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# A Flow Formula For Submerged Rectangular Weirs

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## *Abstract*

***An equation is derived to obtain the discharge over a sharp-edge rectangular weir in the case of free and submerged conditions. A flow equation has been developed for discharge over a sharp crested rectangular weir in such a way that it can be used for both free and submerged flows. The presence of an upstream sloping face increases the discharge. This increase depends on the ratios between the downstream head $H_2$, the upstream head $H_1$ and the slope of the upstream sloping face. This increase in discharge depends on the width of weir in the direction of flow and on the ratio $H_2/H_1$.***

**Notations**

**b** weir breath, normal to flow direction
**B** weir width in the direction of flow
**C** Coefficient of discharge in the weir equation (9)
**$C_1$** Coefficient of discharge for the flow over a weir
**$C_2$** Coefficient of discharge for the flow through a submerged orifice
**g** acceleration due to gravity
**$H_1$** Head on the upstream side of the weir over crest
**$H_2$** Head on the downstream side of the weir over crest
**K** $= \frac{2}{3} C \sqrt{(2g)b}$
**n** Exponent of $H_1^1$ in the free discharge equation
**p** height for the weir above bed level
**$Q_1$** Discharge at head $H_1$ for free discharge case
**Q** Discharge for the submerged case
**r** Radius around the upstream corner for narrow crested weir.
**y** Vertical distance between the weir crest and the sloping face upper edge
**α** Correction in the weir equation (9) for the case of the presence of an upstream sloping face
**β** $= H_2/H_1$
**θ** Angle of the sloping upstream face with vertical correction in the weir equation (9) for the case of narrow crested weir with edge rounding.
**ψ** Ratio between the discharge in the submerge condition to the discharge in the free condition.
**λ** Correction in the weir equation (9) for the case of narrow crested weir with edge rounding.

## 1.0 Introduction

Weirs of different shapes are used as control structures in irrigation system. The flow over the weir is preferred to be a clear overfall flow where the equations relating the discharge passing the weir and the head over the weir are well established. In practice it may sometimes be difficult or impossible to prevent downstream water level from rising above the level of the weir. In this case, the weir is said to be submerged or drowned.

Generally, it is expected that this will reduce the flow, yet the precise relation between the upstream head $H_1$, the downstream head $H_2$ over the weir crest and the discharge Q are not known for most weir shapes. Moreover, the

shape of the weir cross section has a great effect on the relationships between $H_1$, $H_2$ and Q. In this study, a general flow equation was developed to describe the discharge over the vertical sharp-edged rectangular weir for free case as well as the submerged case.

Previous studies have been concerned with the flow over sharp-crested weirs. The general profile of the flow over this type of weir is shown in figure 1 (a-d)

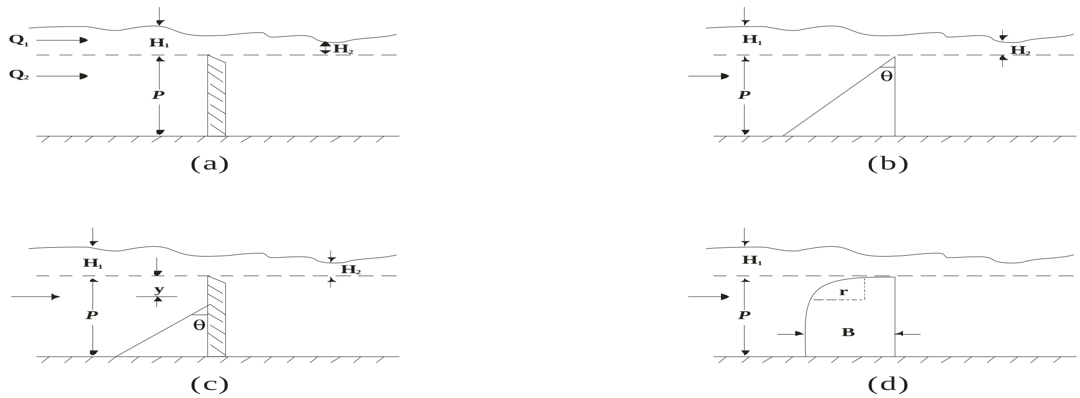



*Fig 1. Weir models used in the study: (a) vertical weir, (b) weir with upstream sloping face equal to P, (c) weir with upstream slope less than P where P is the height of the weir, (d) narrow crested weir.*

