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The Radio And Very Low Frequency (VLF) Electromagnetic Response Of A Layered Earth Media With Variable Dielectric Permittivity

V. C. Ozebo and J. A. Olowofela

Department of Physics, University of Agriculture,
Abeokuta, Nigeria

Abstract

A set of electromagnetic sounding models was formulated using earth parameters such as conductivity, dielectric permittivity, permeability, surface wave impedance; adopting the Maxwell's equation as the basis and assuming the earth to be a layered media.

The amplitude of the modification factor to the wave tilt, Q, the phase of Q and the reflection coefficient, are the computed response parameters studied over a stratified earth media with variable dielectric permittivity. A radio frequency of 125 KHz and a very low frequency (VLF) of 20 KHz were used in the computations and the field parameters studied over a dimensionless induction number, B.

The application of the radio frequency over a two-layer media with conductive basement, revealed that the dielectric permittivity has greater influence on the response parameters, compared with the case when a very low frequency (VLF) was used. However, a remarkable influence was observed with both frequencies over a resistive basement.

Keyword: wave tilt, dielectric permittivity, contaminant, layered earth

1.0 Introduction

Information on the dielectric permittivity of the earth may be useful in problems of electric power transmissions, geological mapping and environmental pollutions. It has been reported that the dielectric permittivity mapping can be particularly important for hazardous waste site characterization because some contaminants may have little effect on the observed resistivity but large effect on the observed permittivity ([4] and [6]). In the recent times, Scholars have dealt with or recognized the existence of dielectric permittivity in electromagnetic data. [8] showed graphically, a means of obtaining resistivity and dielectric permittivity from plane-wave electromagnetic fields. [9] studied the effect of dielectric properties on layered earth measurements for high-frequency ground electromagnetic system. [1], [2] and [3] also carried out similar studies.

In our earlier work [7], we studied the behaviour of some electromagnetic response parameters over homogeneous and layered earth media using both radio and low frequencies. Detailed references were made to similar work done by various scholars, which also serve as a guide to the present study.

1.1. Objective of the study

In consonant with the new world order of pollution – free environment, the work aims at formulating a set of models from earth parameters, and to apply these models over a layered earth media and see the possible influence of variations of dielectric permittivity on the response parameters. Since a layer containing contaminants is better assumed to be dielectric, the study of the response curves from the models may be useful in delineating hazardous waste sites and may help in understanding the characteristics of buried objects or contaminants in such an environment.

Corresponding authors **e-mail:**

2. Materials and methods

The formulation of the theory of the model was made using the Maxwell's equation as the basis. Then, applying the boundary conditions over a stratified medium, the components of both the electric and magnetic fields were derived. These field components aided the computation of some of the field parameters such as the surface impedance, surface admittance, wave tilt, and the reflection coefficient (in both the transverse magnetic and transverse electric modes). These field parameters were used in the formulation of the models, using a very low frequency (VLF) value of 20 kilo Hertz; and then applied over a stratified half space and a homogeneous ground. The modification factor of the wave tilt and its phase, and the reflection behaviour of the wave were studied over a two-layered medium, for various values of conductivity contrasts and dielectric permittivity of the layers. This can however be extended to any number of layers.

2.1 The Formulation of the Model

Consider a plane wave with a time factor $e^{j\omega t}$ incident at an angle θ on a stratified medium of M-layers, where the electric field vector is in the plane of incidence (x-z plane) as shown in figure 1.

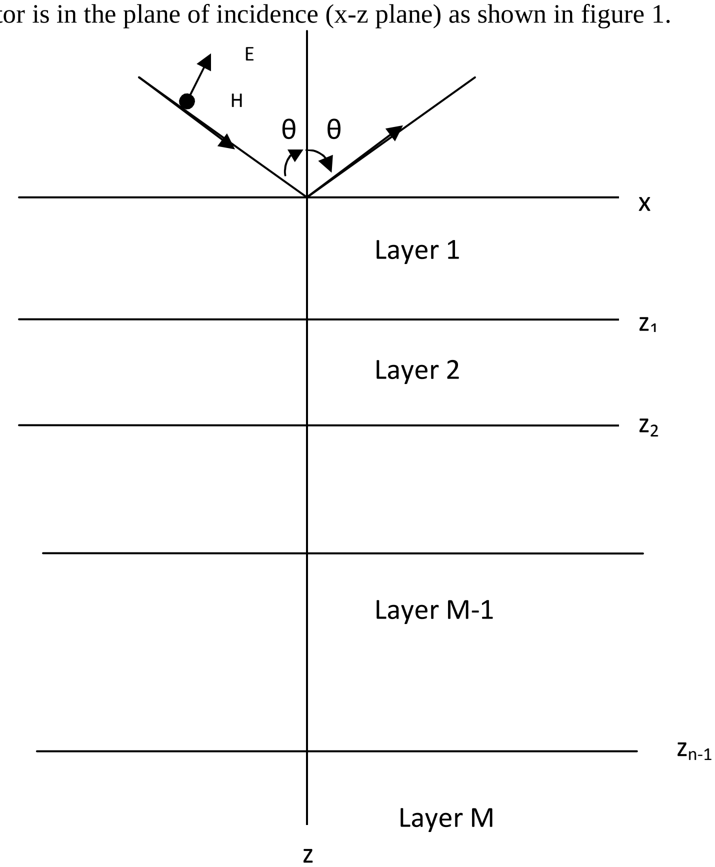


Figure 2.1 Coordinate system for the stratified medium of M-layers.

The magnetic field has only y-component labeled as H_{my}, which is also the solution of the equation:

$$\left(\nabla^2 - \gamma_m^2\right)H_{my} = 0 \tag{2.1}$$

where

$$\gamma_m^2 = j\sigma_m\mu_m\omega - \varepsilon_m\mu_m\omega^2$$

with the real part of γ greater than zero. σ_m, μ_m and ε_m are the electrical conductivity, magnetic permeability and the dielectric constant respectively, of the mth layer and ω is the angular frequency, γ is the wave number.

The general solution of equation (2.1) is of the form;

$$H_{my} = \left[a_m e^{-u_m z} + b_m e^{u_m z}\right] e^{-j\lambda x} \tag{2.2}$$

where

$$u_m^2 = \lambda^2 + \gamma_m^2$$

λ can take any value and $\mathrm{Re}(u_m) \rangle$ 0

Using the Maxwell's equations and derivations from [5], as outlined in [7], the electric field component is given by

$$E_{mz} = \left(j\omega\varepsilon_m + \sigma_m\right)^{-1} \frac{\partial H_{my}}{\partial x} \tag{2.3}$$

The three response parameters: the reflection coefficient in both the transverse magnetic (R_{TE}) and transverse electric (R_{TM}) modes and modification factor to the wave tilt Q, are expressed as follows (detailed derivations in [7]):

$$R_{TM} = \frac{b_0}{a_0} = \frac{K_0 - Z_1}{K_0 + Z_1} \tag{2.4}$$

where,

$$Z_1 = K_1\left(\frac{Z_2 + K_1 \tanh u_1 h_1}{K_1 + Z_2 \tanh u_1 h_1}\right) \tag{2.5}$$

$$Z_2 = K_2\left(\frac{Z_3 + K_2 \tanh u_2 h_2}{K_2 + Z_3 \tanh u_2 h_2}\right) \tag{2.6}$$

$$Z_{M-1} = K_{M-1}\left(\frac{Z_M + K_{M-1} \tanh u_{M-1} h_{M-1}}{K_{M-1} + Z_M \tanh u_{M-1} h_{M-1}}\right) \tag{2.7}$$

$$K_m = \frac{u_m}{\sigma_m + j\omega\varepsilon_m} \tag{2.8}$$

and,

$$Z_M = K_m \tag{2.9}$$

$$R_{TE} = \frac{\overline{b_0}}{\overline{\overline{a_0}}} = \frac{N_0 - Y_1}{N_0 + Y_1} \tag{2.10}$$

where,

$$Y_m = N_m\left(\frac{Y_{m+1} + N_m \tanh u_m h_m}{N_m + Y_{m+1} \tanh u_m h_m}\right)$$

For m = 1, 2, 3, …. M-1 (2.11)

With,

$$Y_M = N_M = \frac{u_M}{j\omega\mu_M} \tag{2.12}$$

$$Q = \frac{K_2 + K_1 \tanh \gamma_1 h_1}{K_1 + K_2 \tanh \gamma_1 h_1} \tag{2.13}$$

The Induction number I_N, is also defined as

$$I_N = (\sigma_1 \mu_0 \omega)^{\frac{1}{2}}.h_1 \tag{2.14}$$

2.2. The model parameters

The model parameters are as shown in table 1.

