

**Calculation of Dose Gamma Ray Build up Factor in Some Homogenous Materials**

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**Abstract**

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*The gamma ray buildup factor was calculated by analyzing the narrow- beam and broad-beam geometry equations using Taylor's formula for isotropic sources and homogeneous materials. The buildup factor was programmed using MATLAB software to operate with any radiation energy (E), atomic number (Z) and the thickness of the absorber in term of the relaxation length ( $\mu x$ ). The model demonstrated capability in the design of an appropriate shielding material for radiation protection, absolute volume and activity measurement and in beam transport studies.*

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**Keywords:** build up factor, radiation shielding, gamma radiation, point isotropic sources, Taylor's formula.

**1.0 Introduction**

Radiations can either be ionizing or non-ionizing. Ionizing electromagnetic radiations such as, Gamma-and X- rays are called photons, and the neutron,  $\alpha$ - and  $\beta$ -radiations are particulates. While electromagnetic waves like Radio waves, infra-red, visible light and RADAR are non-ionizing. The photons are more penetrable in matter than the particulate ionizing radiations. Gamma ray is produced in the nuclei of atoms after emission of either  $\alpha$ - or  $\beta$ -radiations to ground the atoms while X-rays is produced by the de-excitation of electron from higher energy level to lower one. Gamma- and X-rays are widely used in medical imaging, sterilization of surgical instruments and radiation therapy. However, in optimizing the use of these radiations, doses delivered to the unsuspecting population around nuclear facilities need to be minimized in order to prevent radiation hazards. As the energetic radiation passes through living tissue, they ionize the cells thereby disrupting the large complex molecules such as DNA that will immediately damage the cells. At a certain level they can induce genetic mutation and cause cancer such as leukemia and there by shorten life expectancy.

The  $\gamma$ -rays buildup factor accounts for the scattered radiation that did not reach the appropriate target and it is an important shielding parameter. It represents a necessary correction in the design calculations of nuclear reactor, medical field and nuclear laboratories. In order to convert radiation intensities into radiation doses meaningful in risk assessment and management, shielding is an important principle in radiation protection. Equations describing buildup factor are function of attenuation coefficient, which itself a function of photon energy, and distance from the source and geometry. A number of methods and formulae have been used to calculate the dose buildup factor such as; [1], [2], [3], [4], and [5].

The experimental determination of buildup factor for different materials, both homogeneous and heterogeneous, is cumbersome, time wasting, energy consuming and varies with energies of the radiation [6]. Effort is usually directed towards finding the variation of the buildup factor with the absorber atomic number ( $Z$ ), mean free path ( $mfp$ ) and the energy of the radiation ( $E$ ) and also providing an analytic expression for the buildup factor in terms of the atomic number ( $Z$ ), mean free path ( $mfp$ ) and the

energy of the radiation ( $E$ ). In the present work, buildup factor for gamma rays was calculated for some materials ( $Al, Fe, \text{ and } Pb$ ) in a single layer shield using Taylor's method for analyzing the narrow- and broad-beam geometry equations. The analytic expression obtained was computed using MATLAB for parallel beam of monochromatic gamma rays with various energies, 0.5, 1.0, 4.0

and 10.0 MeV as the radiation source, relaxation length ( $\mu x$ ) and atomic number ( $Z$ ).

## 2.0 METHODOLOGY

Taylor's formula gives the buildup factor as a bi-exponential expression:

$$B = A_1 e^{-\alpha_1 \mu x} + A_2 e^{-\alpha_2 \mu x} \quad (2.1)$$

The narrow beam geometry under isotropic source of condition is given as:

$$I = I_0 e^{-\mu x} \quad (2.2)$$

The broad beam condition is given as:

$$I_0 = B(E, \mu x) I_0 e^{-\mu x} \quad (2.3)$$

At  $x = 0$ , equation 2 becomes

$$I = I_0 \quad (2.4)$$

And equation 3 becomes

$$I = BI_0 \quad (2.5)$$

Therefore,  $I_0 = BI_0$

$$\text{Hence, } B = 1 \quad (2.6)$$

The Taylor's formula becomes

$$B = A_1 + A_2 \quad (2.8)$$

$$\text{Hence, } A_1 + A_2 = 1 \quad (2.9)$$

The physical implication of equation 2.9 is that at the point of entering of the radiation into the absorber there is no interaction with the absorber atom. Substituting equation 2.9 into equation 2.1 gives:

