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Application of the Radiative Transfer Equation (RTE) to Scattering by a Dust Aerosol Layer

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Abstract

Incident radiation in its journey through the atmosphere before reaching the earth surface encounters particles of different sizes and composition such as dust aerosols resulting in interactions that lead to absorption and scattering. The Radiative Transfer Equation (RTE) is one of the methods of analysis of how these interactions occur in the atmosphere. This paper explores the Radiative Transfer Equation and shows how it is used to describe the scattered radiative field at points in the atmosphere. A RTE model has been adapted for a dust aerosol layer and results of computations of dust aerosol reflectivity of down-welling radiation in a plane-parallel atmosphere for various incident angles using: Chandrashekhar Isotropic Scattering (CIS) model is presented.

Keywords: Dust Aerosol, Radiative Transfer, Plane-parallel Atmosphere

1.0 Introduction

Radiative transfer refers to the physical phenomena of energy transfer in the form of electromagnetic radiation. The propagation of electromagnetic waves through the atmosphere is affected by absorption, emission and scattering processes. The equation of radiative transfer describes these interactions mathematically. Equations of radiative transfer have application in wide variety of subjects including optics, astrophysics, atmospheric science, and remote sensing. Analytic solutions to the Radiative Transfer Equation (RTE) exist for simple cases but for more realistic media with complex multiple scattering effects numerical methods are often used.

Dust Aerosol and Interaction with Incident Radiation

Dust aerosols result from natural sources such as volcanic activities and dust storms. Anthropogenic sources include land use and agricultural activities. They are dominant in the lower atmosphere and have sizes ranging from 0.1 to 10 microns with lifetimes averaging two weeks [1]. Dust aerosols have a direct solar effect by scattering and absorbing solar radiation and a direct terrestrial effect where large-sized dust aerosols behave like greenhouse gases. Dust aerosols also have an indirect effect by altering cloud properties; changing their reflectivity, droplet size and lifetime. They also affect precipitation efficiency [2].

The Radiative Transfer Equation (RTE)

The Radiative Transfer Equation simply states that as a beam of radiation or electromagnetic waves travels, it looses energy to the atmosphere by absorption and gains energy by atmospheric emission, and redistributes energy by scattering. Solution of the Radiative Transfer Equations therefore enables one to describe the radiative field at each point in the atmosphere [3]. However, due to the complexity of the radiation mentioned earlier, the solution is not so straightforward and certain assumptions have to be made to arrive at the required solution.

2.0 Radiative Transfer without Absorption and Scattering

If a radiation propagates without absorption and scattering, conservation laws require that energy and power are conserved. Since in a homogeneous medium (free space) rays propagate along straight lines, then if ds is an
infinitesimal path length along the propagating ray path, one can write:

$$\frac{dI_\lambda}{ds} = 0$$

(2.1)

This is the RTE for free space propagation. It is independent of position and valid for all ray directions. If it is however assumed that the medium is slightly inhomogeneous but still scattering and absorption is not present (this is possible if the gradient of the real part of the refractive index of the propagating ray is very small and if the imaginary part is negligible) i.e.:

$$|\nabla n'| << k$$
$$n'' = 0$$

(2.2)

In such a case, the rays will follow the rules of Geometric Optics (Snell's Law, Fermat's Principle etc) [4]. The RTE remains as for free space.

Radiative Transfer with Absorption

If the medium were absorbing, then there will be power loss by absorption and by employing Kirchhoff's Law that the absorptivity of any quantity of matter in Local Thermodynamic Equilibrium is equal to the emissivity of the matter, one would expect a corresponding emissivity term. The change in intensity would then be:

$$dI_\lambda = dI_{abs} + dI_{emit}$$

(2.3)

The depletion due to absorption would be:

$$dI_{abs} = -K_{abs} B(T)\, ds$$

(2.4)

Where: B(T) is Planck's function. The RTE for the absorbing medium will then be:

$$\frac{dI_\lambda}{ds} = K_{abs}\left(B - I\right)$$

(2.5)

This equation is known as Schwarzchild's Equation and is the most fundamental description of radiative transfer in a non-scattering medium [5].