Table 1. Model Parameters

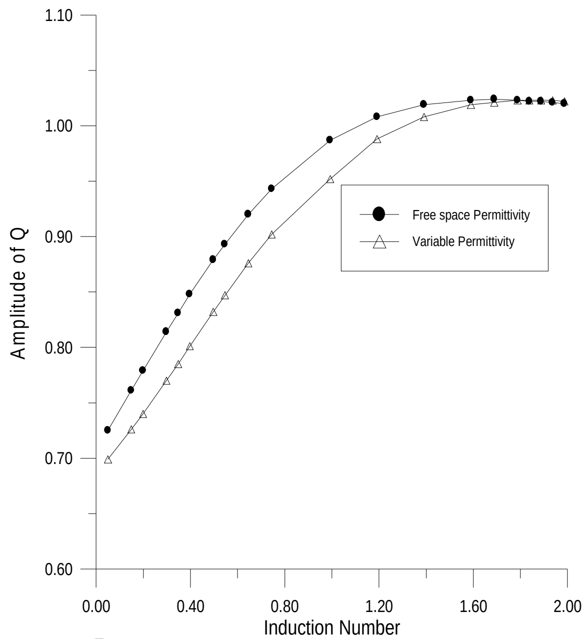
Parameters	Model A	Model B
Frequency	125 *kHz*	20 *kHz*
Upper layer conductivity, σ_1	0.01 mho m^{-1}	0.01 mho m^{-1}
Dielectric constant of air, ε_0	8.854 x 10^{-12} $F\ m^{-1}$	8.854 x 10^{-12} $F\ m^{-1}$
Magnetic permeability, μ_0	4π x 10^{-7} $H\ m^{-1}$	4π x 10^{-7} $H\ m^{-1}$
Range of the upper layer thickness, h_1	2.5 – 50 *m*	2.5 – 50 *m*

We set to study the behaviour of the response parameters when $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$, (free space permittivity) and when $\varepsilon_1 = 100\varepsilon_0$ and $\varepsilon_2 = 1000\varepsilon_0$ (variable permittivity) over various conductivity contrast, $k = \frac{\sigma_2}{\sigma_1}$, of the layers. The response parameters: modification factor to the wave tilt, Q; the phase of Q and the reflection coefficient will be plotted as functions of a dimensionless induction number, using the two frequency values stated above.

3. RESULTS

3.1 Conductive basement

Model A **Model B**



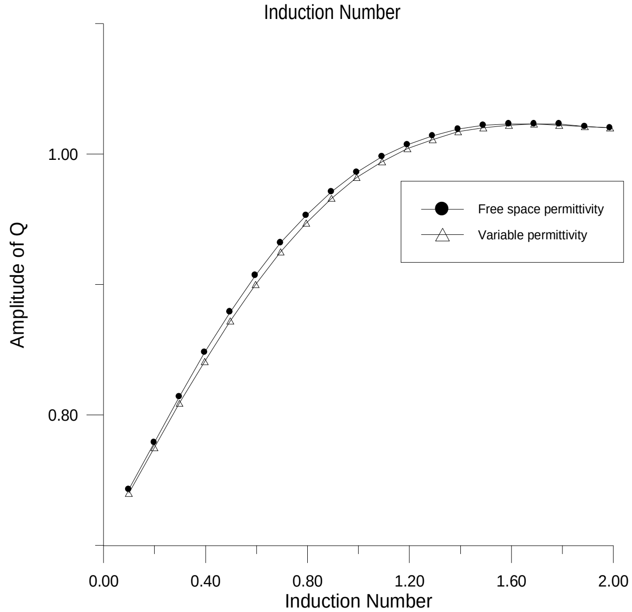
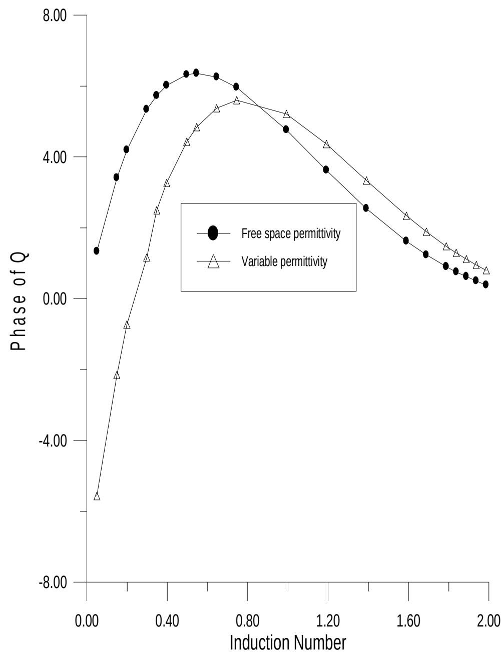


Figure 2. Plot of the Amplitude of Q versus Induction Number (k = 2)

Model A **Model B**



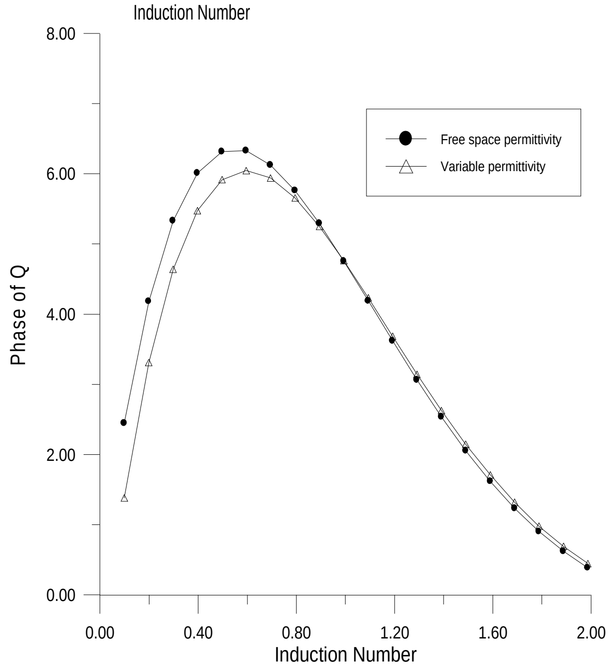
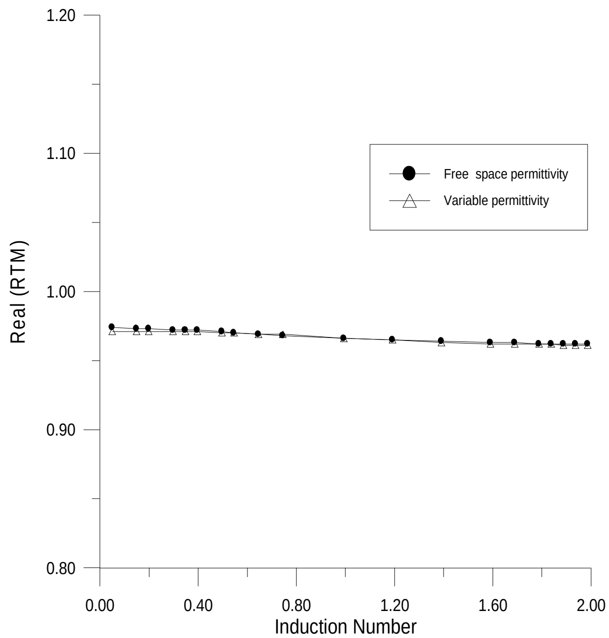
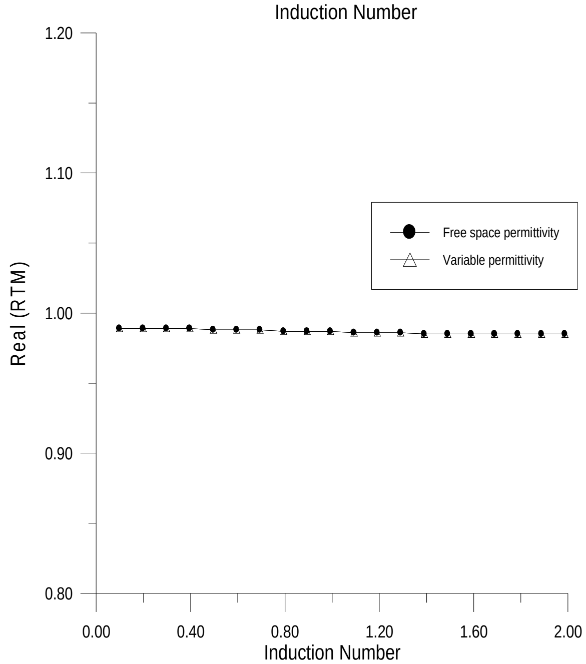