$$B = A_1 e^{-\alpha_1 \mu x} + (1 - A_1) e^{-\alpha_2 \mu x} \quad (2.10)$$

Where  $B$  = Dose buildup factor,  $\mu x$  = mean free path (mfp) or relaxation length and  $A_1, \alpha_1$  and  $\alpha_2$  are parameters of the equation. The values of  $A_1, \alpha_1$  and  $\alpha_2$  for the elements at various energy and mean free path were obtained after curved fitting the buildup factors for low (0.5, 1.0 MeV), intermediate (4.0 MeV) and high (10.0 MeV) energies of three different metals shields, S – block metal, Aluminium (Al), Transition metal, Iron (Fe) and Heavy metal, lead (Pb) obtained from literatures [7] using the MATLAB software with about 95% confidence interval (0.99) and are shown in tables 1-3.

The values of dose  $\gamma$ -ray buildup factor obtained (equation 2.10) using the parameters shown in the tables 1-3 at different atomic numbers,  $z$  and energies,  $E$  are shown in tables 4-6 for the three elements considered.

## 3.0 Results and Discussion

### 3.1 Effect of the Thickness of the Selected Materials on the Buildup Factor

Figures 1, 2 and 3 show the graphs of the buildup factor against the thickness (mfp) of the various materials at the selected energies. The result shows that the buildup factor increases with thickness (mfp) of the materials for all energies. This behaviour is as a result of the increase in the scattering probabilities with small angles when the thickness of the material increases [8].

### 3.2 Effect of the Energy on the Dose Buildup Factor

The figures 4, 5, 6 and 7 show the gamma ray buildup factor against thickness (mfp) for the selected materials at the energies 0.5, 1.0, 4.0 and 10.0 MeV respectively. At gamma energies 0.5 MeV and 1.0 MeV

(Fig. 4 & 5), the buildup factor for Al is greater than that of Fe and Pb, the buildup factor for the low atomic number is higher than that of high Z. At 4.0MeV, figure 6 shows the same behaviour as shown in figures 4 & 5 except for the fact that Fe has the highest buildup factor as against Al in the previous figures. The dose buildup factors at energy 10MeV (Fig. 7) shows completely different behaviour as compared with the previous energies. The dose buildup factor for Pb is higher than that of Fe and Al.

The variations observed in figures 4 - 7 are due to the three processes that come to play when  $\gamma$  - rays

interact with matter. In Al and Fe, the photoelectric effect is small and makes no significant contributions for photon energies above 0.2 MeV, Compton scattering effect is dominant up to 4MeV. Photoelectric effect dominates at energies above 4.0MeV and for materials with high Z. The probability of Compton scattering effect is greater than the probability of photoelectric effect for the material of Al and Fe. Although the probability of Photoelectric effect is greater than Compton scattering for Pb, Compton scattering dominates at energies less than 4.0MeV and for materials with low atomic number. Also, because photoelectric effect and Compton scattering effect cross sections decrease with energy while pair production cross section increases with energy, a minimum in the total attenuation cross section may occur and such a minimum occurs in Pb at around 3 to 4 MeV. This minimum provides an energy window at which radiation leak from a given shield. Hence, the use of heterogeneous materials becomes necessary in shielding application to close such energy windows [9]. At 10MeV, the probability of the pair production effect is greater than probability of Compton scattering effect for high Z.

### 3.3 Effect of Atomic Number on the Buildup Factor

Figure 8 shows the graphs of the dose gamma ray buildup factor against the atomic number Z at constant thickness of 15 mfp. The result shows that the buildup factor decreases with the increase of the atomic number of the shield at the energies 0.5MeV and 1.0MeV. At E = 4.0MeV the dose buildup factor increases to about Z = 27 and then decreases with increasing atomic number. The buildup factor increases with the increase of the atomic number at 10MeV of the gamma ray energy. At this energy, both Compton scattering and pair production play major role with increasing atomic number.

