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Computations Of Critical Depth In Rivers With Flood Plains

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Abstract

Critical flows may occur at more than one depth in rivers with flood plains. The possibility of multiple critical depths affects the water-surface profile calculations. Presently available algorithms determine only one of the critical depths which may lead to large errors. It is the purpose of this paper to present an analytical formulation of a compound-channel Froude number which correctly identifies the occurrence of points of minimum specific energy (Critical points) for flow in rivers with flood plains. A compound-channel Froude number (Eq.16) has been derived and has been shown to accurately predict the critical points in rivers with flood plains. The proposed compound-channel (froude number) can be used in conjunction with existing computer programs for water surface profile computations.

Keywords: (Surface Profile, Computations, Critical Depth, Rivers, Flood Plains, Froude Number, Specific Energy, Compound Channel)

Notation

The following symbols are used in this paper:

A = total cross-section area;
a = subsection area;
E = specific energy;
F = Froude number;
F_e = compound-channel Froude number;
F_i = subsection Froude number;
F_r = weighted Froude number;
$F\alpha$ = Froude number with kinetic energy flux correction;
f = Darcy-Weisbach friction factor;
f_i = subsection friction factors;
g = acceleration of gravity:
K = total cross-section conveyance:
k_i,= subsection conveyance;
n = Manning's n value;
n,= subsection n value;
p_t = subsection wetted perimeter:
Q = total cross-section discharge;
Q_m= average measured discharge;
q_t = subsection discharge;
R = Reynolds number;
r, = subsection hydraulic radius:
S, = slope of energy grade line;
S_0 = bed slope of channel or flume;
T = total cross-section top wide
/,. = Subsection top wide
V = total cross-section mean velocity;
v = mean velocity associated with incremental area, dA :
v, = subsection mean velocity;
a = kinetic energy flux correction coefficient;
Δp = increment of wetted perimeter;
Δy = increment of depth; and
$\sigma 1, \sigma 2, \sigma 3,$ = subsection parameters of compound-channel Froude number.
y = depth of flow;
x = distance along channel;

1.0 Introduction

Analysis of open flow by the application of the energy principle is often examined and supported by the concept of specific energy, which was introduced by Bakmeteff [1] in 1912. Critical depth in open-channel flow occurs when the flow changes from supercritical to subcritical or vice-versa. The computation of critical flow is required for application in several situations. For example

(a) Channels are designed so that the flow is not near critical depth for long distances, since flow is unstable near critical depth.

(b) Gradually varied flow calculations usually become unstable near the critical depth, thereby, necessitating special precautions to avoid it.

(c) Critical depth may be starting point or "control" for computing the steady gradually varied flow water surface profiles.

Okoli [14] has shown that the determination of critical depth in channels with overbank or flood-plain flow (compound channels) can be troublesome. Customary definitions of the Froude number generally do not indicate critical depth at the point of minimum specific energy. In addition, there are some compound-channel geometrics, which produce specific-energy diagrams with two point of minimum specific energy. It is the purpose of this paper to present an analytical formulation of a compound-channel Froude number which correctly identifies the occurrence of points of minimum specific energy for flow in compound open channels. The proposed compound-channel Froude number can be used in conjunction with existing computer programs for water surface profile computations [5, 13, 16] and is necessarily limited by the same simplifying assumptions that are associated with the conventionally used, one-dimensional equation of steady, gradually varied flow [17].

The results of an experimental investigation in laboratory flume are also presented, demonstrating the existence of two points of minimum specific energy and identifying these points by the proposed compound-channel Froude number.

2.0 Froude Number-Flow Regime Discrepancies

For a simple channel of nonrectangular section and uniform cross-sectional velocity distribution, the Froude number F is defined by

$$F = \left(\frac{Q^2 T}{gA^3}\right)^{1/2} \tag{2.1}$$

where Q = water discharge; T = the top width of the water surface; g = acceleration due to gravity; and A = the cross-sectional area of flow. For a compound channel it is customary to include the kinetic energy flux correction coefficient, α, in the definition of specific energy. As a result, it appears as follows in the definition of the Froude number assuming α is constant with depth:

$$F_\alpha = \left(\frac{\alpha Q^2 T}{gA^3}\right)^{1/2} \tag{2.2}$$

For natural channels with overbank flow, it is often assumed that the major contribution to α is the large difference in mean velocity between main channel and overbank sections. By comparison the non-uniformity of the velocity distribution within each subsection can be neglected.

Two major problems arise in the computation of one-dimensional, steady, gradually varied flow profiles in compound channels, as a result of using the Froude numbers F or F_α. First, incorrect solutions are generated when numerical methods are used to solve the gradually varied flow equation written in a form involving the Froude number F_α. Second, incorrect solutions may be accepted when the standard step method is used to compute water-surface profiles near critical depth. These difficulties are the result of neglecting the variation of α with depth in compound-channel flows.

Consider the equation of gradually varied flow in the following form:

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F^2\alpha} \tag{2.3}$$

in which dy/dx = the rate of change in depth of flow with respect to distance along the channel; S_e = the bed slope of the channel; and S_t = the slope of the energy grade line. Prasad [10] has proposed a numerical solution procedure for Eq. (2.3) which can be applied to natural channels. In addition to the assumption that α is constant, the assumptions involved in obtaining Eq. (2.3) include: no lateral flow, a hydrostatic pressure distribution, a constant bed slope, and a straight, very wide channel, or alternatively, an approximately prismatic channel [17]. Because the variation in α with depth and thus with distance along the channel has been neglected, application of Eq. (2.3) to a gradually varied flow in a compound channel will lead to incorrect water-surface elevations. The denominator of the term on the right-hand side in Eq (2.3). arises from a consideration of the variation of specific energy with depth, a portion of which is due to changes in α with variation of specific energy with depth, a portion of which is due to changes in α with depth in compound-channel flow. Furthermore, the use of F_α can cause the right-hand side of Eq. (2.3) to become indefinite at a depth that does not correspond to the actual critical depth.

