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A Cost Control Policy for Material Requirements Planning and Procurements.

Bassey K.J[1]* , Adebola F.B[2] and Okoro C.N[3]
[1,2]Department of Mathematical Sciences,
Federal University of Technology, Akure, P.M.B. 704
[3]Department of Industrial Mathematics/Applied Statistics,
Ebonyi State University, Abakaliki
[*]Corresponding author **e-mail: simybas@yahoo.com.** Tel. +2347031061663

Abstract

It is sometimes difficult to predict that a particular material will be out of stock at a particular time. This uncertainty can be avoided by deriving a probability distribution of stock-out. Let us assume that the stock-out of material occur only at the end of a certain period, say t. This paper derives a probability distribution theory of stock-out at time t which minimizes the total cost involved for the planning and procurements of material within a production cycle. A control policy to sustain the system when the inventory cost is a function of time with a known depreciation value is presented.

Keywords: MRPP, Cost Control, Inventory, Probability, Policy

1.0 Introduction

Material requirements planning (MRP) system became a prominent approach to managing the flow of raw material and components on the floor of industry in the late 20th century. In 1985, it was estimated that between 2000 and 5000 American companies used MRP in production schedules [10]. Since then, research in the area of MRP has received global attention. [5] examined the performance of an extended MRP approach for a hybrid single-stage production/manufacturing system with external return flows. [2] extended the MRP system to include procurements and designed a policy to sustain it. This policy was validated by proving that 'when the average cost of purchasing orders exceeds the purchasing cost of each material (item) at stock-out despite the discounting rates, procurements of each item at stock-out becomes optimal'.

While MRP accuracy depends on the accuracy of stock status, several commentators [3], [4], [6], [7] and [9] expressed concern about the state of MRP in inventory control system. Thus, research in this area is gaining global momentum by the day.

2.0 A Probability Distribution Theory of Stock-out:

When there is material stock-out either caused by the stochastic nature of demand [3], wear-out, or damage, the expected number of stock-out incorporated into the MRP policy will make MRP design more appropriate. When this is done, the procurement policy that will minimize cost of purchasing orders will make MRP optimal and is called Material Requirements Planning and Procurements Policy (MRPP), see for example, [2].

Now, consider the following:

$B(t)$ = number of survivors (materials) at any time t

$B(t-1)$= number of survivors at any time t-1

N= initial number of materials

Then, the probability of stock-out during time t is given by

$$P(t) = \frac{B(t-1)-B(t)}{N} \tag{2.1}$$

The probability that an item has survived to a time t-1 and will be out of stock during the interval t-1 to t can be defined as the conditional probability of stock-out and is given by

$$P(t \mid t-1) = \frac{B(t-1)-B(t)}{B(t-1)} \tag{2.2}$$

Hence, the probability of item survival at time t is given by

$$P_s(t) = \frac{B(t)}{N} \tag{2.3}$$

Let us assume that stock-out occurs just before the period of (k+1) (years or months), where k is an integer. That is to say, the life span of an item lies between t=0 and t=k. We define

$f(t) =$ number of orders at time t

$P(x) =$ probability of stock-out just before a certain period $x+1$

and $$\sum_{x=0}^{k} P(x) = 1$$

Then, if $f(t-x)$ represents the number of orders at time $t-x, t=k, k+1, \ldots$, the new items attaining the period x at time t can be illustrated as

$$\begin{cases} t-x \,\alpha\; 0 \\ t \,\alpha\; .x \\ t+1 \,\alpha\; x+1 \end{cases}$$ where $0; x; x+1$ shows the state of material at time $t-x; t; t+1$.

Hence, the expected number of stock-out of such newly ordered items before getting to the time t+1 is given as

$$E(X) = \sum_{x=0}^{k} P(x) f(t-x); t = k, k+1, \ldots \tag{2.4}$$

But in every production cycle, all stock-outs at time t are replenished to sustain the system. Therefore any replenishment at time (t+1), the expected number of orders are

$$f(t+1) = \sum_{x=0}^{k} P(x) f(t-x); t = k, k+1, \ldots \tag{2.5}$$

Eqn (2.5) is a difference equation in t and the solution can be obtained by putting the value $f(t) = \alpha\beta^{t}$,where α is some constant. Thus, Eqn (2.5) becomes

$$\alpha\beta^{t+1} = \alpha \sum_{x=0}^{k} P(x) \beta^{t-x} \tag{2.6}$$

If we divide Eqn (2.6) by $\alpha\beta^{t-x}$ we get

$$\begin{aligned} \beta^{k+1} &= \sum_{x=0}^{k} P(x)\beta^{k-x} = \beta^{k} \sum_{x=0}^{k} P(x)\beta^{-x} \\ &\Rightarrow \beta^{k+1} - \left\{P(0)\beta^{k} + P(1)\beta^{k-1} + \ldots + P(k)\right\} = 0 \end{aligned} \tag{2.7}$$