Swamee[18] summarized the work of [2] and showed that all the data presented could be put in a relationship between $H_2/H_1$ and $Q/Q_1$; where Q is the discharge at the head H1 computed from the equation for free discharge in the form.

$$Q_1 = KH_1^n \qquad (1.1)$$

*Q* is the discharge for submerged conditions; *k* is the coefficient of discharge. $H_1$ is the head on the upstream side of the weir and $H_2$ is the head on the downstream side of weir. More recent studies such as those by [1] and [10] found that all the data tended to fall on a single curve relating Q2/Q1 against H2/H1, except for high values of H1/P, where P is height of the weir crest above the floor of the channel. (See fig. 2.)

Swamee[16] carried out experiments on rectangular, triangular, parabolic, cusped and proportional vertical weirs. He showed that all his results could be presented by the equation.

$$\frac{Q}{Q_1} = \left[1 - \left(\frac{H_2}{H_1}\right)^n\right]^{0.385} \qquad (1.2)$$

Where n is the exponent for the free discharge as in equation (1.0). This study is different from Swamee [16]from the fact that it concerns flow in a free and submerged weirs.

Bos [4] conducted test on a rectangular, triangular, parabolic, circular, sutro and cusped vertical weirs. He derived a single equation to express the result of his test relating $Q/Q_1$ and $H_2/H_1$. The data presented by [13] were plotted by [12] who found that $Q/Q_1$ is a function of $(H_2/H_1)$” by all weir shape used in their studies. This result was also validated by Myer’s[10].

Other investigators like [9] assumed that flow over the submerged sharp crested weir can now be considered as a summation of two as follows: the flow over the upper part of depth $H_1$-$H_2$ which may be considered as a free discharge over a weir; and that over the lower part of depth $H_2$, which can be considered as a flow through a submerged orifice or

$$Q^1 = 2/3\, C_1 b (2g)^{1/2}\, (H_1 - H_2)^{3/2} \quad . \qquad (1.3)$$

$$Q " = C_2 b(2g)^{½} \ (H_1 - H_2)^{½} \quad (1.4)$$

The total Q equals $Q^1 + Q^n$. $C_1$ and $C_2$ are the coefficient of discharges for the flow over a weir and flow through an orifice respectively, b is the breadth normal to the direction of flow. Values of C with different values of the $H_2/H_1$ were given. Markland [5] quotes Redtenbacher who assumed $C_1 = 0.57$ and $C_2 = 0.62$ and Pestorlozzi who

assumed $C_1 = 0.62$ and $C_2$=0.534 – 0.566 depending on the ratio $H_2/H_1$. The value of $C_2$ increase as $H_2/H_1$ decreases.

## 2.0 Objective of Study

The problems of flow with a sloping face and flow over narrow crested weirs have not been widely studied and the developed models also have not received general acceptance. It is felt that a general equation can be developed to describe the discharge over a sharp edge rectangular weirs for the free over fall case and also for the submerge case.

**The Generalized Flow Equation**

With reference to fig 1a, it may be seen that the discharge over the weir in the submerged condition is the sum of the two discharged; $Q^1$, $Q^{11}$ which represent the free flow over a sharp edged rectangular weir under a head $H_2/H_1$ and $Q^{11}$ which represent a discharge through an orifice of depth $H_2$ and under a head $H_2/H_1$. In this case, the discharge passing over a weir can be written as

$$Q = Q^1 + Q^{11} \quad (2.1)$$

Or

$$Q = 2/3 \ C_1 b(2g)^{½} \ (H_1 - H_2)^{3/2} + C_2 b H_2 \ (2g)½(H_1 – H_2)^{1/2} \quad (2.2)$$