Figure 3: Plot of the Phase of Q versus Induction Number (k = 2)

Model A **Model B**


1.20
1.10
1.00
0.90
0.80
Real (RTM)
Free space permittivity
Variable permittivity
0.00
0.40
0.80
1.20
1.60
2.00
Induction Number


1.20
1.10
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Real (RTM)
Free space permittivity
Variable permittivity
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Induction Number

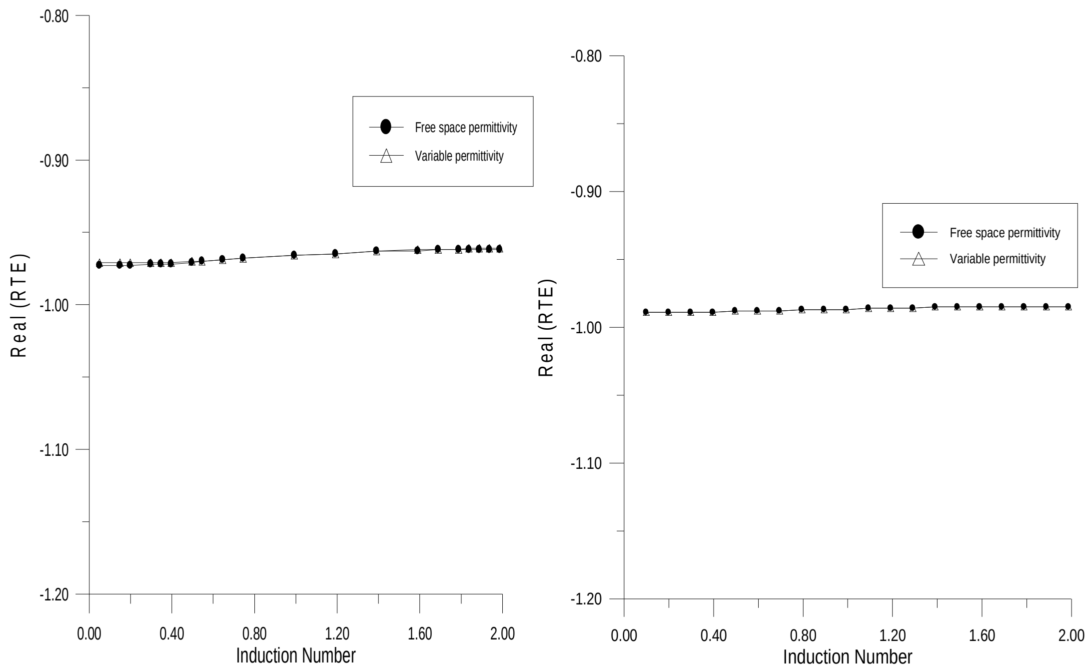


Figure 4. Plots of the Reflection coefficient (TM & TE mode) versus Induction
(k = 2)

Model A **Model B**

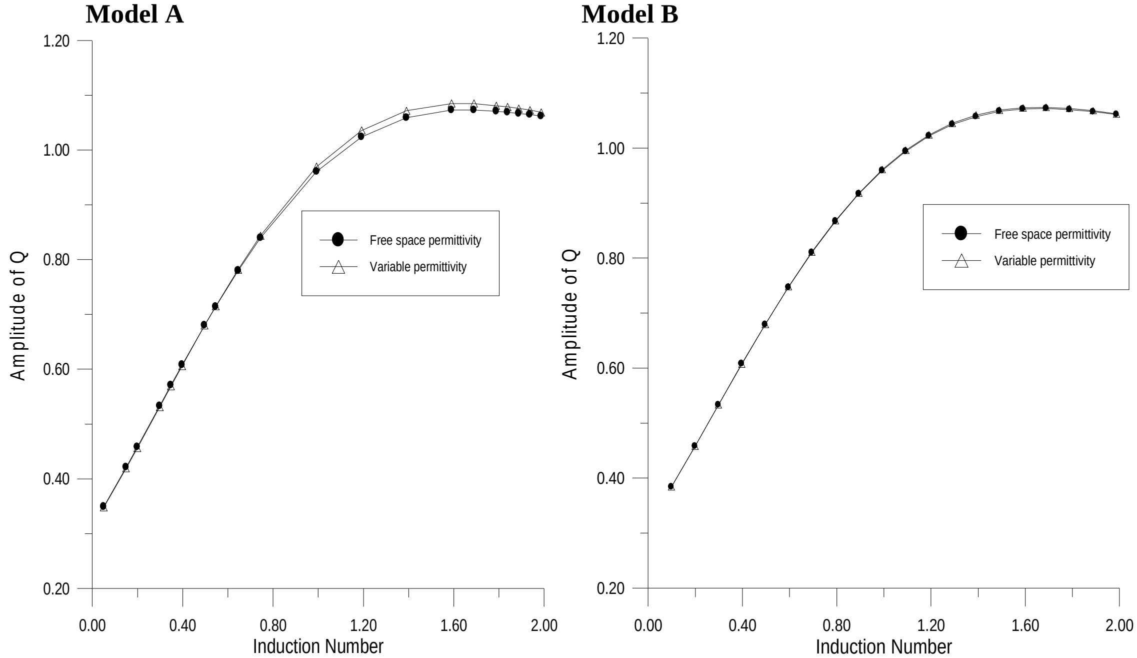
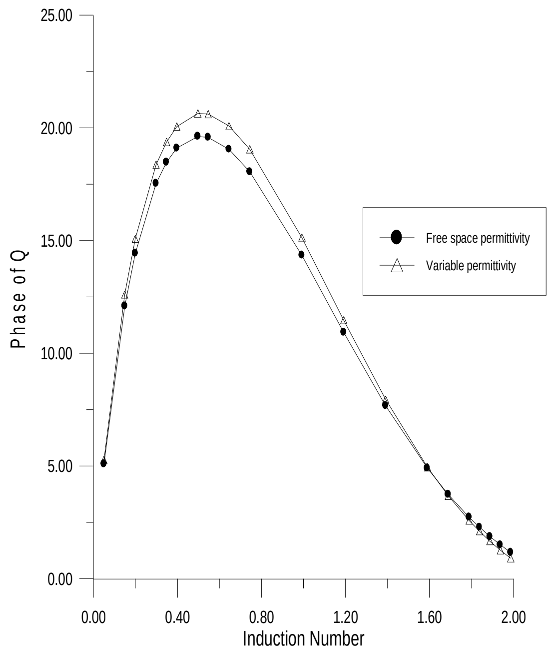


Figure 5. Plot of the Amplitude of Q versus Induction Number (k = 10)