It could be seen that Eqn (2.7) is exactly of roots (k+1), having a (k+1) degree. Let us denote the roots by $\beta_0, \beta_1, \ldots, \beta_k$,then for β=1 we have

$$1-\{P(0)+P(1)+\ldots+P(v)\}=0$$
$$=1-\sum_{x=0}^{v}P(x)=0 \qquad (2.8)$$

It follows that one root of Eqn(2.7) is β=1.If we call this root β_0, in general, Eqn (2.7) will be of the form

$$f(t)=\alpha_0\beta_0^t+\alpha_1\beta_1^t+\ldots+\alpha_k\beta_k^t=\alpha_0+\alpha_1\beta_1^t+\ldots+\alpha_k\beta_k^t \qquad (2.9)$$

where $\alpha_0,\alpha_1,\ldots,\alpha_k$ are some constants.

Again, it follows that since one of the roots in Eqn (2.7) $\beta_0=1$, all other roots, by Descartes' sign rule, will be negative while their absolute value will be less than unity. This means that the value of these roots tends to zero as t tends to infinity ($f(t)=\alpha_0$).

Now, to determine α_0, let

$h(x)=$ probability of item survival for more than x period

Then

$$h(x)=1-[P(0)+P(1)+\ldots+P(x-1)] \qquad (2.10)$$

With assumption that $h(0)=1$, for $f(x)=\alpha_0$

$$E(X)=\alpha_0 h(x) \qquad (2.11)$$

and

$$N=\alpha_0\sum_{x=0}^{k}h(x)\beta_1,\ldots,\beta_k \Rightarrow \alpha_0=\frac{N}{\sum_{x=0}^{k}h(x)}$$

where,

$$\sum_{x=0}^{k}h(x)=\sum_{x=0}^{k}h(x)\Delta(x)=h(x)x]_0^{k+1}-\sum_{x=0}^{k}(x+1)\Delta h(x)$$
$$=h(k+1)(k+1)-h(0).0-\sum_{x=0}^{k}(x+1)\Delta h(x)=h(k+1)(k+1)-\sum_{x=0}^{k}(x+1)\Delta h(x) \qquad (2.12)$$

But we know that $h(k+1)=1-[P(0)+P(1)+\ldots+P(k)]=0$, since no item can survive for more than (k+1) period, then

$$\Delta h(x)=h(x+1)-h(x)=[1-P(0)-P(1)-\ldots-P(x)]-[1-P(0)-P(1)-\ldots-P(x-1)]$$
$$=-P(x)$$

(2.13)

Substituting Eqn (2.13) in (2.12) we get

$$\sum_{x=0}^{k}h(x)=\sum_{x=0}^{k}(x+1)P(x) \qquad (2.14)$$

which is the mean life span of items before getting out of stock in a given production cycle.
Recall that we can compute the expected number of stock-out in a given production cycle [2]. Then, with Eqn (2.14), the value for the probability distribution of material stock-out can be obtained easily.

3.Cost Control Policy for MRPP:

In manufacturing industries, one of the major factors that must be taken into consideration when planning for new material is the cost function. Suppose that the material (item) is available for use over a series of time of equal duration, say, months of a year. We wish to propose a cost control policy in ordering new material when inventory

costs increase with time and the item depreciation value known.
Let

$\delta =$ ordering cost of a new item

$\eta_n =$ handling cost of item at the beginning of nth period

$I =$ annual interest rate

$\partial =$ depreciation value per unit of cash during a period, $\left(\partial = \frac{1}{1+I}\right)$

