$P\{Xct) =) = \lambda^{-\lambda t} \qquad (2.9)$$

Which is the probability from the exponential distribution that the first arrival or service completion occurs after time t. The mean of this distribution is given as

$$E\{X(t)\} = \lambda t \qquad (2.10)$$

Where X(t) is defined as the number of service completions achieved by a continuously busy sever in elapsed time t, where λ = μ.

3.0 The priority Queuing System:

In Nigerian banking system, for instance; some customers may have a higher delay cost than others. The order which customers of the queue are selected for service may sometimes result in this. When this happens, it is a priority rule which determines order of service. Supposing there are m types of customers (as in bank case) labeled type 1, type 2, …., type m. the interarrival times of type I customers (i=1….m), are exponentially distributed with rate λ_i and is assumed to be independent. Also, the service time of a type I customer is described by a random variable S_i (not necessarily exponential). Assume service times are independent of each other, then, with the assumption of independent arrival time, customers are served FCFS, and once service begins, each customer is served to completion without interruption. This is a case of "nonpremptive priority models" (a situation where a higher class special customer cannot be served until the service completion of a prior customer who already, is in service receiving attention of the service facility), [5] and [6].

On the other hand, when a higher class special customer is being served at the expense of a lower class special customer already in system, it is termed "Preemptive queueing system. (see [1]):

4.0 The Complexity:

Many a times, theoretical assumptions may be collapsed by empirical evidence. Although there is no stated distribution (not yet seen in literature if any) of waiting time for FCFS with priority inherent. Our interest is to verify its empirical status on assumption of FCFS waiting time distribution. The work requires serious concentration and time consciousness, in recording the waiting time of each customer and computing average waiting time and the percentage average waiting time caused by priority. The result would be revealing if or not priority consideration have any effect on a FCFS queuing system with regard to waiting time distribution. For accurate record, break time (launch time) of the bank would not be considered. Concentration would be only on queues associated with the deposit and withdrawal section of the bank. The peak period of the bank (viz. first week and last week of the month) was chosen to study the queue process and the five official working days of the week considered.

5.0 The Computational Results

The results of the analysis are as displayed in tables 5.1 to 5.9

The data used (observed data) are so voluminous that they could only be summarized as shown in the tables using frequency distribution.

Table 5.1 Computation of Interarrival Times and their Frequencies

Interarrival Time (1AT) (Mins)

		0-4	**5-9**	**10-14**	**15-19**	**20-24**	**25-29**	**Total**
DAY 1	Freq λ_i	335 330.61	13 18.36	0 0.02	0 0.00001	0 0.0000	1 0.0000	349 348.99
DAY 2	Freq λ_i	265 270.29	21 15.04					286 285.95
DAY 3	Freq λ_i	255 260.43	20 14.47					275 274.90
DAY 4	Freq λ_i	237 242.43	19 13.47					256 255.90
DAY 5	Freq λ_i	267 269.98	12 14.99	5 0.01	1 0.000001			285 284.98

Hypothesis 1:

H_1: Arrival time does not follow Poisson distribution

Table 5.2: Goodness of Fit Test for 1AT

DAY	λ_n	**Var**	χ^2_{cal}	$\chi^2_{0.05}$	$\chi^2_{0.01}$	**Result**
1	2	2.7	49.6	11.1	15.1	S (2)
2	2	1.7	2.49	3.54.	6.64	NS (2)
3	2	1.69	2.23	3.54	6.64	NS (2)
4	2	1.72	2.39	3.54	6.64	NS (2)
5	2	3.48	3588.64	7.81	11.34	S (2)

NS (2) = Not significant not in both levels; S(2) = significant in both levels; S(1) = significant in 1%; NS(1) = Not sig in 5%

Table 5.3: Computation of Service Time and their Frequencies

SERVICE TIME (min)

DAY		0-4	5-9	10-14	15-19	Total
DAY 1-1	Freq λ_i	80 95.11	28 27.26	10 7.80		118 130.17
DAY 1-2	Freq λ_i	70 71.13	17 26.17	11 9.63	4 3.54	102 110.47
DAY 1-3	Freq λ_i	91 103.17	25 29.57	12 8.45		128 141.19
DAY 2-1	Freq λ_i	53 65.56	31 24.12	10 8.87		94 98.55
DAY 2-2	Freq λ_i	40 59.98	36 22.07	9 8.12	1 2.98	86 93.15
DAY 2-3	Freq λ_i	74 83.82	23 24.02	7 6.86		104 114.7
DAY 3-1	Freq λ_i	62 72.54	21 20.29	7 5.94		90 99.27
DAY 3-2	Freq λ_i	45 58.58	30 21.55	9 7.93		84 88.06
DAY 3-3	Freq λ_i	65 75.76	24 21.71	5 6.20		94 103.6
DAY 4-1	Freq λ_i	74 79.79	19 22.87	6 6.53		99 109.19
DAY 4-2	Freq λ_i	43 53.00	17 19.50	11 7.17		76 79.67
DAY 4-3	Freq λ_i	63 67.70	16 19.40	5 5.54		84 92.64
DAY 5-1	Freq λ_i	77 80.60	16 23.10	7 6.60		100 110.3
DAY 4-2	Freq λ_i	67 72.54	18 20.79	4 5.94	1 1.71	90 101.23
DAY 4-3	Freq λ_i	67 74.15	19 21.25	6 6.07		92 101.47

Freq 1-1 = Frequency of Day one with Server one

λ_i 1-1 = Expected freq. of Day one with Server one.