As an alternative to Eq. (2.3), water-surface profiles are compound in natural channels by the standard step method [6] in which the specific energy is computed explicitly. In this case, F_α does not appear in the equation to be solved, but it is used instead to indicate whether the solution is in the supercritical or subcritical flow regime. For compound channels, neither F nor F_α correctly indicates the flow regime. Thus, incorrect solutions of the energy equation can be accepted when the depth is near critical depth.

Compound-Channel Froude Number

Previous Investigations: Previous investigations of the problems associated with defining the Froude number in compound-channel flow have been undertaken, but the focus of these experiments has been the quantification of changes in the boundary shear stress distribution resulting from momentum exchange between the main channel and floodplain. The Federal agencies which maintain and use water-surface profile programs recognizes the Froude number difficulties in compound channel as described in the previous section of this paper, and they examine these difficulties in their user's manuals. The Soil Conservation Service (16), e.g warns of difference of as much as 0.7m between Eq. (2.1) and the critical depth determined by F and the critical depth determined by minimum specific energy.

The American Corp of Engineers presented an algorithm to solve for the depth corresponding to minimum specific energy is compared with the profile depth to check the flow regime rather than using the froude number as a check.

The United States Geological Survey (USGS) proposes the use of an index Froude number based on the Froude number of subsection carrying the greatest discharge. The index Froude cross-section, but it is also recognized better reflect the flow regime consider the index Froude number to be a true Froude number, but rather a warming flag that identifies possible flow-regime problems. A later version of the USGS Water Surface Profile Program incorporates a routine to determine the depth of minimum specific energy.

Chaundhry and Bhallamudi [9] have proposed a discharge-weighed Froude number without experimental corroboration in order to eliminate the computational problems associated with the occurrence of two points of minimum specific energy in compound-channel flows. Although their proposed Froude number succeeds in doing this by identifying only one value of critical depth, it is nevertheless somewhat arbitrary and is divorced from the concept of minimum specific energy.

Clearly, the Froude number should be formulated to reflect the specific energy curve under consideration and should indicate critical depth at the point (or points) of minimum specific energy. Such a Froude number would produce correct numerical solutions of the gradually varied flow Equation; Eq. (2.3) and would eliminate the need for time-consuming routines used to solve for the depth of minimum specific energy in standard step water-surface profile computations.

Derivation and Formulation. — The specific energy. E, for a one-dimensional compound-channel flow is given by

$$E = y + \frac{\alpha Q^2}{2gA^2} \tag{2.4}$$

in which y = the depth of flow. The kinetic energy flux correction coefficient, α, is defined as

$$\alpha = \frac{\int v^3 dA}{V^3 A} \tag{2.5}$$

in which v = the velocity through the element of area, *dA;* and *V* = the mean cross-sectional velocity (3,6). Alpha is thus a measure of the non-uniformity of the velocity distribution. For computational purposes, flow is conventionally divided into channel and overbank subsections by appropriately located vertical lines which are assumed not to transmit shear stress from one section of flow to another, and which do not contribute to wetted perimeter. Wright and Carstens [15] have suggested that the wetted perimeter of the subsection dividing line be retained for the main channel, and that the shear stress applied by the main-channel flow section on the overbank section be considered. Regardless of the manner in which the main flow-flood-plain interaction is treated, the basic assumption in the computation of α, as previously mentioned, is that the contribution of the non-uniformity of the velocity distribution within each subsection is negligible *in* comparison to the variation in mean velocity between subsections. If Eq. 2.5 is applied with this assumption to a compound channel, which has been divided into subsections, the kinetic energy flux correction coefficient becomes

$$\alpha = \frac{\sum_i \left(\frac{k^3{}_i}{a^2{}_i} \right)}{\frac{K^3}{A^2}} \tag{2.6}$$

in which k_i = the conveyance of the *i*th subsection; a_i = the area of the *i*th subsection; and $K = \sum k_i$ = the conveyance of the total cross section (3.6). The subsection conveyance is computed from the Manning's equation as follows:

$$K_i = \frac{AR^{2/3}}{n_i}$$

$$K_i = \frac{Q}{\sqrt{Si}} = QSi^{-1/2}$$

$$Q = \frac{1}{n} a_i r_i^{2/3} S^{1/2}$$

$$Ki = \frac{1}{n} a_i r_i^{2/3} \tag{2.7}$$

in which, n = Manning's Coefficient, γ_2 = hydraulic radius = a/p and p = wetted perimeter.

The point (or points) of minimum specific energy is obtained by differentiating Eq. 2.4 with respect to *y* and setting the derivative equal to zero. Because both α and area are functions of depth, the differentiation produces [14]

$$\frac{dE}{dy} = 1 - \frac{\alpha Q^2}{gA^3}\frac{dA}{dy} + \frac{Q^2}{2gA^2}\frac{d\alpha}{dy} = 0 \tag{2.8}$$

Noting that *dA/dy = T*, and that by rearranging terms, the following expression is obtained:

$$\frac{\alpha Q^2 T}{gA^3} - \frac{Q^2}{2gA^2}\frac{d\alpha}{dy} = 1 \tag{2.9}$$

The left-hand side of Eq. 2.9 is unity at the point of minimum specific energy; therefore, a compound-channel Froude number F_c can be defined from Eq. 2.9 as

$$F_c = \left(\frac{\alpha Q^2 T}{} - \frac{Q^2}{2gA^2}\frac{d\alpha}{dy} \right)^{1/2} \tag{2.10}$$

At the point of minimum specific energy F_c will have a value of 1.