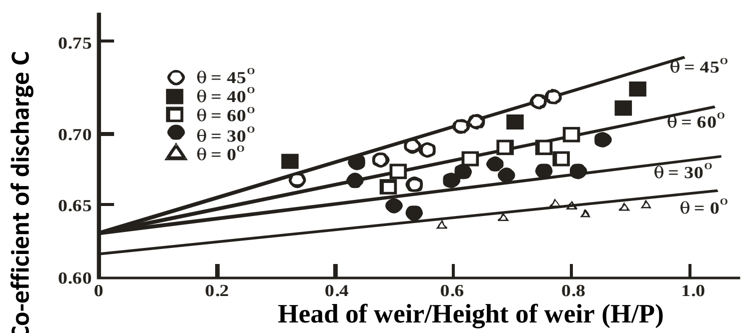



**Fig. 2: Effect of Sloping face of height less than *P* on *C***

In equation (2.2) the effect of the velocity of approach is neglected. If it is assumed that $C_1 = C_2$ =C, after rearranging the terms equation (2.2) can be put in the form

$$Q = 2/3 \ C_1 b(2g)^{½} \ (H_1)^{3/2} \quad [(1-\beta)^{1/2} \ (1+\beta/2)] \quad (2.3)$$

Where $\beta = \dfrac{H_2}{H_1}$ or in general

$$Q = 2/3 \, \psi c b(2g)^{½} \ H_1^{\ 3/2} \quad (2.4)$$

Where: $\psi = (1-\beta)^{1/2} \ (1+\beta/2)$ (2.5)

For the free condition $H_2/H_1 = 0$ and $\psi = 1$. Equating (2.4) for the free condition is

$$Q = 2/3 c b(2g)^{½} \ H_1^{\ 3/2} \quad (2.6)$$

$\psi$ represent the ratio between Q for the submerged condition and $Q_1$ for the free flow condition under same head $H_1$.

It is believed that equation (2.7) represents a general equation describing the flow over rectangular sharp-edge weirs. In equation (2.4) and (2.6), the values of C can be calculated using Rehbock's formular:

$$C = 0.611 + 0.08 H_2/P \quad (2.7)$$

It has been shown that the presence of an upstream sloping face of height P increases the discharge. Fig 1b shows the mode of the weir in this case. The relationship between the value of C, calculated from equation (2.8) and $H_1/P$ for the free condition and for different slopes is shown in figure 2. The slope 1:1 gives the maximum value of C and consequently the maximum value of Q.
For submerged flow, it is expected that equation (2.3) will be applicable after introducing a coefficient $\alpha$ to account for the modified flow field caused by the presence of the slope; equation (2.3) is then put in the form:

$$Q = \frac{2}{3} C\psi\alpha b\left(2g\right)^{\frac{1}{2}} H_1^{\frac{3}{2}} \tag{2.8}$$

If the height of the slope is less than full height of the weir, as in Fig 1c, Equation (2.8) can be applied with a different value of $\alpha$. The coefficient $\alpha$ is expected to be a function of $H_2/H_1$ and the angle of the slope $\theta$. This function can be obtained experimentally.