Model A **Model B**



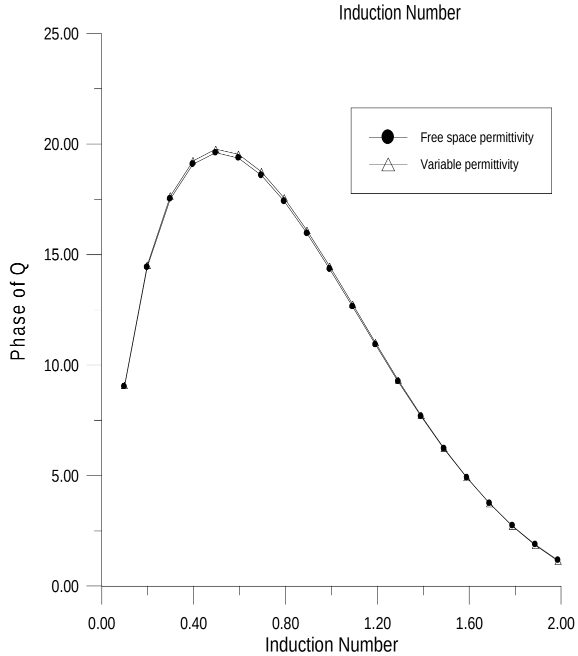
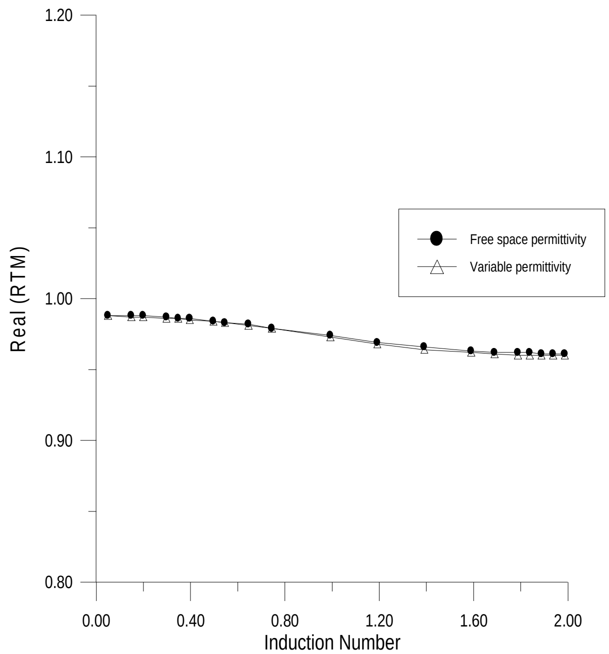


Figure 6. Plot of the Phase of Q versus Induction Number (k = 10)

Model A **Model B**


1.20
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Real (RTM)
Free space permittivity
Variable permittivity
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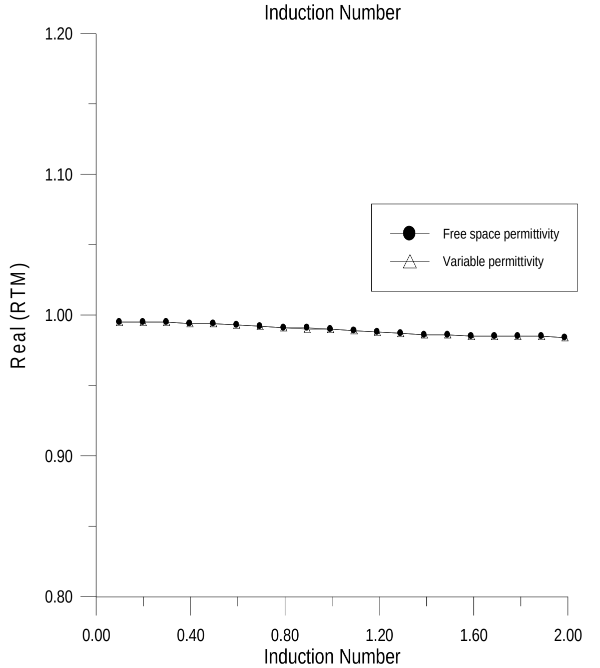
1.60
2.00
Induction Number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Free space permittivity
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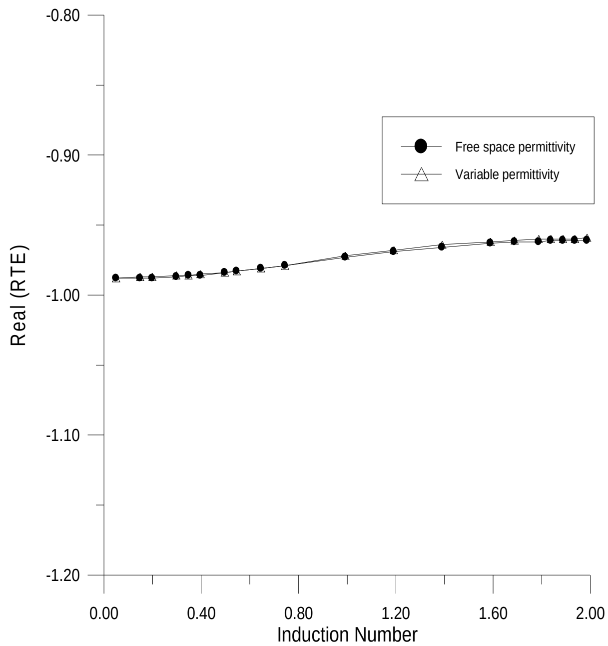
Induction Number



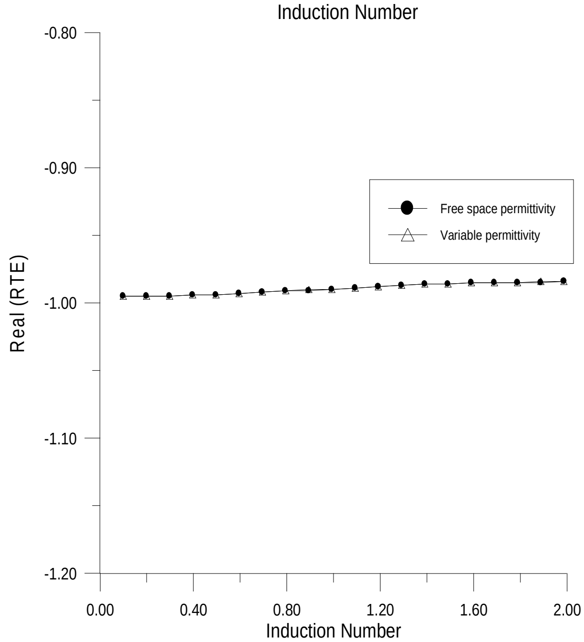


Figure 7. Plots of the Reflection coefficient (TM & TE mode) versus Induction (k = 10)

Model A **Model B**

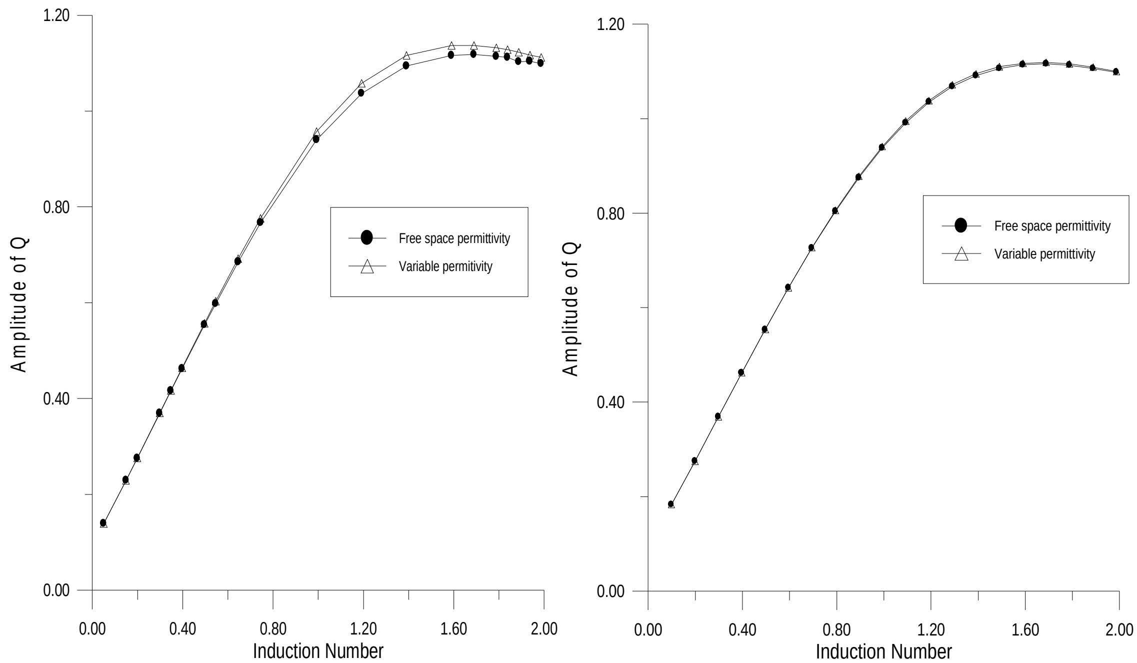


Figure 8. Plots of the Amplitude of Q versus Induction Number (k = 100)