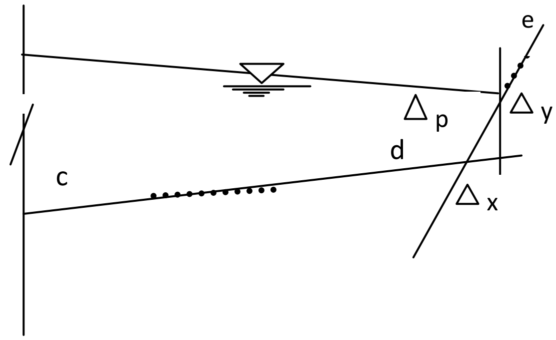

Fig. 1 Definition Sketch for Evaluation of dp_i/dy rate of change in the wetted perimeter with respect to depth of flow in the subsection

With the exception of $d\alpha/dy$, all of the terms on the right-hand side of Eq.2.10 are routinely, determined in water-surface profile computations. Evaluation of $d\alpha/dy$ can be achieved by differentiating Eq. 2.6 with respect to *y*. As shown in Appendix I, the derivative becomes

$$\frac{d\alpha}{dy} = \frac{A^2\sigma_1}{2gA^2} + \sigma_2\left(\frac{2AT}{K^3} - \frac{A^3\sigma_3}{K^4}\right) \tag{2.11}$$

in which
$$\sigma_1 = \sum_i \left[\left(\frac{k_i}{a_i}\right)^3\left(3t_i - 2r_i\frac{dp_i}{dy}\right)\right] \tag{2.12}$$

$$\sigma_2 = \sum_i \left(\frac{k^3{}_i}{a^2{}_i}\right) \tag{2.13}$$

$$\sigma_3 = \sum_i \left[\left(\frac{k_i}{a_i}\right)\left(5t_i - 2ri\frac{dp_i}{dy}\right)\right] \tag{2.14}$$

In Eqs. 12-14, t_1 = the top width of the *i*th subsection; and dpi/dy = the rate of change in wetted perimeter with respect to depth of flow in the ith subsection. Evaluation of dp_i/dy is simplified by the fact that the cross-section lines. The definition sketch in Fig.1 (which is a portion of a right overbank subsection) shows the water-surface intersecting the segment $\overrightarrow{de}$. This line makes a contribution of Δp to the subsection rate in wetted perimeter. The rate of change in wetted perimeter with respect to depth is a constant. Therefore can be evaluated as

$$\frac{dp}{dy} = \frac{\Delta p}{\Delta y}$$

(2.1 5)

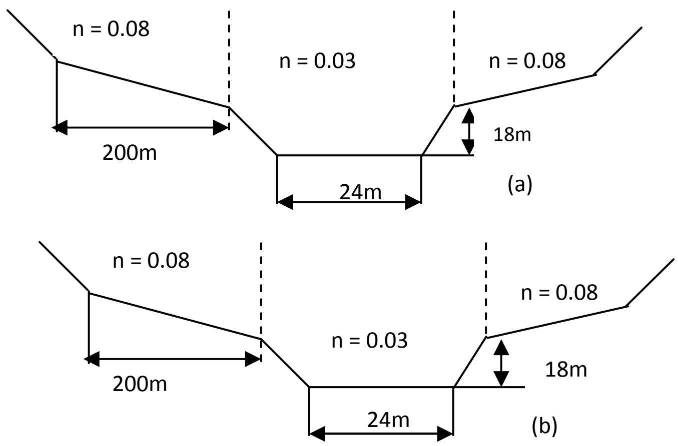

Fig. 2. Channel Cross sections for evaluation of specific energy and Froude Numbers (a) Cross Section A: (b) Cross Section B.

The terms Δp and Δy are generally determined when computing the geometric properties of a cross section for use in a water-surface profile program. It should be noted that if the water surface is at point *e*, *dp /dy* should be evaluated for the line segment $\overrightarrow{de}$, but if the water surface is at point *d*, *dp/dy* should be evaluated for the line segment *cd*. In situations where the water surface does not intersect the wetted perimeter of a subsection (e.g., the boundary between the main channel and overbank stage), dp_i/dy is the sum of $\Delta p_i / \Delta y$ for each of the banks.

The working equation for the compound-channel Froude number can be obtained by substituting Eq. 2.11 into Eq. 2.10 and simplifying:

$$F_c = \left[\frac{Q^2}{2gK^3} \left(\frac{\sigma_2 \sigma_3}{K} - \sigma_1 \right) \right]^{1/2} \tag{16}$$

If the Manning's *n* value is considered to vary with depth of flow in any subsection, σ_1 and σ_3 can be written to reflect the variation:

$$\sigma_1 = \sum_i \left[\left(\frac{k_i}{a_i} \right)^3 \left(3t_i - 2r_i \frac{dp_i}{dy} - \frac{a_i}{n_i} \frac{dn_i}{dy} \right) \right] \tag{2.17}$$

$$\sigma_3 = \sum_i \left[\left(\frac{k_i}{a_i} \right) \left(5t_i - 2r_i \frac{dp_i}{dy} - \frac{a_i}{n_i} \frac{dn_i}{dy} \right) \right] \tag{2.18}$$

in which *dn, /dy* = the rate of change in n, with respect to depth of flow.

3.0 EVALUATION

The behavior of the compound-channel Froude number, F_c, may be evaluated by examining the specific-energy diagrams of two idealized, symmetric cross sections, each conveying 142 $m^{3}/^{5}$. Cross section A (Fig. 2(a)) is from [9]. In Fig. 3, the specific-energy curve of flow cross section reveals two points of minimum specific energy at depths of flow approx 2.07 m and 1.62 m. These points are indicated by C_1 and C_2, respectively, in Fig. 3.

F_c for this cross section is plotted in Fig. 4 along with F and F_α. As expected, all three equations produce the same curve below top of bank (simple channel situation), but only Eq. (2.16) for F_c correctly locates C_l, the upper depth of minimum specific energy 2.07m, and connects with the lower curve at the top of bank depth.