1: The curve fitted values of dose gamma ray buildup factor of Al

E (MeV)	Thickness (mfp)						A <sub>1</sub>	$\alpha_1$	$\alpha_2$
	1	2	4	7	10	15			
0.5	1.3018	1.6899	2.8186	5.8749	11.5442	26.7076	14.34	- 0.3145	- 0.3182
1.0	1.3139	1.7258	2.9747	6.7146	15.1064	57.8532	9.926	- 0.2892	- 0.291
4.0	1.1622	1.3481	1.8023	2.7321	4.012	6.7591	- 9.211	- 0.1951	- 0.1908
10.0	1.260	1.526	2.0768	2.9513	3.8868	5.5903	40.17	- 0.01495	- 0.00877

Table 2: The curve fitted values of dose gamma ray buildup factor of Fe

E (MeV)	Thickness (mfp)						A <sub>1</sub>	$\alpha_1$	$\alpha_2$
	1	2	4	7	10	15			
0.5	1.2146	1.475	2.1751	3.8928	6.9618	18.3129	- 9.981	- 0.2032	- 0.2024
1.0	1.2326	1.5172	2.2882	4.1816	7.4854	18.4448	- 7.821	- 0.2477	- 0.2434
4.0	1.4414	1.9023	2.885	4.5221	6.3728	9.9924	65.2	- 0.025551	- 0.01918

10.0	1.1537	1.3309	1.7706	2.7153	4.1604	8.4549	10.66	0.1517	- 0.1526
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Table 3: The curve fitted values of dose gamma ray buildup factor of Pb

E (MeV)	Thickness (mfp)						A <sub>1</sub>	α <sub>1</sub>	α <sub>2</sub>
	1	2	4	7	10	15			
0.5	1.149	1.2846	1.5189	1.7895	1.9787	2.1532	16.16	0.03429	0.04684
1.0	1.2976	1.5742	2.0736	2.7173	3.2712	4.0751	- 2.184	0.1076	- 0.02321
4.0	1.1335	1.2849	1.6508	2.4041	3.5011	6.5499	7.241	- 0.1272	- 0.1275
10.0	1.30	1.69	2.86	6.27	13.78	51.16	0.498	-0.2624	-0.2624

Table 4: Comparison of dose gamma ray buildup factor values for Al with the reference value

E (MeV)	Work	Thickness (mfp)					
		1	2	4	7	10	15
0.5	Present work	1.30	1.69	2.82	5.87	11.54	26.71
	Ref.	(2.37)	(4.24)	(9.47)	(21.5)	(38.9)	(80.80)
1.0	Present work	1.31	1.73	2.97	6.71	15.11	57.85
	Ref.	(2.02)	(3.31)	(6.57)	(13.1)	(21.2)	(37.90)
4.0	Present work	1.16	1.35	1.80	2.73	4.01	6.76
	Ref.	(1.53)	(2.08)	(3.22)	(5.01)	(6.88)	(10.1)
10.0	Present work	1.26	1.53	2.08	2.95	3.89	5.59
	Ref.	(1.28)	(1.55)	(2.12)	(3.01)	(3.96)	(5.63)

Table 5: Comparison of dose gamma ray buildup factor values for Fe with the reference value

E (MeV)	Work	Thickness (mfp)					
		1	2	4	7	10	15
0.5	Present work	1.21	1.48	2.18	3.89	6.96	18.31
	Ref.	(2.07)	(2.94)	(4.87)	(8.31)	(12.4)	(20.6)
1.0	Present work	1.23	1.52	2.29	4.18	7.49	18.44
	Ref.	(1.92)	(2.74)	(4.57)	(7.81)	(11.6)	(18.9)
4.0	Present work	1.44	1.90	2.89	4.52	6.37	9.99
	Ref.	(1.48)	(1.90)	(2.95)	(4.61)	(6.46)	(9.92)
10.0	Present work	1.15	1.33	1.77	2.72	4.16	8.45
	Ref.	(1.22)	(1.44)	(1.95)	(2.89)	(4.07)	(6.7)

Table 6: Comparison of dose gamma ray buildup factor values for Pb with the reference value