Model A **Model B**

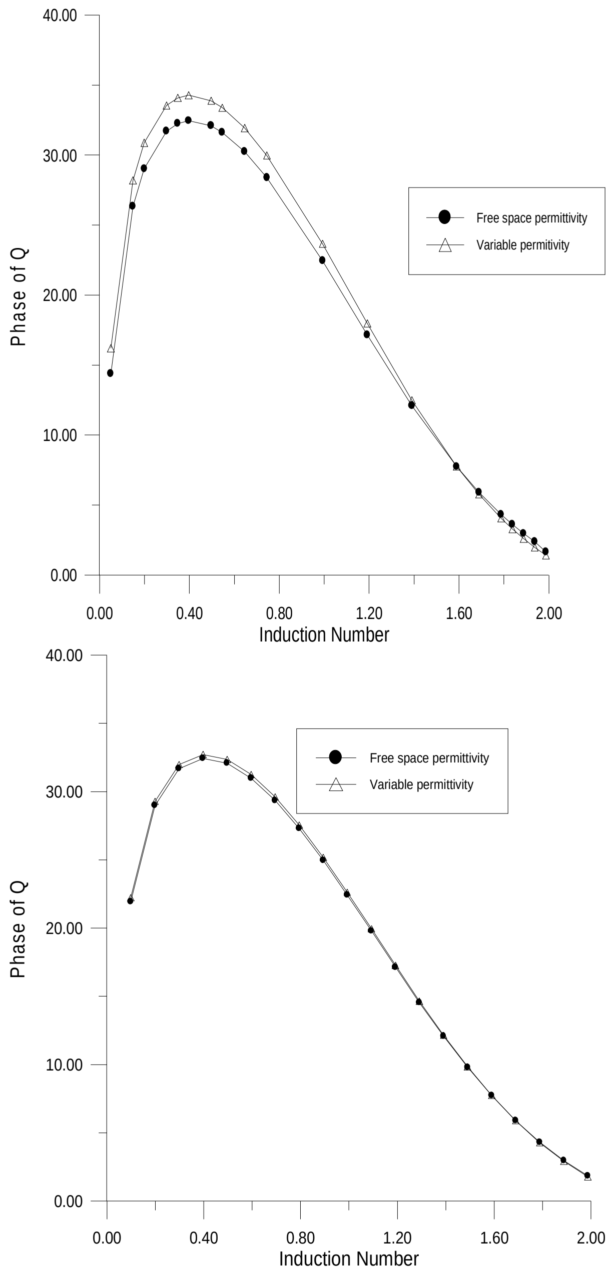
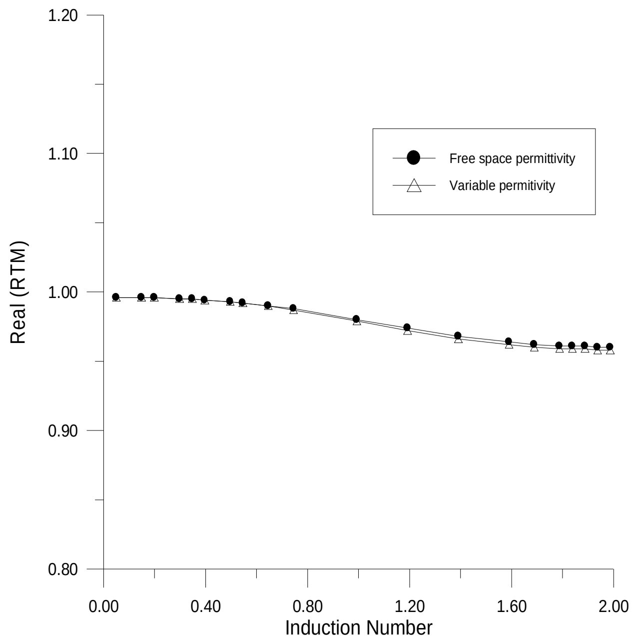
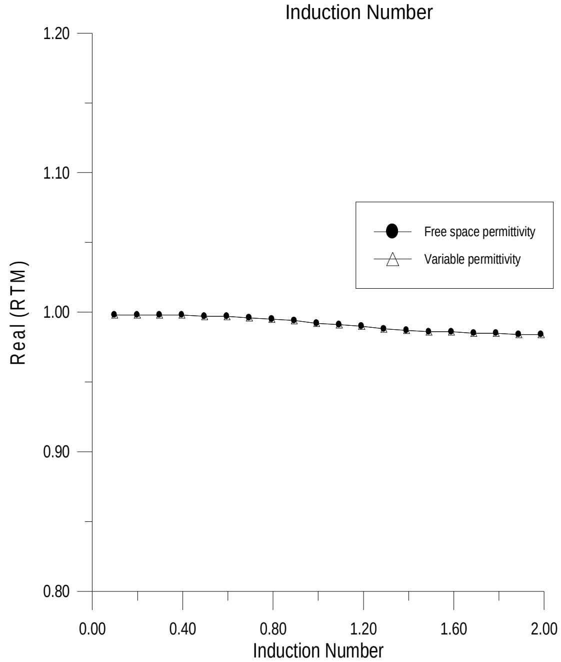


Figure 9. Plot of the Phase of Q versus Induction Number (k = 100)

Model A **Model B**





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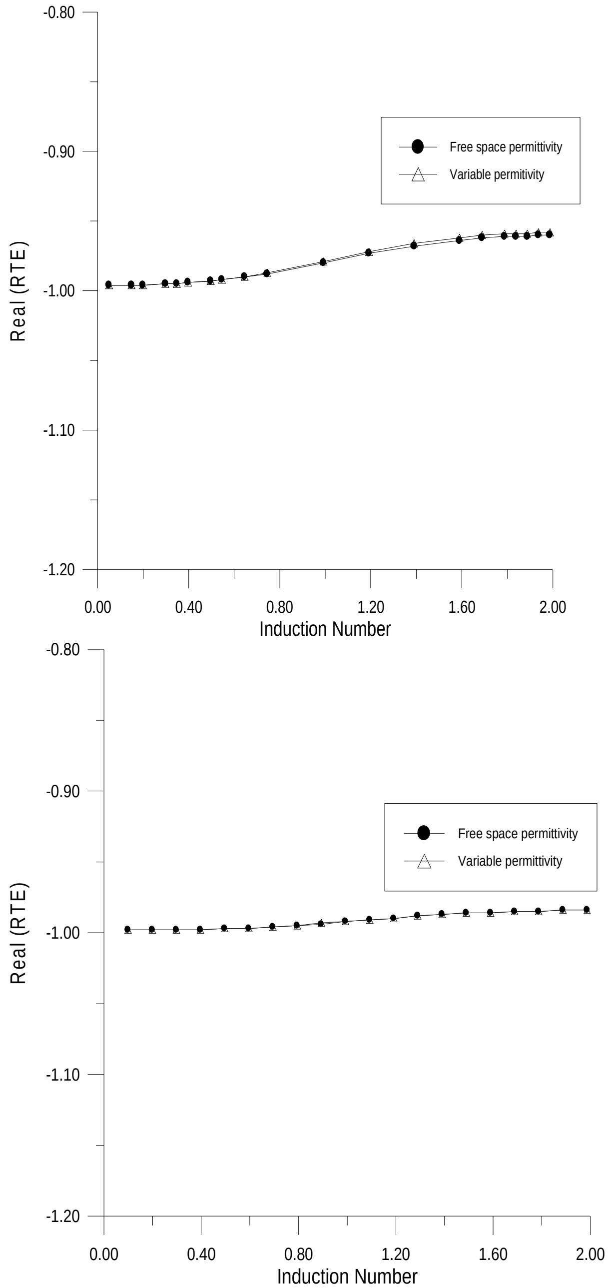


Figure 10. Plots of the Reflection coefficient (TM & TE mode) versus Induction (k = 100).