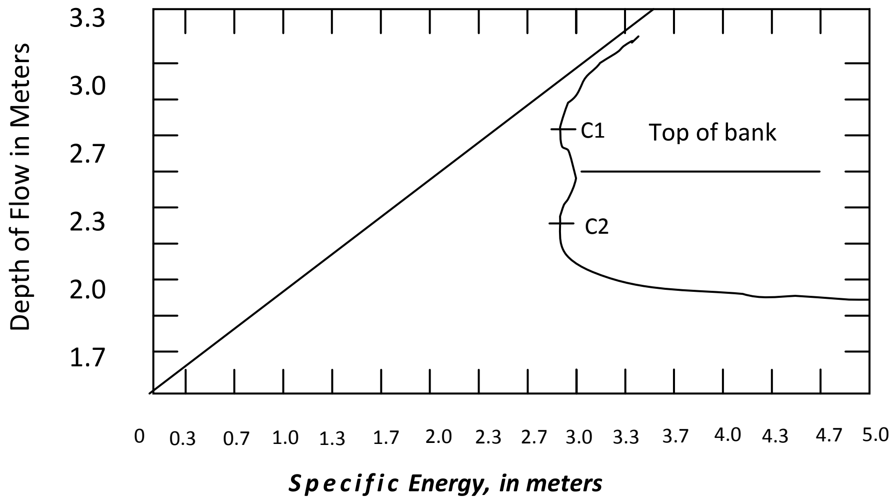

Fig. 3. Specific Energy for Cross Section A Conveying 140m^3/s

The shape of the Froude number curve is independent of the discharge, and the fiducial point (F_c = 1) can be shifted left or right by varying the discharge. This means that once F_c is plotted for a particular cross section and discharge, points of minimum specific energy for other discharges may be determined without the necessity of constructing new specific-energy diagrams. In effect, the variable F_C/Q provides a universal horizontal scale for Fig. 4 which depends only on the conveyance and geometric properties of the particular cross section. Thus, for a given depth of flow, the critical discharge, Q_c, can be computed by taking the reciprocal of the corresponding value of F_c/Q, because F_c/Q for the given depth equals I/Q_c for the critical condition.

Cross section B is presented in Fig.2 (b) and differs from cross section A only in that the flood plains have a 100:1 slope toward the channel. The specific-energy diagram cross section B (Fig. 5) reveals a single point of minimum specific energy below top of bank at the same depth of flow as for cross section A (point C_2).

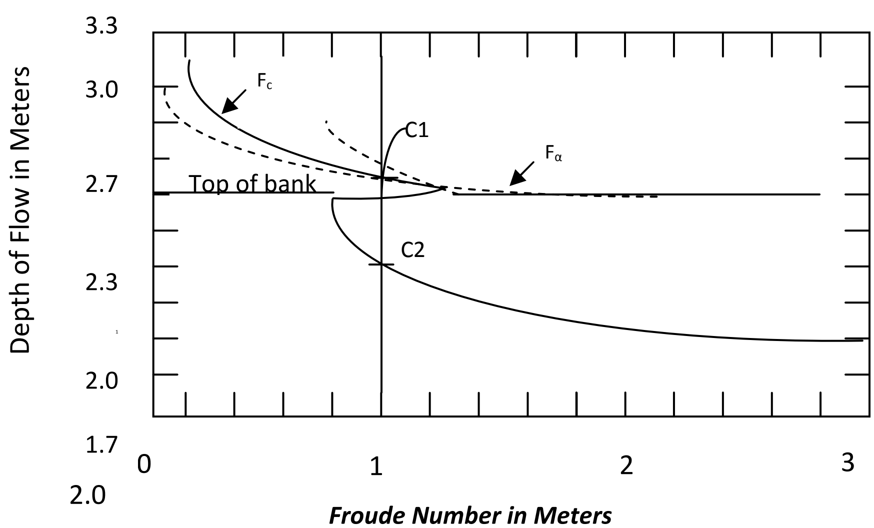

Fig. 4.—Froude Numbers for Cross Section a Conveying 140m^3/s

The three Froude number curves shown in Fig. 6 for cross section B are again identical below top of bank, but F each indicate another point of minimum above top of bank at depths of flow of 1.98m and 2.07m, respectively.

The occurrence of these false points of minimum specific energy is a more serious deficiency of Eqs. 2.1 and 2.2 than the errors in crucial depth shown in Fig. 4.

It is evident from these two examples that the Froude numbers generated by Eqs. 2.1 and 2.2 are not acceptable for use in the gradually varied flow equation Eq. 2.3. Neither definition of Froude number faithfully reflects the specific-energy diagram in overbank flow situations, and either would produce divergence from a correct profile solution. It is equally evident that Eqs. 2.1 and 2.2 are not satisfactory for checking the flow regime in the standard step method. Only F_c in Eq. 2.16 accurately reflects the specific-energy diagram and indicates the correct flow regime. The experimental investigation into the occurrence of two points of minimum specific energy in the following portion of this paper offers guidance for the interpretation of the flow regime between the two points of minimum specific energy, Cl and C2, in cross section A (Fig. 3).

4.0 Experimental Investigation

The experimental investigation consisted of measuring point velocities in a compound-channel cross section which was formed by constructing a single rectangular overbank section in a laboratory flume. Sufficient point velocity measurements were made at eight different depths of flow (at approximately the same discharge for each depth) to compute the discharge, mean velocity, Kinetic energy flux correction coefficient, and specific energy for each complete details of the experimental procedure are given by the writer.

The experiments were conducted in a tilting steel flume (24.38 m) long, (10.7m) wide, and (0.46 m) deep. The flume was provided at William Fraiser Hydraulic Laboratory of the University of Strathclyde, Glasgow. Uk. This flume was also used by [14] and details of its construction is given below. The overbank section was constructed of 27cm in exterior plywood and two-by-six fir framing lumber, resulting in the channel dimensions shown in Fig.7. All wooden components were coated with sand sealer and exterior acrylic-latex paint. The overbank section was attached to the flume with silicon adhesive.

Point velocities were measured with a (1.83-mm) outside diameter pitot-static tube operated in conjunction with a differential pressure transducer. Data collection, reduction, and analysis were accomplished with an HP9825A desktop computer controlling a digital voltmeter which measured the voltage output from the pressure transducer and preamplifier. Point velocity measurements were made at a station 19.81m downstream of the flume entrance. Preliminary measurements were made at a station 18.29m downstream. Comparison of dimensionless profiles of velocity between the two stations indicated that the flow was fully developed.