Hypothesis 2:

H_1: Service time does not follow exponential distribution.

TABLE 5.4: GOODNESS OF FIT TEST FOR SERVICE TIME (ST)

DAY	λ_n	μ	**Var**	χ^2_{cal}	$\chi^2_{0.05}$	$\chi^2_{0.01}$	**Result**
1	1	4.29	10.33	3.04	5.99	9.21	NS (2)
	2	4.5	4.19	3.48	7.81	11.34	NS (2)
	3	3.9	10.59	3.63	5.99	9.21	NS (2)
2	1	4.7	11.5	4.51	5.99	9.21	NS (2)
	2	5.3	12.56	16.86	7.81	11.34	S (2)
	3	3.8	9.10	1.20	5.99	9.21	NS (2)
3	1	3.9	9.8	1.72	5.99	9.21	NS (2)
	2	4.9	11.48	6.59	5.99	9.21	NS (2)
	3	3.8	8.4	2.00	5.99	9.21	S (2)
4	1	3.6	8.4	1.12	5.99	9.21	NS (2)
	2	4.6	13.5	2.84	5.99	9.21	NS (2)
	3	3.5	8.3	1.74	5.99	9.21	NS (2)
5	1	3.5	8.3	2.37	5.99	9.21	NS (2)
	2	3.6	9.3	1.37	7.81	11.34	NS (2)
	3	3.7	10.5	4.85	5.99	9.21	S (2)

Table 5.5:Computation of Waiting Times (W.T) and their Frequencies for Servers 1-3

5.5.1: W.T DAY 1

(Mins)	**F_1**	λ_{i1}	**F_2**	λ_{i2}	**F_3**	λ_{i3}
1-10	11	17.16	9	14.52	12	18.34
11-20	13	14.62	8	12.33	8	15.63
21-30	0	12.46	6	10.54	3	13.31
31-40	15	10.62	13	9.10	18	11.38
41-50	10	9.05	7	7.65	8	9.66
51-60	16	7.71	13	6.53	19	8.25
61-70	6	6.57	9	5.56	11	7.03
71-80	13	5.60	9	4.74	13	5.99
81-90	6	4.77	1	4.04	8	5.10
91-100	4	4.07	5	3.45	3	4.38
101-110	3	3.46	2	2.93	2	3.70
111-120	5	2.95	2	2.49	5	3.15
121-130	8	2.52	9	2.13	5	2.69
131-140	7	2.14	6	1.81	10	2.29
Total	111	103.7	99	87.75	125	1109

5.5.2: W.T DAY 2

(Mins)	**F_1**	λ_{i1}	**F_2**	λ_{i2}	**F_3**	λ_{i3}
1-10	15	22.15	5	19.44	6	22.83
11-20	2	16.74	3	14.98	4	17.59
21-30	11	12.66	12	11.56	19	13.57
31-40	22	9.56	29	8.67	31	10.18
41-50	27	7.23	20	6.54	18	7.68
51-60	7	5.46	10	4.85	16	5.70
61-70	8	1.92	7	1.54	7	1.81
Total	**92**	**75.72**	**86**	**67.58**	**101**	**79.36**

5.5.3: W.T DAY 3

(Mins)	F_1	λ_{i1}	F_2	λ_{i2}	F_3	λ_{i3}
1-10	16	30.29	20	30.27	19	32.57
11-20	24	19.51	23	19.11	21	21.27
21-30	25	12.56	20	11.96	26	14.06
31-40	01	8.09	10	7.61	17	9.24
41-50	12	5.21	11	4.81	14	6.07
Total	**87**	**75.66**	**84**	**73.76**	**97**	**83.21**

5.5.4: W.T DAY 4

(Mins)	F_1	λ_{i1}	F_2	λ_{i2}	F_3	λ_{i3}
1-10	31	35.78	24	30.64	26	31.12
11-20	20	22.81	17	19.73	17	20.25
21-30	21	14.70	18	12.71	18	13.04
31-40	10	9.27	20	8.18	14	8.24
41-50	18	5.72	8	4.92	15	5.03
51-60	1	3.77	1	3.10	1	3.01
Total	**101**	**92.05**	**88**	**79.28**	**91**	**80.69**