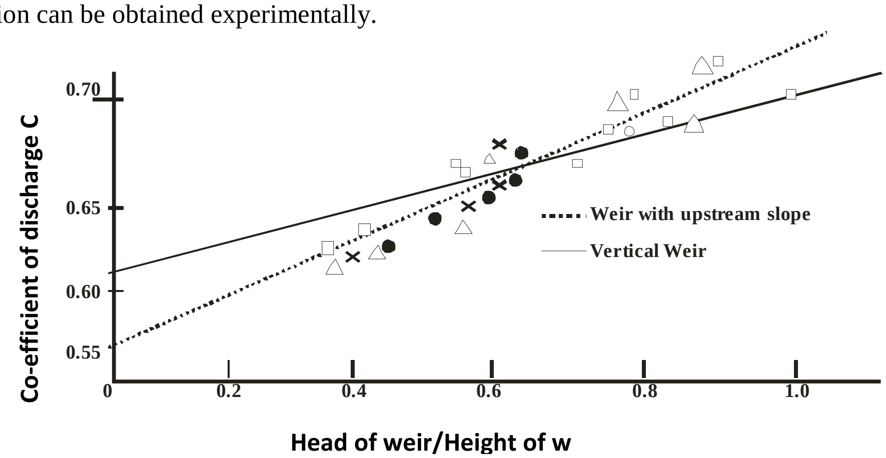


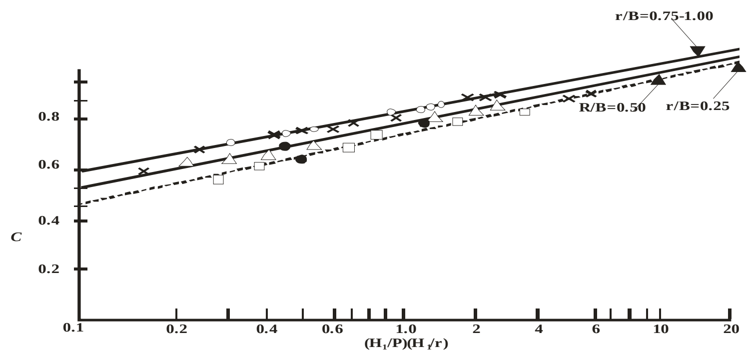

**Fig. 3: Effect of Sloping Face of Height P on C**

The value of C in Equation (2.8) may be taken from Fig. 2 depending on $H_1/P$ and the angle of slope $\theta$. For the case of a slope whose height is less than P, Fig. 1c, the value of C is taken from Fig. 3.

**Narrow Crested Weir with Edge Rounding**

Sharp crested weirs used for irrigation projects need continuous maintenance. The crests become rusted or may be damaged by sand particles moving with water or by floating debris. The use of a weir of a certain width in the direction of flow may provide the solution of such problems. Previous studies for free flow condition proved that when rounding the upstream edge of the weir, the discharge is increased. Fig 1(d) shows the shape of this weir with the variables involved in its study. Fig 4 shows the relationship between $H_1^2/Pr$ and C for different values of r/B. It can be seen that the maximum value of C is obtained when $r/B \geq 0.75$, where r is the radius of the curve and B is the weir breadth in the direction of flow. The submerged case where the downstream depth is higher than the height P of the weir was studied for $r/B \geq 0.75$. It was expected that Equation (2.3) could be written in form

$$Q = 2/3\, C\Psi\lambda b(2g)^{1/2} H_1^{3/2} \tag{2.9}$$




**Head of weir/Height of weir (H/P)**

**Fig 4: Variation of C with $H_1^2$/Pr for narrow crested weir**

Where $\lambda$ is a coefficient expected to be a function of $H_2$,$H_1$ and B. This function was obtained experimentally

## 3.0 Materials and Method

Experiments were performed in a horizontal, rectangular flume, 9.10m long, 30.48cm wide and 30.48cm deep. The depth was controlled by a sluice gate at the downstream end. The flume had a smooth bed and flexi-glass side walls. The weir models were made of brass for the vertical weirs with upstream sloping face. Water is pumped from the main laboratory sump through a 15cm diameter pipe to the channel entrance. A value in the delivery pipe is used to regulate the flow of water. The head over the weir is measure by a point gauge fitted to a traveling bridge. The downstream depth is measured by a piezometer connected to the channel bed. The discharge leaving the channel is measured by weight in a weighing tank. The upstream depth was measured at a distance five times the weir height P from weir location. The downstream distance chosen was just beyond the turbulence caused by the nappe.

Three groups of test were carried out to test the validity of Equation (2.3) and to find the values of $\psi$ in Equation (2.2). The first group dealt with the submerged flow over a sharp-edged rectangular weir. Two weir heights were used: 15.27cm and 11.37cm. The second group of tests dealt with model weirs with an upstream sloping face.