With the assumption that the item is procured after every n period of operation, we based our policy on the period for which the total cash is minimum, making it the best policy for optimal control of cost. Now, let τ_n be the discounted value of all future costs of purchasing orders and handling cost of item, then with a policy to replenish item after every n period we have

$$\tau_n = \{[\delta + \eta_1] + \partial\eta_2 + \partial^2\eta_3 + \ldots + \partial^{n-1}\eta_n\} \quad \text{(for 1 to n period)}$$
$$+ \{\partial^n[\delta + \eta_1] + \partial^{n+1}\eta_2 + \ldots + \partial^{2n-1}\eta_n\} \quad \text{(for n+1 to 2n period)}$$

$$+ \ldots = \{[\delta + \eta_1][1 + \partial^n + \partial^{2n} + \ldots] + \partial\eta_2[1 + \partial^{n-1} + \partial^{2n} + \ldots] + \ldots + \partial^{n-1}\eta_n[1 + \partial^n + \partial^{2n} + \ldots]\}$$
$$= \{[\delta + \partial_1] + \partial\eta_2 + \ldots + \partial^{n-1}\eta_n\}\{1 + \partial^n + \partial^{2n} + \ldots\}$$
$$= \left\{\delta + \sum_{i=1}^{n}\partial^{i-1}\eta_i\right\}\left\{\frac{1}{1-\partial^n}\right\}; \tau < 1 \tag{3.1}$$

Thus, the value of τ_n is the cost required to take care of future planning with a policy of every nth period procurements. If n is an optimal replenishment interval, then τ_n will be minimum following the inequality

$$\tau_{n+1} > \tau_n < \tau_{n-1} \tag{3.2}$$

This implies that $\tau_{n+1} - \tau_n > 0$ and $\tau_n - \tau_{n-1} < 0$ (3.3)

Proof:

We want to show that the inequalities in Eqn (3.3) holds. From Eqn (3.1), since $\tau_n = \frac{\varphi(n)}{1-\partial^n}$,then

$$\tau_{n+1} = \frac{\varphi(n+1)}{1-\partial^{n+1}} = \frac{(1-\partial^n)\tau_n + \partial^n\eta_{n+1}}{1-\partial^{n+1}} = \frac{1-\partial^n}{1-\partial^{n+1}}\tau_n + \frac{\partial^n\eta_{n+1}}{1-\partial^{n+1}}$$

where $\varphi(n) = \delta + \eta_1 + \partial\eta_2 + \ldots + \partial^{n-1}\eta_n$ (3.4)

Considering Eqn (3.3),

$$\tau_{n+1} - \tau_n = \frac{\varphi n + 1}{1-\partial^{n+1}} - \frac{\varphi(n)}{1-\partial^n} = \frac{\varphi(n+1)(1-\partial^n) - \varphi(n)(1-\partial^{n+1})}{(1-\partial^n)(1-\partial^{n+1})}$$

$$= \frac{[\varphi(n+1) - \varphi(n)] + \partial^{n+1}\varphi(n) - \partial^n\varphi(n+1)}{(1-\partial^n)(1-\partial^{n+1})} = \frac{\partial^n\eta_{n+1} + \partial^{n+1}\varphi(n) - \partial^n[\varphi(n) - \partial^n\eta_{n+1}]}{(1-\partial^n)(1-\partial^{n+1})}$$
$$= \frac{\partial^n(1-\partial^n)\eta_{n+1} - \partial^n(1-\partial)\varphi(n)}{(1-\partial^n)(1-\partial^{n+1})} = \frac{\partial^n(1-\partial)}{(1-\partial^n)(1-\partial^{n+1})}\left[\frac{1-\partial^n}{1-\partial}\eta_{n+1} - \varphi(n)\right] \tag{3.5}$$

Since $\partial < 1$ and $1-\partial^n > 0$,then $\tau_{n+1} - \tau_n$ is always positive and has the same sign as the quantity in the bracket of Eqn (3.5). Similarly, putting n-1 for n in (3.5) will give

$$\tau_n - \tau_{n-1} = \frac{\partial^{n-1}(1-\partial)}{(1-\partial^{n-1})(1-\partial^n)}\left[\frac{1-\partial^{n-1}}{1-\partial}\eta_n - \varphi(n-1)\right] = \frac{\partial^{n-1}(1-\partial)}{(1-\partial^n)(1-\partial^{n-1})}\left[\frac{1-\partial^{n-1}}{1-\partial}\eta_n\left(\varphi(n)-\eta_n\partial^{n-1}\right)\right]$$

$$= \frac{\partial^{n-1}(1-\partial)}{(1-\partial^n)(1-\partial^{n-1})}\left[\frac{1-\partial^n}{1-\partial}\eta_n - \varphi(n)\right] \tag{3.6}$$