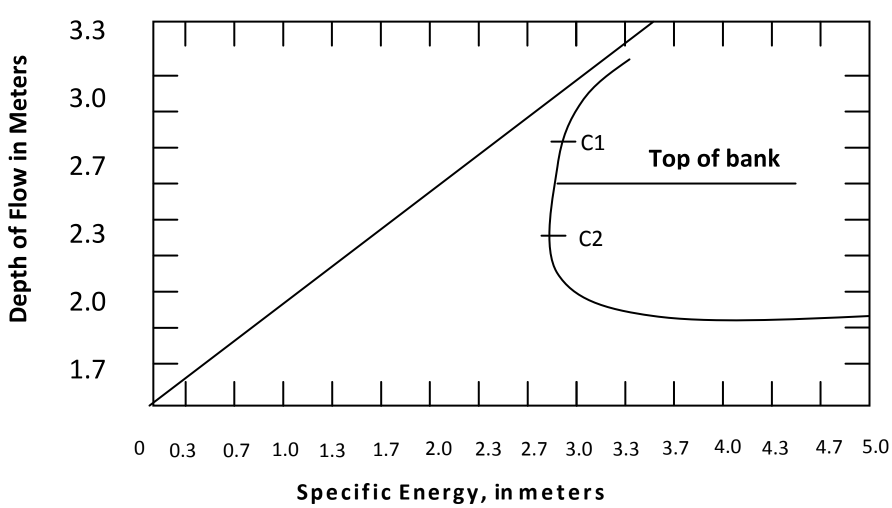

Fig. 5. Specific Energy for Cross Section B Conveying 140m^3/s

The preliminary experiments indicated that a discharge of (0.048 m^3/s) would produce a specific-energy curve with two points of minimum specific energy. An estimate of the error in setting the discharge to (0.048 m^3/s) included the calibration error of the Venturi-meter used to measure the discharge and also included an estimate of the error introduced by observed fluctuations in the Venturi-meter manometer during the course of an experimental run. The estimated error in discharge was of the order of ±3%, which was the same range of error observed between individual discharges determined from the Venture meter and the discharges determined by integration of the point velocity measurements.

Establishing a truly uniform flow profile for the experimental runs proved impossible. Any discharge flowing near the depth corresponding to minimum specific energy, as these were, could be expected to be inherently unstable. The instability was exacerbated by the variations in the overbank surface, which were of the order of (0.3 cm). Standing waves in and a cross-hatched water surface in the channel thwarted efforts to achieve a uniform water-surface profile. As a result, the adopted experimental procedure was to establish a profile as close to uniform as possible such that the desired depth of flow was obtained where the point velocities were to be measured. The maximum observed change in depth for overbank-flow runs was approx (1.5 cm) between the channel entrance and the measuring station where the flow depth was (17.3 cm). For larger depths of flow, the water-surface profiles tended to be more stable and more nearly uniform. A profile at a depth of flow of (21.3 cm) was established to demonstrate that a uniform profile could be obtained in the downstream reach of the flume if the depth of flow was sufficiently greater than the depth corresponding to minimum specific energy.

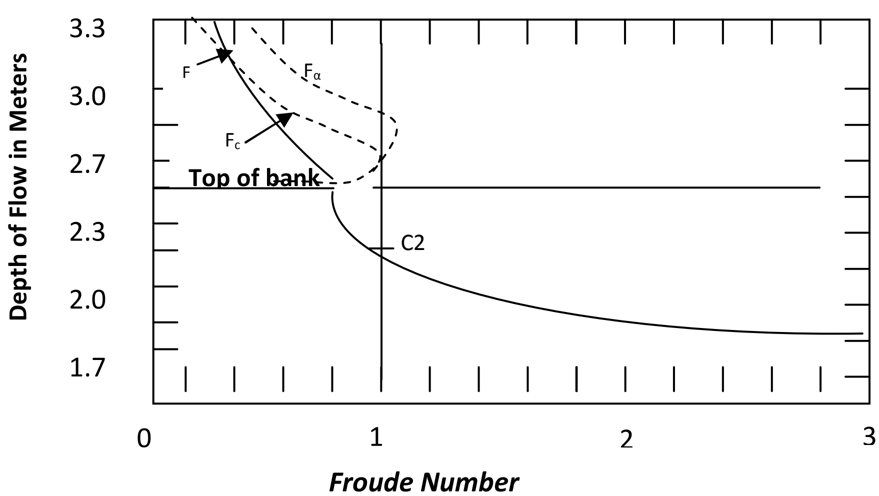

Fig. 6.—Froude Numbers for Cross Section B Conveying 140m^s/s

5.0 Results And Discussion

Table 1 presents the values of area, discharge, kinetic energy flux correction coefficient, and specific energy computed from experimental measurements for each of the eight reported runs. Runs 5 and 6 are not reported in the table because of operational difficulties during each run. It is apparent from the results presented in Table 1 that as the depth increased for those experimental runs with overbank flow, the proportion of the total discharge in the overbank section increased. It should also be noted that the values of α for the main channel alone are measurably larger than 1.0 because of the narrowness of the main channel section.

Observations of the water surface for the four experimental runs with overbank flow indicated greater instability as the depth of flow decreased. The water-surface instability was manifested by standing waves in the overbank section and a choppy, cross-hatched water surface in the channel section. Beginning at the upper depth of minimum specific energy (run 2) and continuing with decreasing depth, the standing wave fronts in the overbank section were perpendicular to the mean flow direction and then were bent downstream into a cross-hatched pattern in the channel section characteristic of supercritical flow. The surface instability continued to increase for the experimental runs as depth decreased below top of bank. The fact that, the water surface was

unstable for experimental runs 7 and 8, the first two runs below top of bank in Table 1, suggests that the upper point of minimum specific energy could be considered the limit of subcritical flow for situations in which two points of minimum specific energy occur in water-surface profile computations.