In the first case, three heights and three slopes were tested. The heights were 15.27cm, 11.70cm and 18.0cm and the slope $30^O$, $45^{O}$ and $60^{O}$ to the vertical. One experiment was carried out for the submerged condition in the case of a broad crested weir for comparison.

## 4.0 Results and Discussions

Table 1 present the results of measurements on the rectangular notch. The first 3 columns show readings obtained in the laboratory. The head H shown in the fourth column is obtained by subtracting the initial gauge in the second line of the table, for example
H = 62.1 – 3.93 = 58.68mm.

| **QTY (kg)** | **t(s)** | **Gauge Reading(mm)** | **Head H ($m^3/s$)** | **$10^4$ Q ($m^3/s$)** | **$10^2$ $H^{3}/_{2}$ ($m^3/s$)** | **Cd** |
|---|---|---|---|---|---|---|
| - | | 3.93 | 0.0 | - | - | - |
| 30 | 39.4 | 62.61 | 58.58 | 7.61 | 1.442 | 0.605 |
| 30 | 44.7 | 57.12 | 53.19 | 6.71 | 1.227 | 0.618 |
| 30 | 55.3 | 49.92 | 45.99 | 5.42 | 0.986 | 0.621 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 30 | 70.1 | 43.07 | 39.41 | 4.28 | 0.774 | 0.624 |
| 30 | 81.6 | 39.98 | 35.14 | 3.68 | 0.666 | 0.623 |
| 30 | 116 | 31.98 | 28.05 | 2.59 | 0.470 | 0.621 |
| 15 | 72.6 | 28.01 | 24.08 | 2.07 | 0.374 | 0.624 |
| 7.5 | 51.9 | 22.38 | 18.45 | 1.45 | 0.251 | 0.651 |
| 7.5 | 95.5 | 17.06 | 13.13 | 0.79 | 0.150 | 0.589 |

**Table 1: Results with rectangular notch.**

The discharge rate Q is obtained from time t required to collect quantity Qty. for example,

$$Q = \frac{30}{39.4} = 0.761kg/s = 0.761 = 7.61x10^{-4}m^3/s$$

So that, $10^4Q = 7.61m^3/s$

Since, we expect that Q will vary as $H^{3/2}$ values of $H^{3/2}$ are also tabulated. For example, in the second line of the table,

H = 58.68mm = 0.05868m

Hence, $H^{3}/_{2} = 0.01422m^{3/2}$

So that, $10^2 H^{3/2} = 1.422m^{3/2}$

**Verification of the General Flow Equation**

The testing of the validity of the general flow equation is shown in Fig. 4. The experimental points obtained from the tests using sharp-edged vertical weirs are shown together with the values obtained from Equation (2.5). The curve shows the relationship between y and $H_2/H_1$. It can be seen that there is a god agreement between the experiment and the derived equation. Thus the hypothesis used to derive Equation (2.4) us fairly well established.

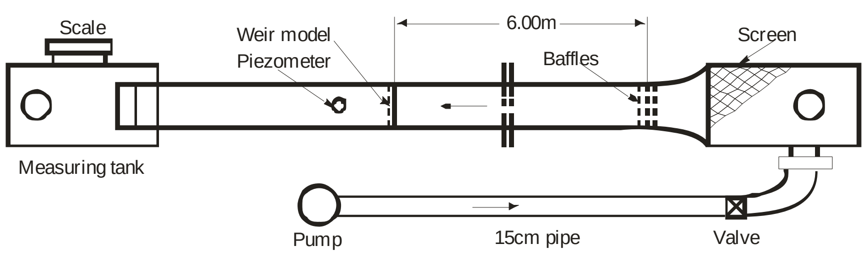



**Fig. 5. Experimental Flume**

**Effect of Upstream Slope Whose Height Is less than P**

Three different slopes were used for different values of y/P. It was found that the presence of these slopes does not have any effect on the flow over sharp edged weir in submerged case, Fig. 5 as shown in Fig. 3.