Radiative Transfer with Absorption and Scattering

The equation of transfer with scattering and absorption recognizes that depletion of the radiation occurs due to both absorption and scattering. Therefore, it is extinction coefficient that must appear in the depletion term rather than absorption. Moreover, one needs to add a source term that describes the contribution of radiation scattered into the beam from other directions. Thus:

$$dI_{\lambda} = dI_{ext} + dI_{emit} + dI_{sca} \quad (2.6)$$

Where:

$$dI_{ext} = -K_{ext} I_{\lambda} ds \quad (2.7)$$

Therefore, in a scattering atmosphere, the complete and general three-dimensional form of the equation of radiative transfer for un-polarized incident radiation is:

$$dI_{\lambda}(\vec{r}.\hat{r}) = -K_{ext}(\vec{r}) I_{\lambda}(\vec{r},\hat{r}) + K_{abs}(\vec{r}) I_{B.\lambda}(\vec{r},\hat{r}) + \frac{K_{sca}}{4\pi}\int_{4\pi} I_{\lambda}(\vec{r}.\hat{r}) g(\vec{r},\hat{r}) \hat{r} d\Omega \quad (2.8)$$

The differential form of the equation in the z-direction is:

$$\frac{dI_{\lambda}}{dz} = -I_{\lambda}K_{ext} + I_{B,\lambda}(T)K_{abs} + \frac{K_{sca}}{4\pi}\int_{4\pi} I'_{\lambda} g(\theta,\phi) d\Omega' \quad (2.9)$$

Where:

The intensity of the incident radiation at a given wavelength is I_{λ}

The blackbody radiation intensity at a given wavelength as a function of the temperature (T) is $I_{B,\lambda}$

The notation used here for the scattered intensity is I'_{λ} but I_{sca} can also be used to denote the same.

The gain of the radiative transfer is $g(\theta,\phi)$, while the element of back-scattering solid angle is $d\Omega'$

The total rate at which energy is radiated by a source is called the flux and denoted by: Φ with the unit in watts. The flux of the radiation transfer in the atmosphere is given by the equation:

$$\frac{1}{K_{ext}(\vec{r})c}\frac{\partial I(\vec{r},\hat{r},\lambda,t)}{\partial t} + \frac{1}{K_{ext}(\vec{r})}(\hat{r}.\vec{\nabla}) I(\vec{r},\hat{r},\lambda,t) = -I(\vec{r},\hat{r},\lambda,t) + J(\vec{r},\hat{r},\lambda,t) \quad (2.10)$$

Where $\hat{r}$ is a unit vector in the direction of scattered radiation, $(\vec{r})$ is the position vector, J is the radiation source function and I is the radiation intensity. K_{ext} is the radiation extinction coefficient and is related to the extinction cross-section by:

$$K_{ext} = NC_{ext} \quad (2.11)$$

Similar relationships exist for the scattering and absorption coefficients. The last term of the R.H.S. of equation (2.10) is the scattering source function vector. The first term is an attenuation term and represents the attenuation of the incident wave due to absorption and scattering of the radiation beam as it propagates through the atmosphere. The radiation beam in the case of solar radiation results from photons being scattered from the path of propagation in all directions. It is the presence of the scattering source function term that ensures that the propagation field is a function of the entire atmospheric radiation field and thus ensures transport over a large distance [6]. Figure 1 shows the approximate geometry of radiative transfer for the incident radiation in the atmosphere:

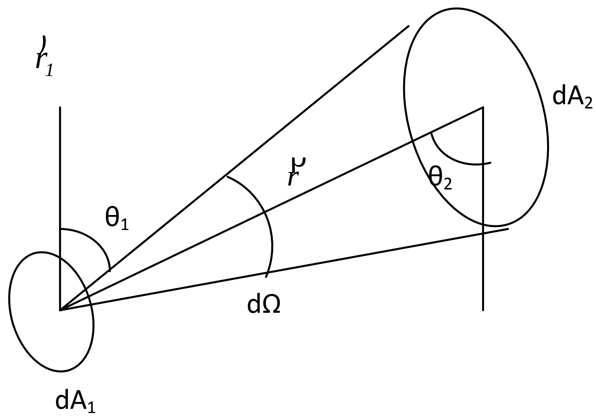

Figure1: Radiative Transfer Geometry for an incident Radiation

1.4 Plane-Parallel Atmosphere

In the atmosphere, for the solution of the Radiative Transfer Equation, the atmosphere is approximated as vertically stratified and horizontally homogeneous. It is then possible to reduce the spatial dimension from three to one and thus find solutions as a function of altitude only. In doing this, one replaces the position vector $\vec{r}$ by the scalar directional unit z to achieve what is called *plane-parallel geometry* [7]. The unit vector is then replaced by the

natural plane-parallel coordinates (the spherical polar angles θ, Φ). Thus equation (2.10) is simplified to:

$$\cos\theta \frac{dI(z,\theta,\phi)}{K_{ext}dz} = I(z,\theta,\phi) + J(z,\theta,\phi) \tag{2.12}$$

In order to take into effect the optical properties of the atmosphere, an alternate vertical coordinate system is introduced:

$$\frac{d\tau}{dz} = -K_{ext} \Rightarrow \tau(z) = \int_z^\infty K_{ext}dz' \tag{2.13}$$

Where τ is called the optical depth. The *optical depth* is a measure of transparency and is defined as the negative logarithm of the fraction of radiation that is scattered or absorbed as it propagates through a medium. For incident radiation wave propagating through the atmosphere:

$$\frac{I_{sca}}{I} = e^{-\tau} \tag{2.14}$$

In the earth's atmosphere, a tilted path is observed and the optical depth is defined by:

$$\tau' = m\tau \tag{2.15}$$

Where: *m* is called the *air mass factor*. For the approximated *plane-parallel* atmosphere, therefore:

$$m = \frac{1}{\cos\theta} \Rightarrow \cos\theta = \frac{1}{m} = \mu \tag{2.16}$$

At the top of the atmosphere τ =0 and it increases with decreasing altitude. The optical depth for the atmosphere is usually measured by a sun photometer.

Using equation (2.14) & (2.16) in (2.13):

$$\mu \frac{dI(\tau,\mu,\varphi)}{d\tau} = I(\tau,\mu,\varphi) - J(\tau,\mu,\varphi)$$
(2.17)

The change in sign is as a result of the negative sign in the definition of the optical depth (τ).

3.0 The Radiative Transfer Equation (RTE) for the Dust Aerosol Layer

The dust aerosol layer is assumed to have a total height h consisting of layers of irregularly spaced spherical dust particles as shown in the diagram, figure 2. Scattering is limited to the directions of reflected and transmitted radiation with the same incidence angle (θ): A wave with intensity I_2 incident from above the dust aerosol layer would be transmitted, absorbed or reflected, while the angle of incidence in air remains constant.

The down-welling wave, therefore gives rise to a transmitted wave in the same direction and a reflected wave in the direct opposite up-welling direction.