2.2. Resistive basement

Model A **Model B**

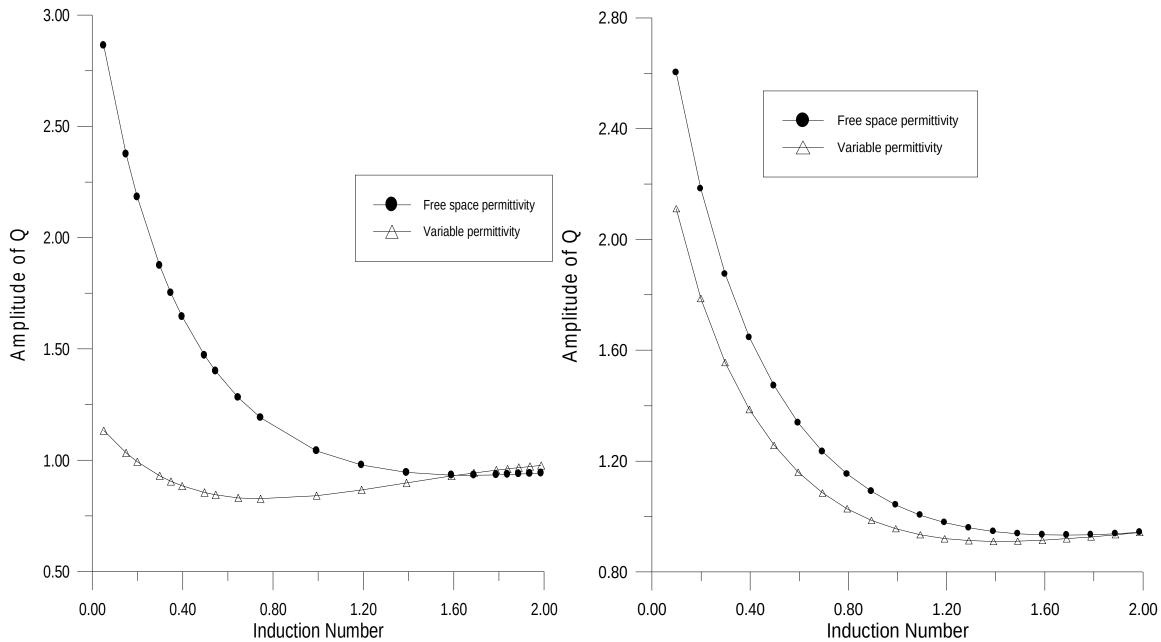
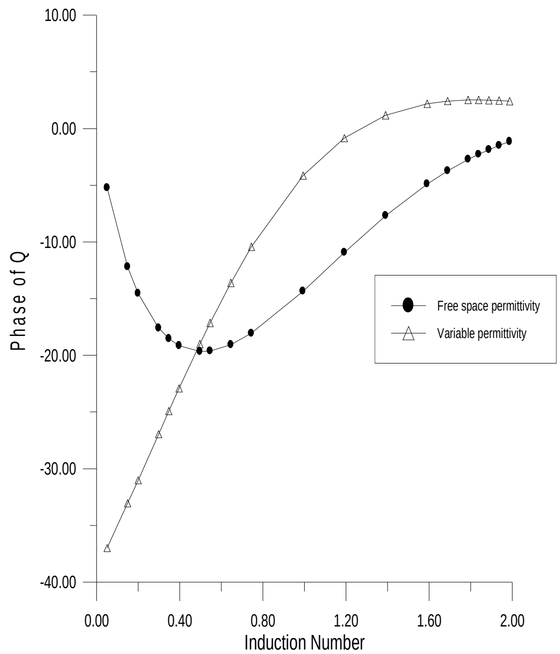


Figure 11. Plot of the Amplitude of Q versus Induction Number (k = 0.1)

Model A **Model B**



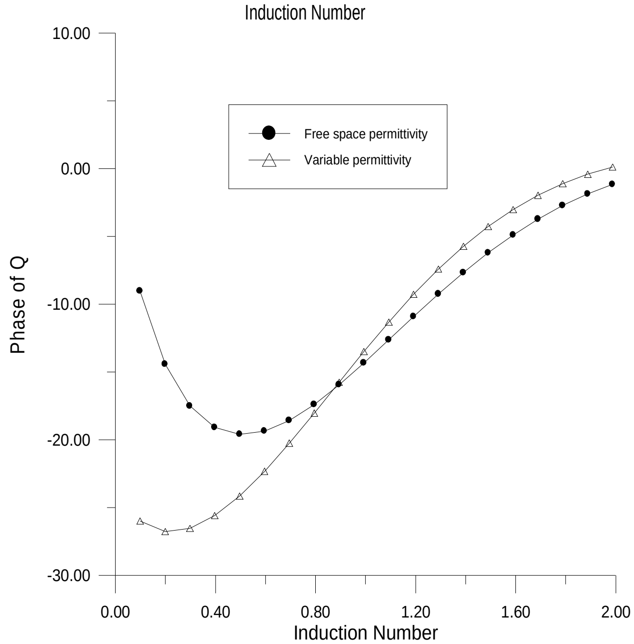
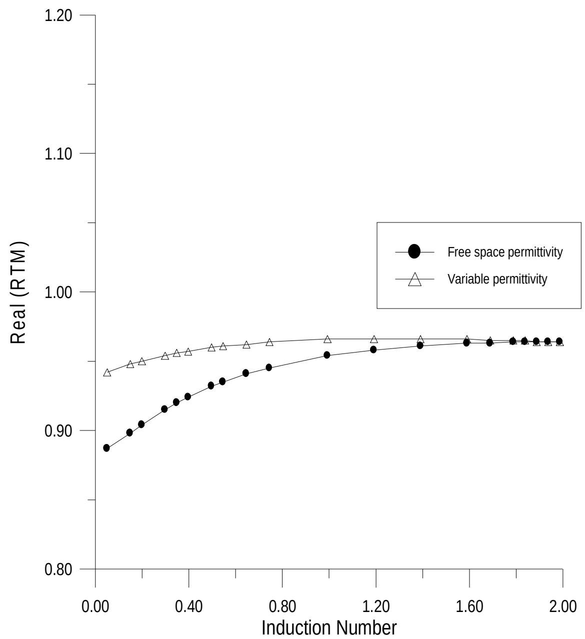


Figure 12. Plot of the Phase of Q versus Induction Number (k = 0.1)

Model A **Model B**



1.20
1.10
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0.90
0.80
Real (RTM)
Free space permittivity
Variable permittivity
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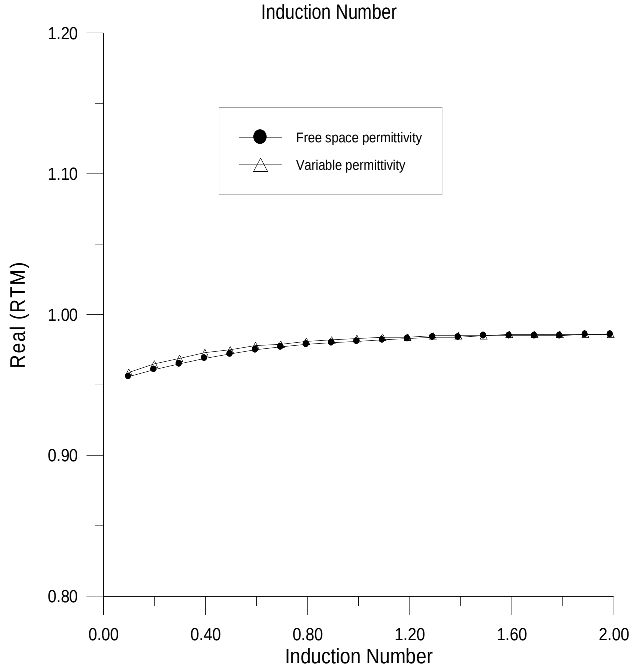
1.20
1.60
2.00
Induction Number



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Real (RTM)
Free space permittivity
Variable permittivity
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Induction Number