**Effects of Sloping Face of Height P**

The effect of the three slopes used, namely $30^O$, $45^O$ and $60^O$ to the vertical can be seen in Fig. 4. The $30^O$ slope has slight or no effect on the discharged and the experimental points lie on the curve for vertical weirs. For 45 $^{O}$ and 60 $^{O}$ slopes, an increase in the discharge was observed. This increase depends on the ratio $H_2/H_1$.

To obtain the value of $\alpha$ in Equation (2.8) experimentally, and enlarged chart was drawn to find the difference between the experimental points for the $45^{O}$ and $60^{O}$ slopes. $\alpha$ is the ratio between $\psi$ where there is a slope and $\psi$ for the vertical case. The change of $\alpha$ with $H_2/H_1$for difference slopes used is shown in Fig. 6. It can be concluded that using Equation (2.5) together with Equation (2.8) and Fig 6 gives the discharge for the case of the submerged flow condition for a vertical weir an upstream sloping face of height P.

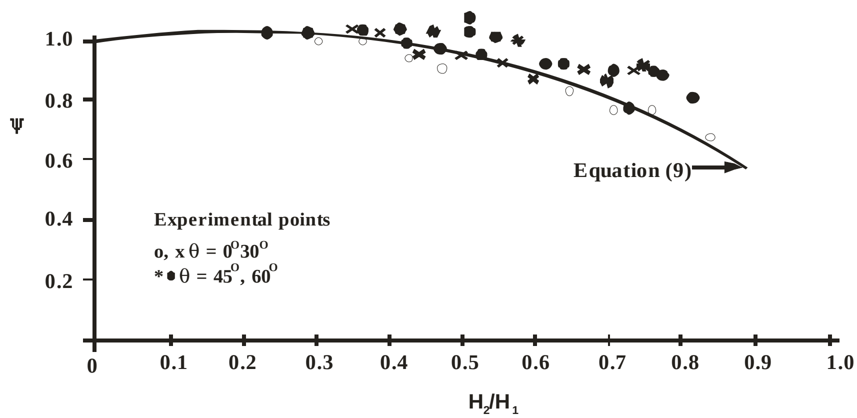



**Fig. 6: Testing the Validity of the Weir Equation**

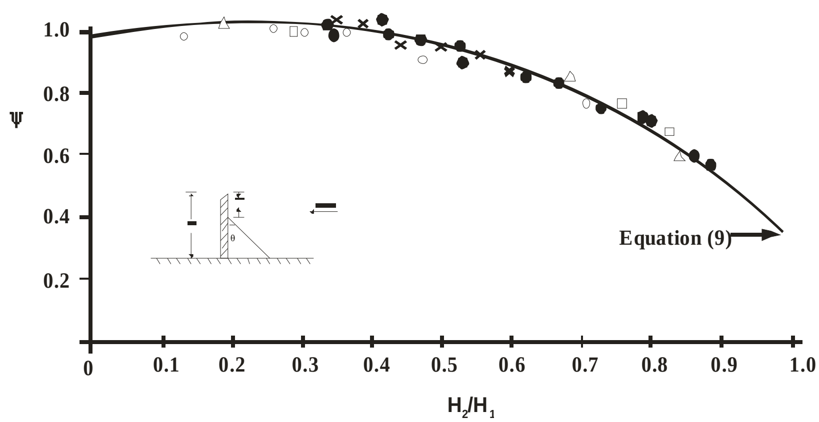



**Fig. 7: Change of $\psi$ with $H_2/H_1$ for all values of $\theta$, y and P**

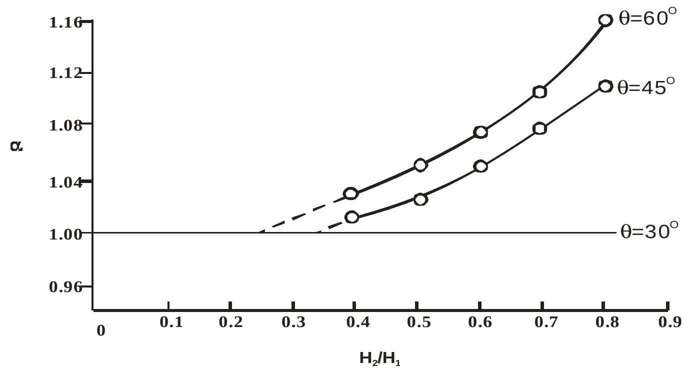