And the necessary condition for minimum value of τ_n is given by

$$(\tau_n - \tau_{n-1}) < 0 < (\tau_{n+1} - \tau_n) \equiv \eta_n < \frac{\delta + (\eta_1 + \partial\eta_2 + \ldots + \partial^{n-1}\eta_n)}{1 + \partial + \partial^2 + \ldots + \partial^{n-1}} < \eta_{n+1} \tag{3.7}$$

The expression between η_n and η_{n+1} in (3.7) define the weighted average of all costs up to the period (n-1) with corresponding weights of $1, \partial, \partial^2, \ldots, \partial^{n-1}$. Therefore, the value of n satisfying this relationship will be the best period for replenishing inventory.

Lemma 3.1:

Given a discrete time inventory with stochastic demand, the mean annual cost of inventory will be minimized by replenishment when the next period's cost becomes greater than the current cost.

Proof:

Let $\delta =$ ordering cost of a new item

$\eta_t =$ handling cost for the t period, and

n= period in which an order for a new item is due(ordering age of an item)

when t is a discrete variable, the mean cost incurred over the period n is given by

$$K\delta_n = 1/n\left[\delta - \sum_{t=0}^{n}\eta_t\right] \tag{3.8}$$

Supposed that δ and $\sum_{t=0}^{n}\eta_t$ are monotonically decreasing and increasing respectively, then there will exist a value of n for which $K\delta_n$ is minimum. That is to say

$$K\delta_{n-1} > K\delta_n < K\delta_{n+1} \Rightarrow K\delta_{n-1} - K\delta_n > 0 \text{ and } K\delta_{n+1} - K\delta_n > 0 \tag{3.9}$$

Thus, for any period n+1 we will have

$$K\delta_{n+1} = \frac{1}{n+1}\left[\delta - \sum_{t=1}^{n+1}\eta_t\right] = \frac{1}{n+1}\left[\delta - \sum_{t=1}^{n}\eta_t + \eta_{n+1}\right]$$

$$= \frac{n\left\{\delta - \sum_{t=1}^{n}\eta_t\right\}}{(n+1)(n)} + \frac{\eta_{n+1}}{n+1} = \frac{n}{n+1}K\delta_n + \frac{\eta_{n+1}}{n+1} \tag{3.10}$$

Thus,

$$K\delta_{n+1} - K\delta_n = \frac{n}{n+1}K\delta_n + \frac{\eta_{n+1}}{n+1} - K\delta_n = \frac{\eta_{n+1}}{n+1} + K\delta_n\left[\frac{n}{n+1} - 1\right]$$

$$= \frac{\eta_{n+1}}{n+1} - \frac{K\delta_n}{n+1} \tag{3.11}$$

Hence, $K\delta_{n+1} - K\delta_n > 0$ implies that

$$\frac{\eta_{n+1}}{n+1} - \frac{K\delta_n}{n+1} > 0 \equiv \eta_{n+1} - K\delta_n > 0 \text{ or } \eta_{n+1} > K\delta_n \qquad (3.12)$$

Similarly, $K\delta_{n-1} - K\delta_n > 0$ implies that

$$\eta_{n+1} < K\delta_{n-1} \cdot \qquad (3.13)$$

In summary, if the next period cost, η_{n+1},is more than the mean cost of nth period, $K\delta_n$, then the best policy is procure items at the end of the nth period otherwise, do not procure;
That is:

Control Policy $\left\{ \eta_{n+1} > \frac{1}{n}\left[\delta - \sum_{t=0}^{n} \eta_t \right] \right.$ Place order on items at the end of nth period, or

Control policy $\left\{ \eta_n < \frac{1}{n-1}\left[\delta - \sum_{t=0}^{n-1} \eta_t \right] \right.$,the present period running cost is less than the previous period's mean cost, $K\delta_{n-1}$, then do not place order. The policy is called *cost control policy* for MRPP.

4.Conclusion:

In this work, a cost control policy has been formulated and presented for MRPP. A probability distribution theory of stock-out has also been derived and presented. By these expositions, insight into the empirical demonstration has been established. Nevertheless, the empirical demonstrations are left for further research. The need for doing this for the development of statistical theory for the control of inventory cost in MRPP need not be over-emphasized.

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