E (MeV)	Work	Thickness (mfp)					
		1	2	4	7	10	15
0.5	Present work	1.149	1.28	1.52	1.79	1.98	2.15
	Ref.(1)	(1.24)	(1.39)	(1.63)	(1.87)	(2.08)	(2.20)
1.0	Present work	1.30	1.57	2.07	2.72	3.27	4.08
	Ref.(1)	(1.38)	(1.68)	(2.18)	(2.80)	(3.40)	(4.20)
4.0	Present work	1.13	1.28	1.65	2.40	3.50	6.55
	Ref.(1)	(1.28)	(1.56)	(2.18)	(3.29)	(4.69)	(7.70)
10.0	Present work	1.30	1.69	2.86	6.27	13.78	51.16
	Ref.(1)	(1.11)	(1.24)	(1.54)	(2.27)	(3.54)	(7.70)
	Ref.(2)	(1.51)	(2.01)	(3.42)	(7.37)	(15.4)	(50.8)

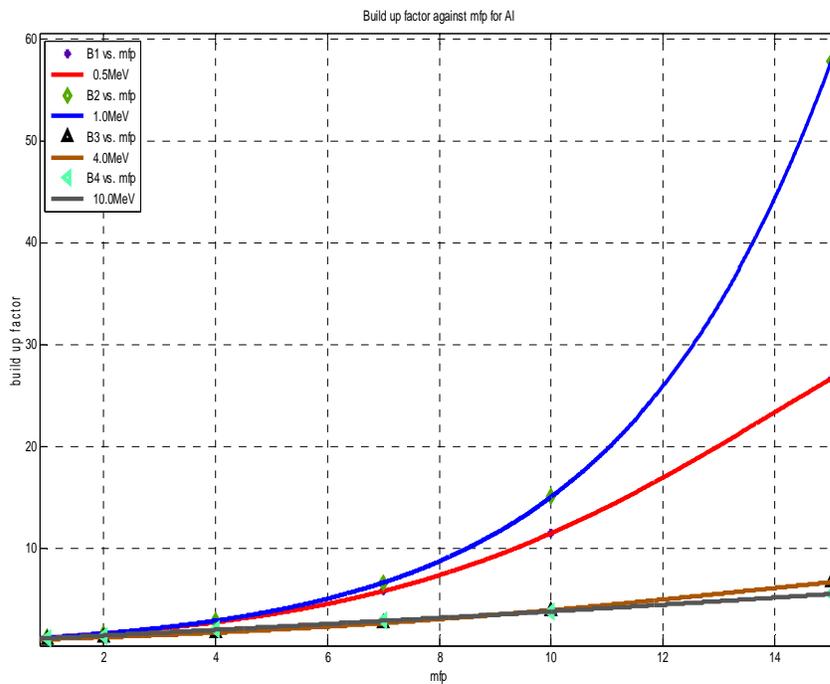


Figure 1: graph of buildup factor against mfp for Al

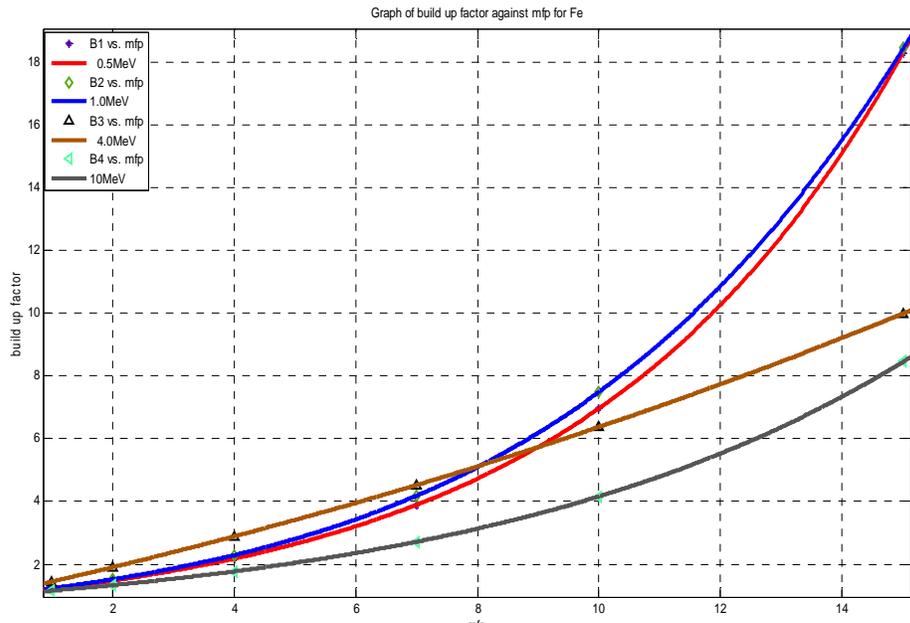