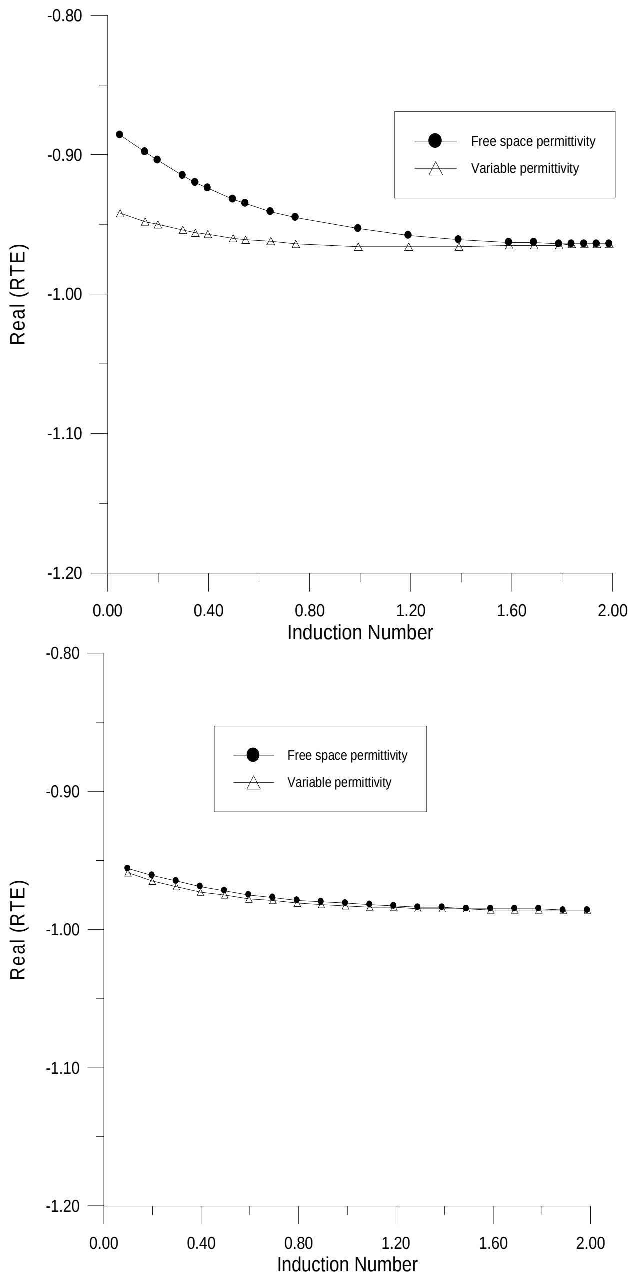
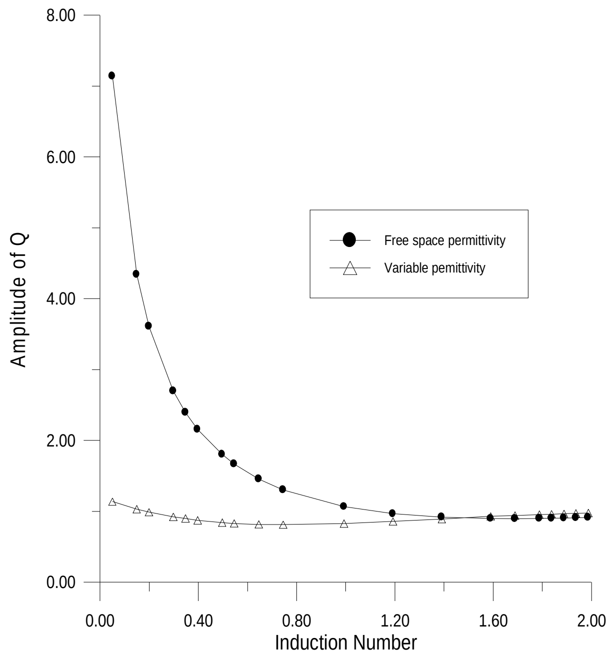


Figure 13. Plots of the Reflection coefficient (TM & TE mode) versus Induction (k = 0.1)

Model A **Model B**



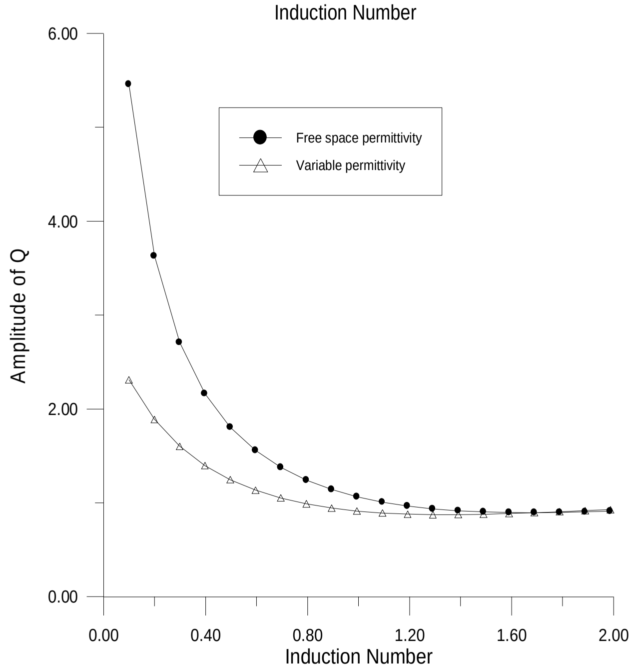


Figure 14. Plots of the Amplitude of Q versus Induction Number (k = 0.01)

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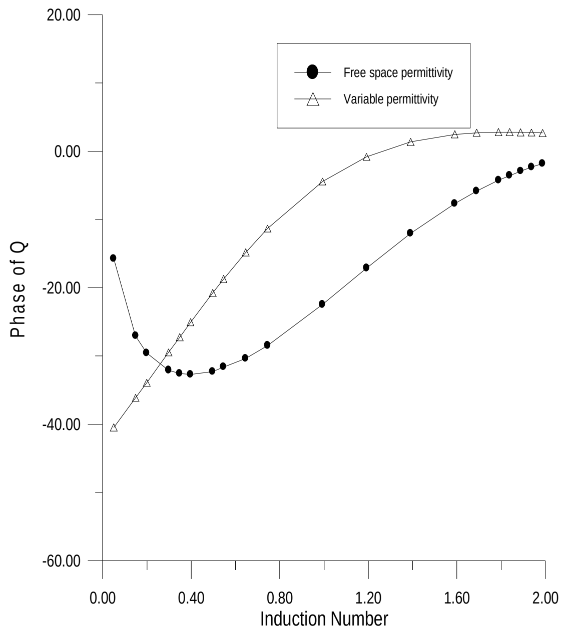
Model A **Model B**



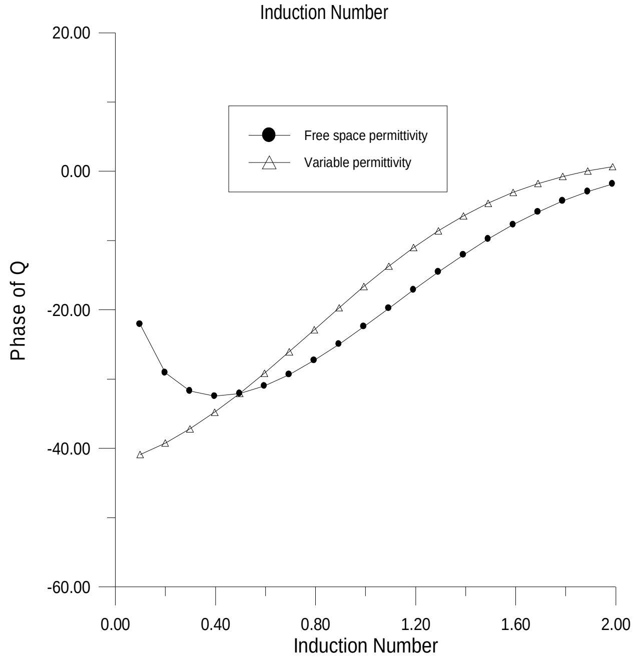
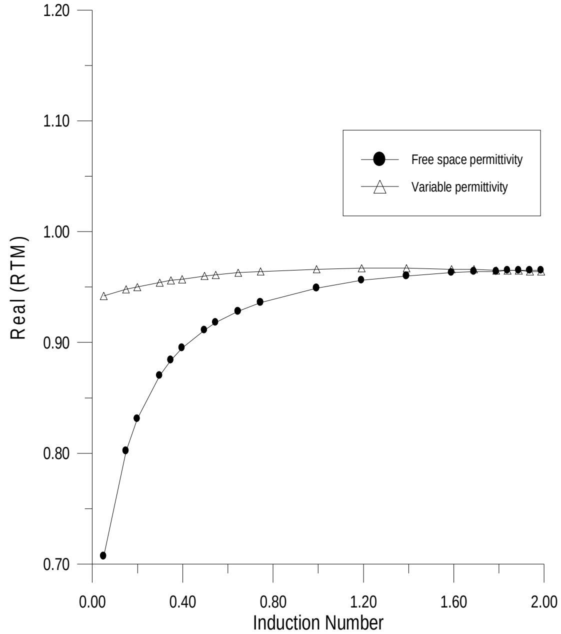