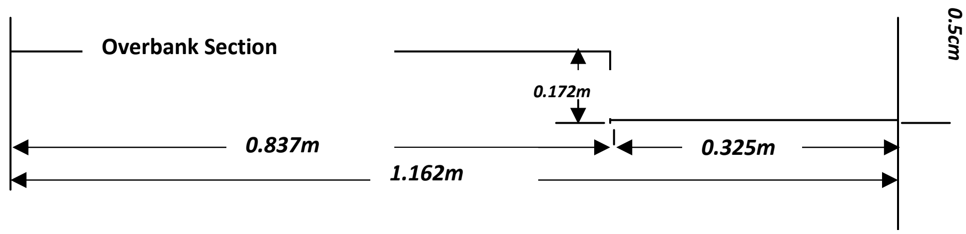

Fig. 7. —Cross Section of Flume and Overbank Section, Looking Downstream

The experimental specific-energy data in Table 1 are plotted in Fig. *8(a)*. Although the variation in discharge from run to run causes some scatter in the plot, there is evidence of two points of minimum specific energy. The experimental values of α plotted in Fig. 8(b) show little scatter and indicate that (alpha α) is primarily a function of depth of flow. This observation suggests that a specific-energy diagram for a single value of discharge can be constructed by substituting the average discharge of eight runs 0.048 m^3/s into Eq. 2.4 while using the experimental data for all other variables. Fig. 9 presents the resulting average specific-energy diagram. The two points of minimum specific energy are more clearly apparent in this figure.

The concept of computing a Froude number for the flow in a subsection of a compound channel has already been mentioned with regard to the USGS index Froude number [12]. The subsection Froude numbers (computed with Eqs. 2.1 and 2.2) for the experimental data of this investigation are presented in Table 2. The Froude number of the channel (Col. 3 or 4 of Table 2) is the index Froude number of these experimental runs because the channel is the subsection with the largest discharge. All four depths of flow above top of bank are subcritical based on the index Froude number, but as shown in Fig. 9, the two lower overbank depths are not subcritical. For this experimental investigation, the index Froude number does not correctly indicate the flow regime of compound-channel flow.

Table 1: Values of Area, discharge, kinetic energy, flow correction coefficient and specific energy computed from experimental measurements.

Run (1)	Y,(m) (2)	S_o (3)	E,in (m) (4)	A in m^2 (5)	Q in m^3/sec (6)	α (7)	A in m^2 (8)	Q in m^3/sec (9)	α (10)	A in m^2 (11)	Q in m^3/sec (12)	(13) α
1	0.1983	0.001018	0.2190	0.0589	0.0386	1.0840	0.0274	0.0117	1.108	0.0863	0.0503	1.192
4	0.1907	0.001128	0.2142	0.0558	0.0393	1.083	0.0215	0.0093	1.132	0.0782	0.0486	1.198
2	0.1830	0.001485	0.2135	0.544	0.0424	1.082	0.0158	0.0060	1.169	0.0701	0.0489	1.224
3	0.1730	0.002096	0.2142	0.0514	0.0451	1.088	0.0078	0.0006	1.340	0.0592	0.0476	1.238
10	0.1425	0.003300	0.2105	0.0424	0.0466	1.096	-	-	-	0.0424	0.0466	1.096
7	0.1525	0.002118	0.2135	0.0454	0.0475	1.087	-	-	-	0.0454	0.0475	1.087
8	0.1425	0.003300	0.2105	0.0424	0.0466	1.096	-	-	-	0.0424	0.0466	1.096
9	0.1321	0.004455	0.2142	0.0393	0.0474	1.100	-	-	-	0.0393	0.0474	1.100

[9] apply the concept of a subsection Froude number to obtain their weighted Froude number F_r, which is given by

$$F_r = \frac{\sum_i (q_i F_i)}{Q} \tag{5.1}$$

in which q_t = the subsection discharge; and F, = the subsection Froude number computed by Eq. 2.1. Values of F, for the experimental data are presented in Col. 7 of Table 2. As in the case of the index Froude number, the weighted Froude number does not correctly indicate the flow regime.

Analysis

The proposed compound-channel Froude number cannot be directly determined from the experimental data. Attempts to use Eq. 2.10 fail because it is difficult to determine $d\alpha/dy$ from the limited number of experimental data points. Eq. 16 fails because the slope of the energy grade line is not precisely known, which means that the subsection resistance coefficient and thus the conveyance, k_i, cannot be determined from the experimental data. If it had been possible to establish a uniform flow condition for each run, the energy gradient would parallel the flume slope, and the conveyance for each subsection could be computed from the experimental data alone. The compound-channel Froude number can only be determined indirectly through an independent prediction of the experimental results.

Working in the same flume as used in the present investigation, [14] experimentally determined a friction-factor relationship for smooth rectangular channels of the form

$$\frac{1}{\sqrt{f}} = 2.03\log(R\sqrt{f}) - 1.30 \tag{5.2}$$

in which f = the Darcy-Weisbach friction factor; and R = the Reynolds number. If it is assumed that Eq. 5.2 is valid when applied independently to each channel subsection, the friction factor, f_i, can be determined for the ith subsection. The mean velocity in the *i*th subsection, v_i, is then given by

$$v_i = \left(\frac{8gr_iS_e}{f_i}\right) \tag{5.3}$$

in which r_i= the hydraulic radius of the ith subsection; and S_e = the slope of the energy grade line. Because the values of *f*, and v_i obtained from Eqs. 5.2 and 5.3 must be such that the subsection discharges sum to the average measured discharge, Q_m, of (0.048 m^3/s), the following equation must be satisfied:

$$S_e = \frac{Q_m{}^2}{\left[\sum_i \left(\frac{8g}{f_i} r_i a_i{}^2\right)^{1/2}\right]^2} \tag{5.4}$$

It has been implicitly assumed that S_e is the same for all subsections. Eqs. 5.2, 5.3, and 5.4 can be solved iteratively for the friction factor and velocity in each subsection for a given total discharge and depth. The iterative solution procedure is given in detail by [2] and [9]. The velocities, v_i, were calculated by the procedure just described for the mean measured discharge of Q_m (0.048 m^3/s). It was assumed that the imaginary vertical boundary between the main channel and overbank section made no contribution to wetted perimeter. Furthermore, the friction factors determined for each subsection were converted to Manning's *n* values because the formulation for the compound Froude number, F_e, is in terms of *n*. The *n* values so obtained exhibited a slight variation with depth; however, to facilitate the computations, constant *n* values of 0.03 and 0.08 were adopted for the channel and overbank sections, respectively. From the velocities and *n* values for each subsection, the specific energy and compound Froude number were computed for a series of depths within the range of measured depths In the computation of the specific energy and F_c, it was assumed that *α- Alpha* of each subsection had the value 1.0 rather than the measured value. In this way, the computational procedure remained independent of the measured data and was executed in the same manner as would be expected when determining F_c for a natural river channel in the course of a water-surface profile computation.