Figure 2: graph of buildup factor against mfp for Fe

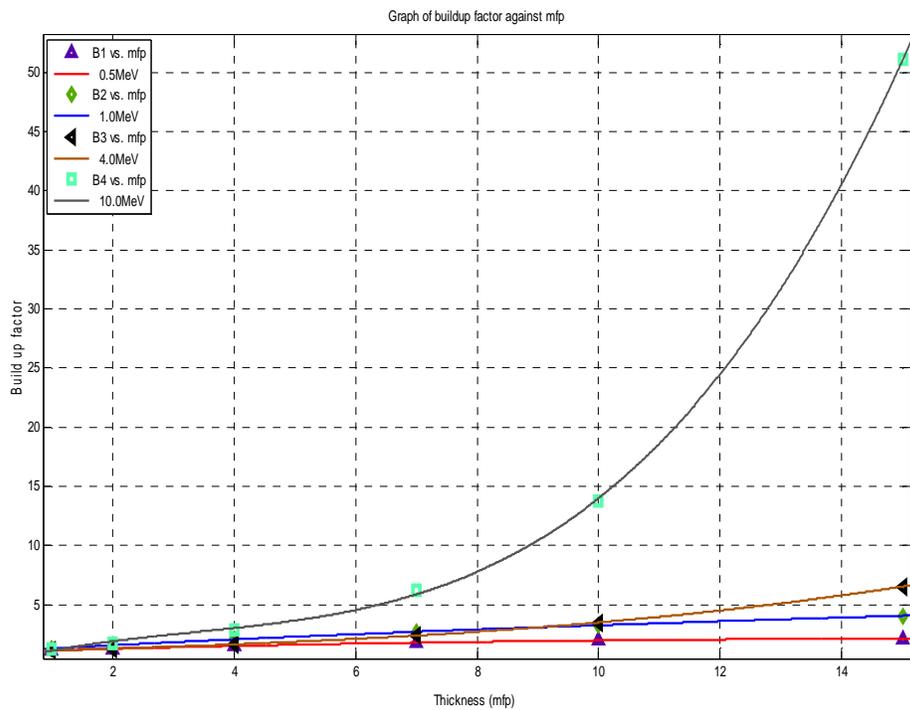


Figure 3: The graph of buildup factor against mfp for Pb

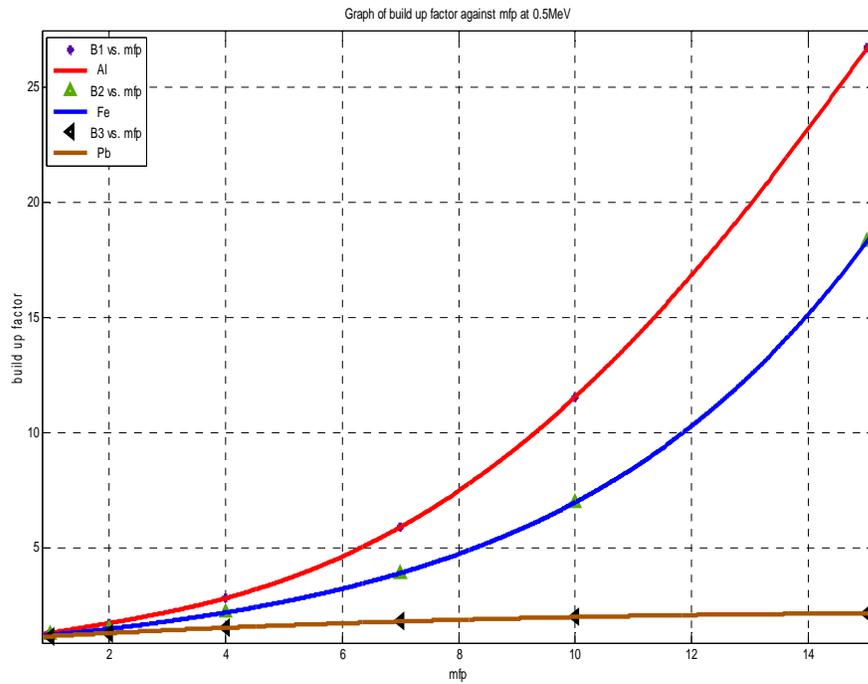


Figure 4: The graph of buildup factor against mfp at 0.5MeV

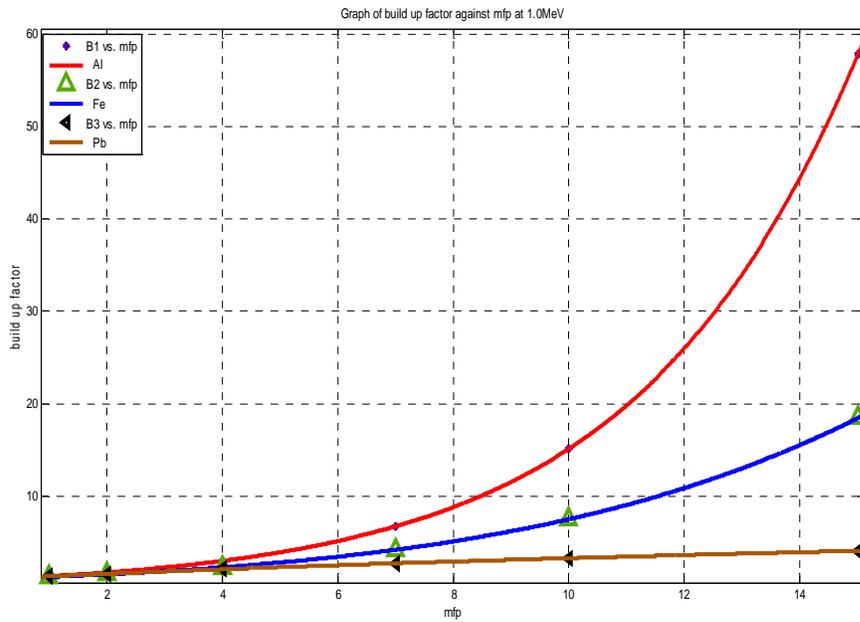


Figure 5: graph of buildup factor against mfp at 1.0MeV

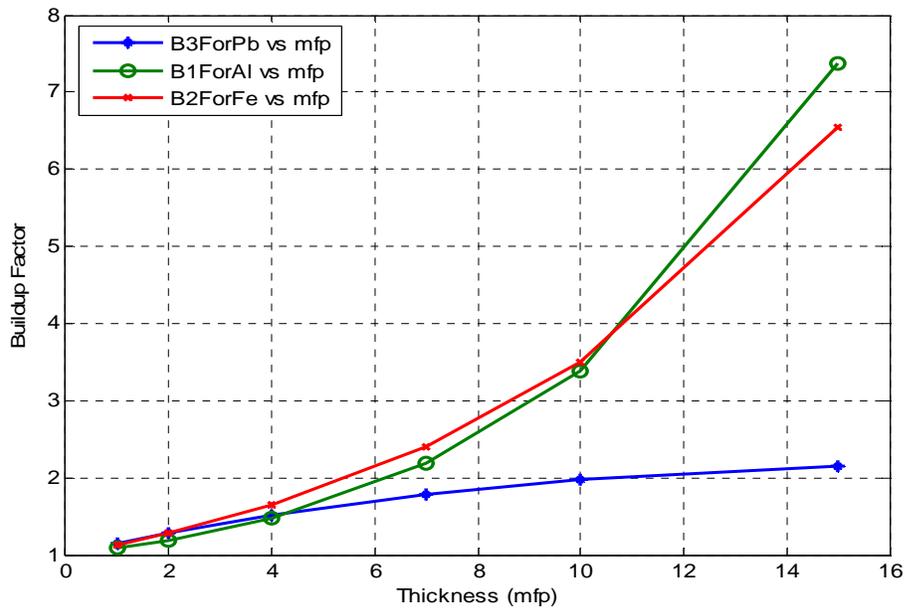


Figure 6: The graph of buildup factor against mfp at 4.0MeV

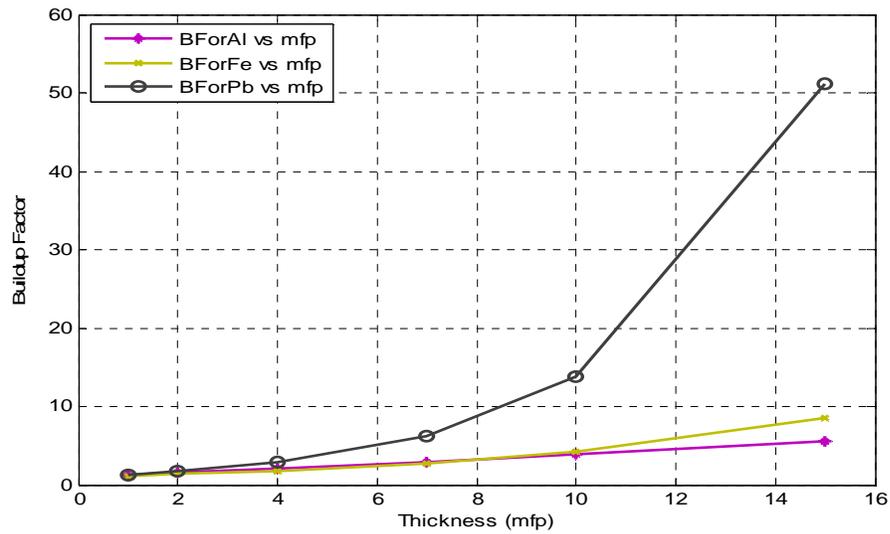


Figure7: The graph of buildup factor against mfp at 10.0MeV

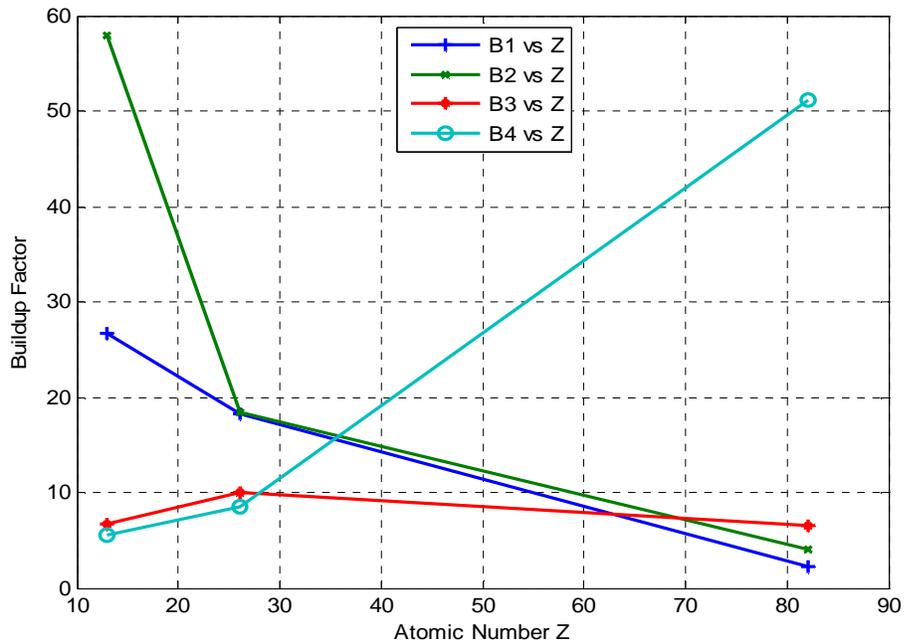


Figure 8: The graph of buildup factor against atomic number

#### 4.0 Conclusion

The build-up factor for gamma rays was calculated for some materials (*Al, Fe, and Pb*) in a single layer shield using Taylor's method for analyzing the narrow- and broad-beam geometry equations. The dose build-up factor of gamma ray increases with increasing thickness of the selected materials. The dose build-up factor of low atomic number  $Z$  is higher than that of high  $Z$  at energies below 4.0MeV. At 10.0MeV the dose build-up factor for high atomic number,  $Z$  is higher than that of low  $Z$  apparently due to the competing gamma interaction mode of Compton and pair production effect. The results obtained in this work compared well with what obtained in the literature [2, 3]. Although Taylor's method was employed in this work, the present model enables theoretical estimation of build-up factor for materials with various atomic numbers at different energies and mean free path. The paper therefore elucidates a novel approach to the calculation of build-up factor for materials in the design of an appropriate shielding material for radiation protection, absolute volume and activity measurement and in beam transport studies.

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