Figure 15. Plots of the Phase of Q versus Induction Number (k = 0.01)

Model A **Model B**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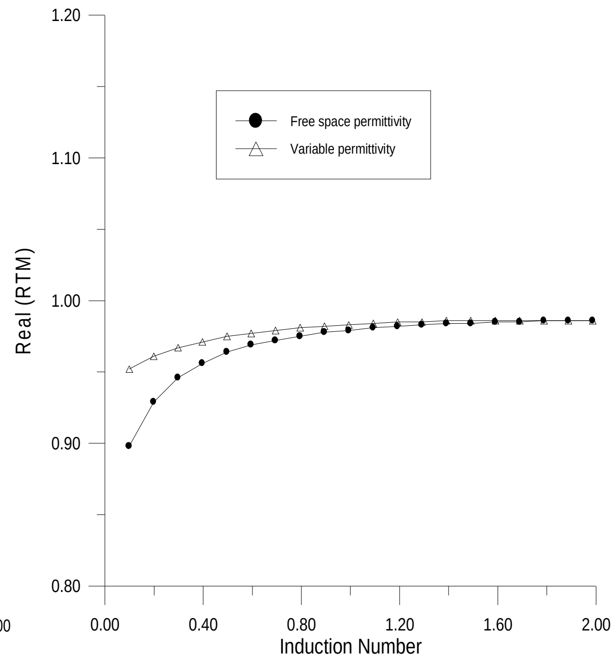
Induction Number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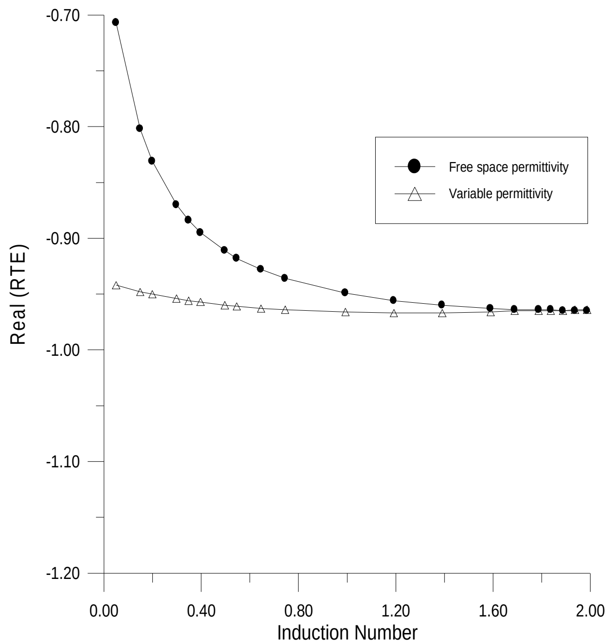
Induction Number



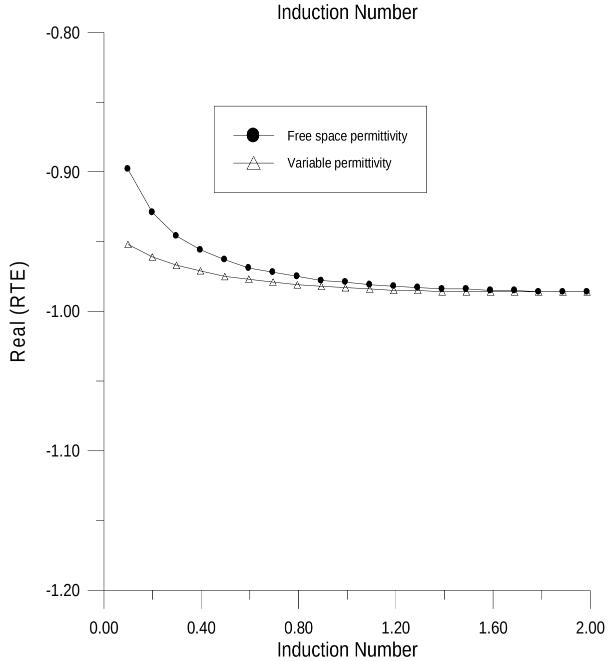


Figure 16. Plots of the Reflection coefficient (TM & TE mode) versus Induction
(k = 0.01)

4. Discussion of results

4.1 Conductive basement

When the value of the conductivity contrast, k is 2, figures 2 to 4 depict the response curves for amplitude of Q, phase of Q and the two modes of reflection coefficient respectively (transverse magnetic, TM and transverse electric, TE). In a conductive basement, the variation in the permittivity of the layers produces little effect on response parameters with the exception of the phase of Q, when a very low frequency (VLF) of 20 kHz is applied (Model B). It is obvious from the three figures that model **A** shows a better resolution of the curves in each figure, especially at lower values of the induction number. Again, the phase of Q seems the best diagnostic parameter while the two modes of the reflection coefficients are mirror image of each other. It can also be observed that for the variable permittivity case, the phase shift is reduced compared to the free space permittivity considerations.

.Figures 5 through 7 show the response curves when the value of k is 10. The resolution of the curves is observed to diminish compared to when k is 2. Model A still maintain a better resolution of the curves compared to model B. However, a higher value of the phase shift was achieved with a variable permittivity situation. The resolution of the two modes of reflection coefficient is observed to be poor but they still maintained the mirror image relationship.

When the value of k is 100, the response curves of the field parameters are shown in figures 8 through 10. The only obvious difference with the case when k is 10 seemed to be in the value of the phase shift which is higher here. In all the cases considered above (conductive basement), model A (with radio frequency of 125 kilo Hertz) tends to exhibit high resolution of the curves

4.2Resistive Basement

The response curves of the layers in a resistive basement show a high level of resolution Figures 11 through 13 show the case when the value of k is 0.1. The curves of the amplitude of Q, phase of Q and the two modes of reflection coefficient show remarkable differences between the two permittivity considerations. In particular, curves of the phase of Q exhibits great distinctions for the two cases under study (see figure 12). The two curves of the two modes of the reflection coefficient are easily distinguishable from each other especially with the radio frequency value shown in Model A of figure 13.

When the value of k is 0.01, figures 14 through 16 depict the response curves. The curves in both models (A and B) show a high degree of resolution but with model A exhibiting a better resolution. A greater phase shift is always observed as the permittivity increases compared to free space situations.

In general, the modification to which these curves are subjected to when the variable permittivity case is considered becomes very obvious as the conductivity contrast, k reduces, that is, as the ground becomes more resistive. We can therefore infer that the application of the models will assist in delineating the character of a foreign body (buried objects or contaminants) in between the layers; that is, whether the body is resistive or conductive, from the nature of the response curves. Also, for shallow exploration of non conductive – ore bodies, the assumption that $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$ is not advisable.

5. Conclusions

Ideally, each stratum of the layered earth model is considered to be a conducting medium however, some real life situations suggest otherwise. This may be attributed to the presence of some hazardous wastes or contaminants in one or more of the layers.

The set of models in A and B reveal in the first instance, that dielectric permittivity has observable influence on the response pattern of the field parameters in a layered medium. This effect becomes more obvious with a higher value of frequency as can be seen from the curves in Model A. In general, the influence of the dielectric permittivity on the response parameters becomes more pronounced on resistive media.

The models developed in this work can thus aid in delineating sites for hazardous wastes or contaminants as these have influence on observed permittivity.

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