The predicted specific-energy diagram is shown in Fig. 10 (a), and two depths of minimum specific energy are apparent, although each depth is approximately 0.0067m smaller than the corresponding depths in Fig. 8(a) or Fig. 9. The entire specific-energy curve in Fig. 10 (a) is skewed slightly downward and to the left when compared with the measured curve in Fig. 8(a) or the average curve in Fig. 9.

Table 2: Froude Numbers for Experimental Data

Run (1)	y, in metre (2)	Channel F (Eq. 1) (3)	F_α (Eq. 2) (4)	Overbank F (Eq. 1) (5)	F_α (Eq. 2) (6)	Weighted F_r (Eq. 19) (4)
1	0.650	0.471	0.490	0.721	0.759	0.529
4	0.625	0.508	0.529	0.821	0.873	0.586
2	0.600	0.583	0.606	0.925	1.001	0.629
3	0.567	0.675	0.704	1.017	1.177	0.692

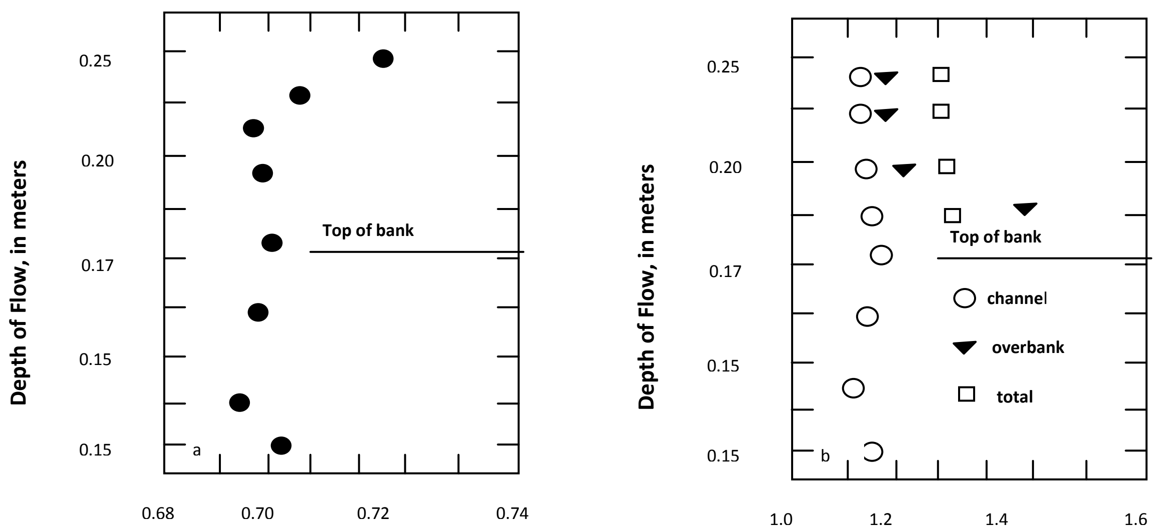

Fig. 8- Specific Energy and Kinetic Energy Flux Correction Factor from Experimental Data

(a) Specific energy, in (b) Alpha.

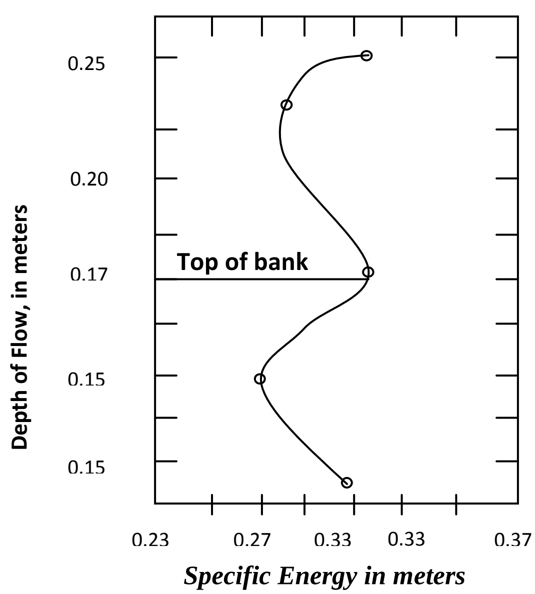

Fig. 9 Specific Energy in meters

The predicted compound-channel Froude number curve in Fig.10 (b) exhibits the behavior typical for two points of minimum specific energy, and is in correspondence with the predicted specific-energy curve as expected.

To investigate the role that neglecting the transfer of linear momentum to the overbank section plays in the skew of the predicted specific-energy curve, the correction suggested by (15) was considered. Although the correction improved the agreement between the measured and computed discharges in the overbank section, especially at the larger depths, the effect on the computed specific-energy curve was minimal because of the relatively small changes in α which resulted from the correction.