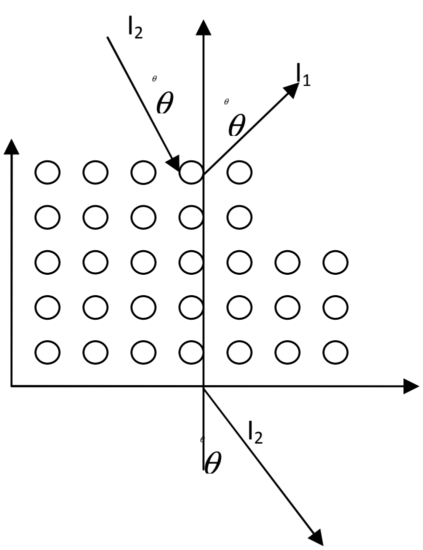

Figure 2: Scattering by a dust aerosol layer

One can then formulate two interacting transfer equations for the up-welling and down-welling intensities $I_1(z)$ and $I_2(z)$:

$$\uparrow +\frac{dI_1}{dz} = -K_{abs} I_1 + K_{abs}\left(I_2 - I_1\right)$$
$$\downarrow -\frac{dI_2}{dz} = K_{abs} I_2 + K_{sca}\left(I_1 - I_2\right)$$
(3.1)

Here, the term with the integral in the RTE (2.8) has been replaced by the coupling terms: $K_{sca}I_2$ and $K_{sca}I_1$ and emission has been omitted.

$(I_1 - I_2)$ =k is the scattering reduction term.

Another pair of coupled equation can be obtained by using the transformation sum $J = I_1 + I_2$ to obtain:

$$\frac{dJ}{dz} = -(K_{abs} + 2K_{sca})k \tag{3.2}$$

And:

$$\frac{dk}{dz} = K_{abs}J \tag{3.3}$$

Where J is the total intensity and k is the net radiation in the upward direction whose change is not affected by scattering. The second equation corresponds to the flux equation.

All the equations can be solved analytically by solutions of the type:

$$I_j = A_j \exp(+\gamma_2 z) + B_j \exp(-\gamma_2 z);\ j = 1, 2 \tag{3.4}$$

Where: Aj and Bj are coefficients that can be determined from boundary conditions. Illumination is assumed to be from above, thus defining the incident radiation as: $I_0 = I_2$ (at the top of the dust aerosol layer) and the damping coefficient is:

$$\gamma_2 = \sqrt{K_{abs}^2 + 2K_{abs}K_{sca}} \tag{3.5}$$

While, its inverse value is the effective penetrating depth given by:

$$d = \frac{1}{\gamma_2} \tag{3.6}$$

For an infinite layer: B_j=0 and I_1 and I_2 within the layer are proportional to $\exp(-\gamma_2 z)$. This means that the intensities diminish exponentially with thickness of the dust aerosol layer. Boundary conditions [8] for the dust aerosol layer between z=0 and z=h are:

$$I_1(z=0) = 0; I_2(z=0) = t.I_2(z=h);$$
$$I_1(z=h) = r.I_2(z=h) \tag{3.7}$$

Where: I_2 (z=h) is the incident radiation.

The reflectivity and transmitivity of the dust aerosol can then be computed to yield the results:

$$r = r_0 \frac{1 - t_0^2}{1 - r_0^2 t_0^2}$$
$$t = t_0 \frac{1 - r_0^2}{1 - r_0^2 t_0^2} \tag{3.8}$$

Where: t_0 (exponential transmission function) and r_0 (reflectivity at infinite h) are defined by:

$$t_0 = \exp\left(-\gamma_2 h\right)$$

$$r_0 = \frac{K_{sca}}{K_{sca} + k_{abs} + \gamma_2}$$

(3.9)

For a thick aerosol layer, only the topmost interface would contribute to reflection. Matzler [10] plotted the reflectivity (Figure 3) and transmitivity (Figure 4) using equation (3.8) for four absorption coefficients:

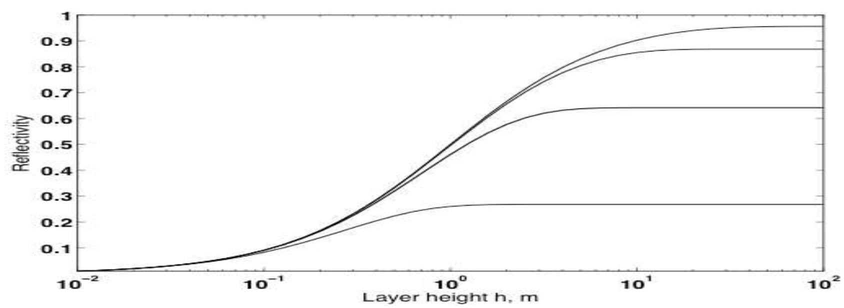

Figure 3: Reflectivity versus layer thickness of a volume scattering medium

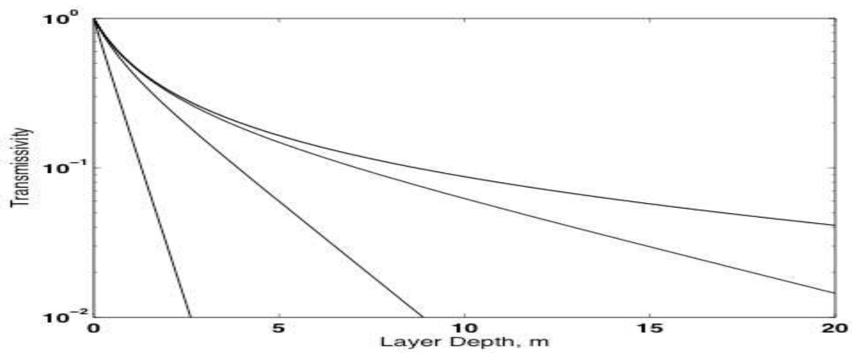

Figure 4: Transmitivity versus layer thickness (Matzler, 2000)

4.0 Reflectivity for a dust aerosol layer using (CIS) model

Radiative Transfer in Isotropic scattering media is extensively discussed in Chandrasekhar [9]. Following the solution of the RTE for isotropic scattering, a model was developed using MATLAB functions. The computer programme using the numerical MATLAB computation functions developed by Matzler (2008) [10] has been used following the equations of reflectivity. The model computes the reflectivity of down-welling radiation in plane-parallel atmosphere for various observation directions (θ) given by the zenith angle. Computations have been made for a dust layer composed of (a): high quantity of hematite (a strongly absorbing dust aerosol constituent) with approximate single scattering albedo of 0.52. (b): dust aerosol with undetermined constituents with representative single scattering albedo of (0.80) and (c): dust composed of mainly clay minerals (illite, kaolinite and montmorillonite with single scattering albedo of 0.95).

5.0 Discussion of Results

In all the three cases, one notes an increasing reflectivity with increasing angle of incidence. For a high hematite constituent dust aerosol, reflectivity at low incidence angle is much lower than for an undetermined dust aerosol and dust aerosol composed of greater amount of clay minerals. This is usual since hematite is highly absorbing and one should expect that its reflectivity would be correspondingly low. Dust aerosol containing a high amount of clay mineral constituent exhibit higher reflectivity for all incident angles. This is because the representative single scattering albedo used in the computation is close to

1(0.95). A single scattering albedo of 1 means high reflectivity.