5.5.5: W.T DAY 5

(Mins)	F_1	λ_{i1}	F_2	λ_{i2}	F_3	λ_{i3}
1-10	13	22.60	3	14.22	9	17.50
11-20	23	17.42	12	11.31	14	13.76
21-30	17	13.44	16	8.99	17	10.82
31-40	11	10.05	5	7.18	8	8.64
41-50	1	7.60	2	5.49	2	6.70
51-60	0	5.64	0	4.21	0	5.31
61-70	15	4.79	13	3.39	18	3.93
71-80	14	3.66	12	1.93	13	2.54
81-90	6	2.82	4	0.47	2	1.15

Hypothesis 3: H_1: The Waiting Time distribution is not exponential

Table 5.6: Goodness of Fit Test for Waiting Time

DAY	$1/\mu$	χ^2_{cal}	$\chi^2_{0.05}$	$\chi^2_{0.01}$	Result
1-1	0.016	61.85	22.36	27.69	S (2)
1-2	0.016	55.06	22.36	27.69	S (2)
1-3	0.016	74.38	22.36	27.69	S (2)
2-1	0.028	105.44	12.59	16.81	S (2)
2-2	0.026	120.53	12.59	16.81	S (2)
2-3	0.026	115.02	12.59	16.81	S (2)
3-1	0.044	29.39	9.49	13.28	S (2)
3-2	0.046	18.33	9.49	13.28	S (2)
3-3	0.042	32.67	9.49	13.28	S (2)
4-1	0.045	32.15	11.07	15.09	S (2)
4-2	0.044	24.45	11.07	15.09	S (2)
4-3	0.043	28.38	11.07	15.09	S (2)
5-1	0.026	72.78	15.51	20.09	S (2)
5-2	0.023	128.59	15.51	20.09	S (2)
5-3	0.024	115.02	15.51	20.09	S (2)

Table 5.7: Computation of Priority Waiting Time and their Frequencies

Priority waiting Time (PWT) (Mins)

DAY		1-5	6-10	11-15	16-20	TOTAL
DAY 1	Freq	22	7	1		30
DAY 2	Freq	3	22	3		28
DAY 3	Freq	5	11			16
DAY 4	Freq	11	4			15
DAY 5	Freq	22	16	1	1	40

Table 5.8: Percentage Average Waiting Time from Priority Customers

DAY	SERVER	Priority AV.WT	Total Mean WT	% Cont
1	1	4.5	62	7.2
	2	4.5	61.01	7.4
	3	4.5	62.14	7.2
2	1	8.0	36	22
	2	8.0	38.76	20.6
	3	8.0	38.07	21.0
3	1	6.4	22.97	27
	2	6.4	21.81	29.3
	3	6.4	24.06	26.6
4	1	4.3	22.23	19.3
	2	4.3	22.55	19.1
	3	4.3	23.08	19.3
5	1	5.6	38.5	14.5
	2	5.6	44.07	12.7
	3	5.6	41.04	13.6

Table 5.9: rate of Arrivals Per Hour, Rate of Service Per Hour and Traffic intensity Utilized.

	Day 1			Day 2			Day 3			Day 4			Day 5		
Service	λ	μ	l	λ	μ	l	λ	μ	l	λ	μ	l	λ	μ	l
1	30	15	0.7	30	12	0.8	30	15	0.7	30	15	0.7	30	15	0.7
2	30	12	0.8	30	12	0.8	30	12	0.8	30	12	0.8	30	12	0.8
3	30	15	0.7	30	15	0.7	30	15	0.7	30	15	0.7	30	15	0.7

6.0 Concluding remarks:

Tables 5.2 and 5.4 collapsed the alternative hypothesis tested showing that the arrivals and services times of the bank satisfy Poisson and exponential processes. With table 5.6, it was observed that FCFS with inherent priority discipline dragged the waiting time away from being exponential. This could be seen in the larger variances caused by the priority rule given that the waiting time of customers in the priority class tends to be much smaller. The larger variance under the assumption of FCFS justifies the test result that waiting time distribution is not exponential. Table 5.8 determines the percentage average priority waiting time that caused the deviation of the waiting time distribution from the assumed distribution. Table 5.9 shows that the arrival rate was constant for the period under study. Since the queue was a "single queue-three servers" model, the arrival time was approximately twice the service time, thus $\lambda>\mu$, and the service facilities were used to capacity causing the waiting line to be indefinite.

Finally, this research has opened a ground for more researches on the type of distribution a FCFS with inherent priority waiting time will follow. The need for more service facilities, especially during peak periods was suggested for the bank, to reduce unnecessary priority treatment.

REFERENCES

[1] Cooper, R. (1981): Introduction to Queueing Theory (2nd Ed) North-Holland, New York.

[2] Gross, D. and Harris, C. (1985): Fundamentals of Queueing Theory (2nd Ed). Wiley, New York.

[3] Hastings K. (1989): Introduction to The Mathematics of Operations Research. Marcel Dekker, New York.

[4] Hillier, S. and Lieberman, J. (1995): Introduction to Operations Research (6th Ed). McGraw-Hill, Singapore.

[5] Smith, D. (1977): Quantitative Business Analysis. Wiley, New York.

[6] Winston, W. (1994): Operations Research: Applications and Algorithms. Wadsworth, California.