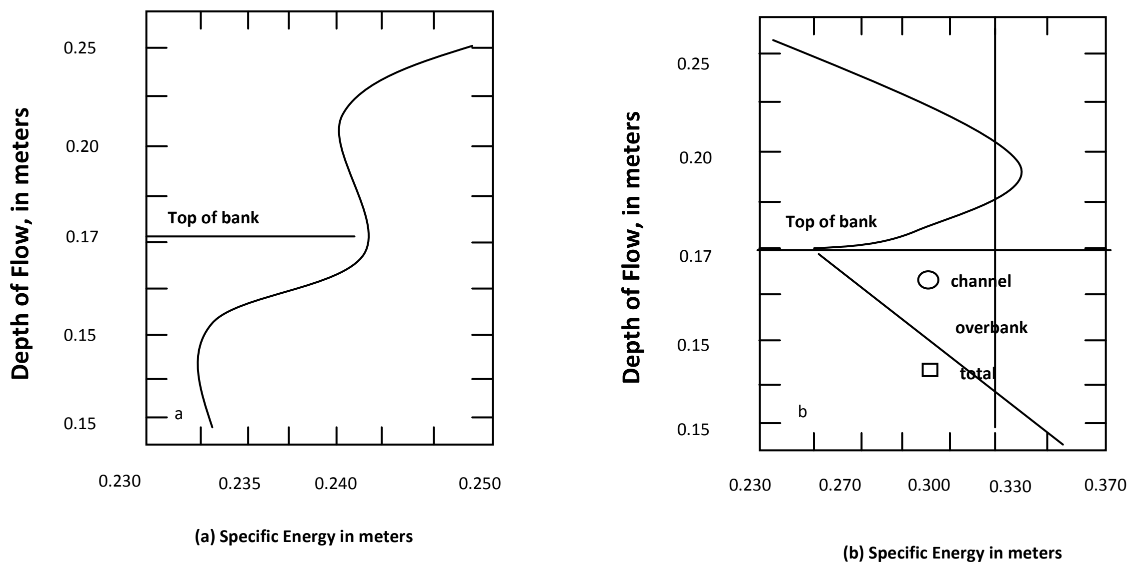

Fig. 10.—(a) Predicted Specific Energy in Experimental Flume for 0.047m^2/s (b) Compound Channel Froude Number for Fig. 10(a)

The skew in the specific-energy curve is most pronounced below top of bank depth where transfer of linear momentum to the overbank does not occur. The skew in this portion of the curve can be attributed to selecting subsection a values of unity in computing specific energy. It should be noted that the depths of flow in the flume were small compared to depths of flow normally found in field situations. For this reason, the velocity head in the flume makes a large relative contribution to specific energy, and any adjustment to velocity head (such as subsection α) has far more effect on specific energy in the flume than it would in the field.

The same analysis can be applied to subcritical and supercritical flow regimes in field situations where kinetic energy correction coefficients can be as much as 1.4 or more in the main channel. For subcritical flow where the velocity head is small, an adjustment to velocity head would be insignificant. For supercritical flow, the velocity head can be 50% or more of the depth, and an α-adjustment to velocity head would have a significant effect on specific energy. This reasoning explains the increasing leftward shift in Fig. 10(a) as the depth of flow decreases, and the implication is that predicted specific energies and Froude numbers in field channels under subcritical flow conditions would be closer to measured values.

Conclusions

Existing formulations of the Froude number Eqs.2.1 and 2.2 do not accurately reflect the specific-energy curve for flow in a compound open channel and do not correctly locate points of minimum specific energy. A compound-channel Froude number Eq.2.16 is derived and is shown to accurately reflect the specific-energy curve of flow in a compound open channel by correctly locating points of minimum specific energy. When applied to a simple channel with uniform velocity distribution, the compound channel Froude number is identical to Eq. 2.1, the conventional definition of Froude number.

The compound-channel Froude number is appropriate for use with the gradually varied flow equation Eq. 2.3 and provides the proper check of the flow regime when used in conjunction with the standard step method of water-surface profile computation. The proposed Froude number is subject to the same assumptions that apply to the equation of gradually varied flow commonly employed in water-surface profile computations.

For some compound-channel geometries characterized by wide, level flood plains, two points of minimum specific energy can be computed for certain discharges. Laboratory investigation of a one-dimensional flow demonstrates that this phenomenon can in fact occur, and indicates that the upper point of minimum specific energy may be considered the proper limit of subcritical flow.

Appendix I. Derivation of *dα/dy*

Writing Eq. 2.6 as

$$\alpha = \frac{A^2}{K^3}\sum_i\left(\frac{k^3}{a^2{}_i}\right) \tag{A1}$$

and differentiating with respect to *y* produces

$$\frac{d\alpha}{dy} = \frac{A^2}{K^3}\sum_i\left[3\left(\frac{k_i}{a_{\ i}}\right)^2\frac{dk_i}{dy} - 2\left(\frac{k_i}{a_{\ i}}\right)^3\frac{da_i}{dy}\right] + \sum_i\left(\frac{k^3{}_i}{a^2{}_i}\right)\left[\frac{2A}{K^3}\frac{dA}{dy} - \frac{3A^2}{K^4}\frac{dK}{dy}\right] \tag{A2}$$

Noting that $da_i/dy = t_i$, $dA/dy = T$, and $dK/dy = \sum_i (dk_i/dy)$, the following is obtained:

$$\frac{d\alpha}{dy} = \frac{A^2}{K^3}\sum_i\left[3\left(\frac{k_i}{a_{\ i}}\right)^2\frac{dk_i}{dy} - 2t_i\left(\frac{k_i}{a_{\ i}}\right)^3\right] + \sum_i\left(\frac{k^3{}_i}{a^2{}_i}\right)\left[\frac{2AT}{K^3} - \frac{3A^2}{K^4}\sum_i\left(\frac{dK_i}{dy}\right)\right] \tag{A3}$$

Evaluate dk_i/dy by writing Eq. 2.7 as

$$ki\left(\frac{1.49}{n_i}\right)\frac{a_i^{5/3}}{p_i^{2/3}} \tag{A4}$$

and differentiate with respect to y to obtain

$$\frac{dk_i}{dy} = \left(\frac{1.49}{n_i}\right)\left[\frac{5}{3}\left(\frac{a_i}{p_i}\right)^{2/3}\frac{da_i}{dy} - \frac{2}{3}\left(\frac{ai}{p_i}\right)^{5\backslash 3}\frac{dp_i}{dy}\right. \tag{A5}$$

Again noting that $d\alpha_{,}/dy = t_{,}$, and multiplying and dividing by a_n the following is obtained:

$$\frac{dk_i}{dy} = \frac{1}{3}\left(\frac{k_i}{a_i}\right)\left[5t_i - 2r_i \frac{dp_i}{dy}\right] \tag{A6}$$

Substituting Eq. A6 into Eq. A3 and simplifying, results in Eq. 2.11.

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