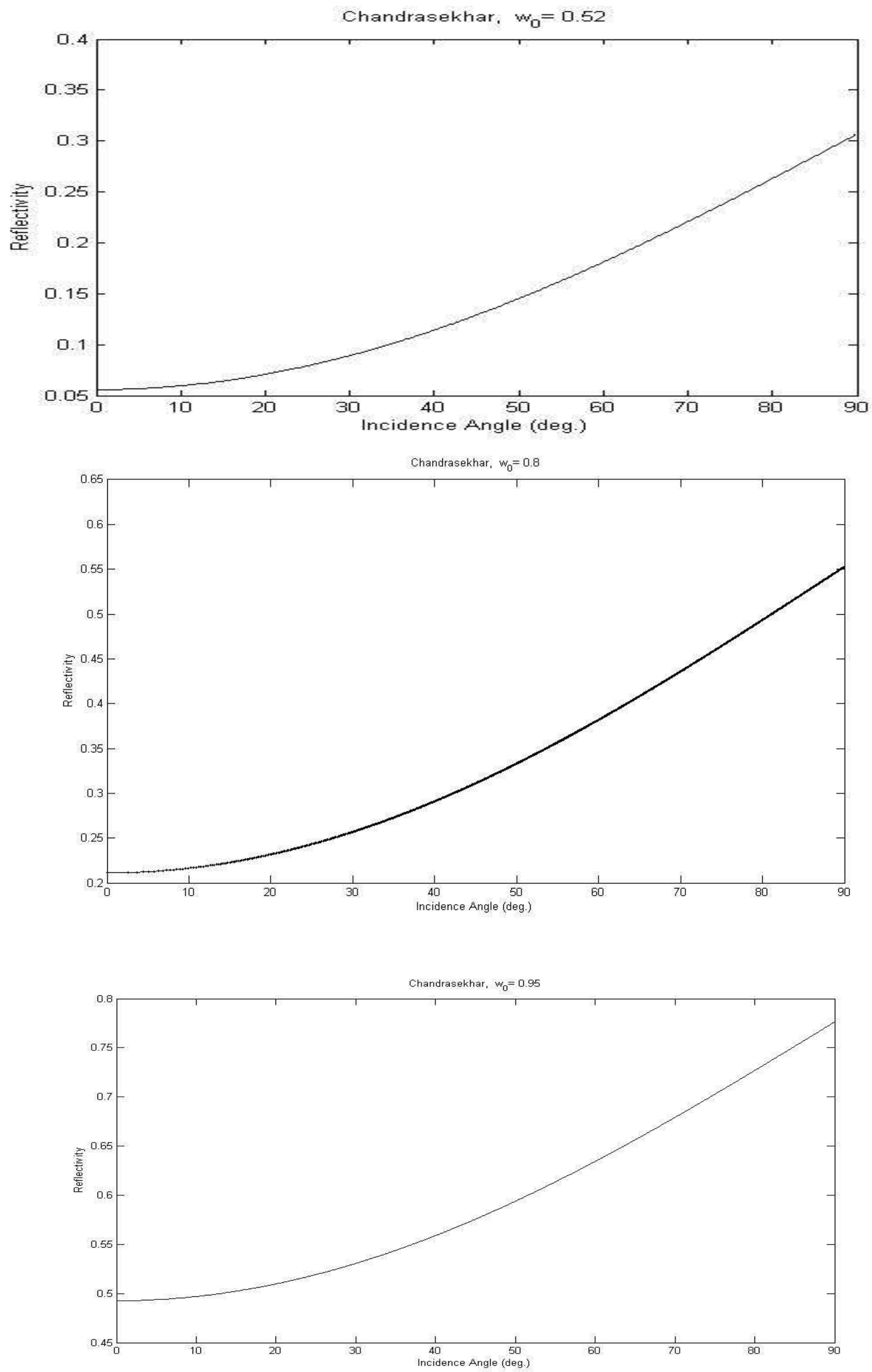

Figure 5: (a) Dust aerosol with high hematite content (b) dust aerosol of mixed constituent
(c) High clay mineral constituent

6.0 Conclusion

The Radiative Transfer Equation (RTE) as a model of analysis of interactions that occur in the atmosphere has been discussed. Its role in the description in description of the scattered radiative field was also explored. Its application to a dust aerosol layer has been analyzed and using a RTE model

(Chandrasekhar Isotropic Scattering) model, reflectivity for three types of dust aerosol layer was computed for down-welling radiation in a plane-parallel atmosphere for different incident angles. Results show that for all three cases, there is an increasing reflectivity with increasing angle of incidence. Dust aerosols high in clay mineral constituent have the highest reflectivity.

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