**Fig. 8: Change of α with $H_2/H_1$ weirs of sloping face**

**Narrow Crested Weir with Edge Rounding**

Previous studies showed that the edge rounding of the narrow crested weir increases the discharge. The maximum value of discharge was obtained when the ratio between the radius of rounding to the weir breadth in the direction of flow was equal to or greater than 0.75. The model used in this study with r/B=0.75 showed an increase of the discharge with the increase of the breadth B in the direction of flow. The increase depends on the ratio $H_2/H_1$. Fig. 9 shows a plot of $Q/Q_1$ against $H_2/H_1$ for the vertical sharp crested weir represented by Equation (2.5) together with the results obtained from the four models used for narrow crested weirs. The curve from the tests of the broad crested weir is also shown for comparison.

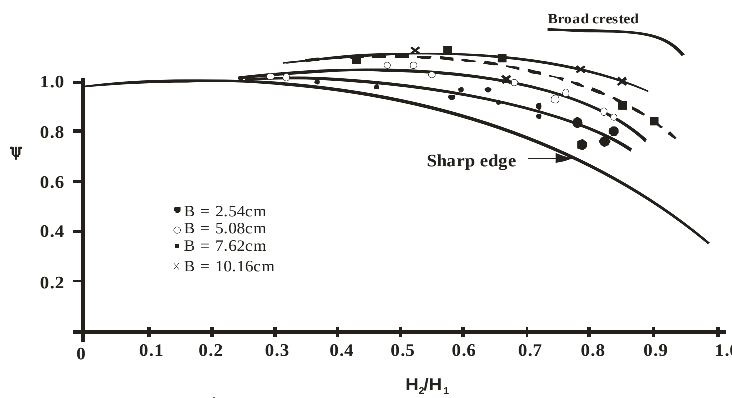



**Fig. 9. $Q/Q^1$ against $H_2/H_1$ for narrow crested weirs**

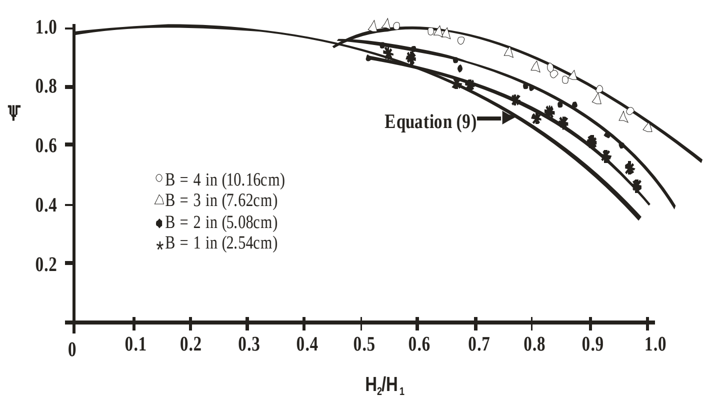



**Fig. 10. λ against $H_2/H_1$ for narrow crested weirs**

Fig 10 shows ψ against $H_2/H_1$ for different values of B. The increase of B increases the ψ value and consequently the discharge. The value of λ is a function of value of λ in Equation (2.9) as shown in Fig 11.

The value of $\lambda$ is a function of $H_2/H_1$ and B/b. It is believed that Equation (2.5) together with fig 6 and 11 gives the discharge in submerge condition for a narrow crested weir with the ratio r/B=0.75.
These results for the rectangular weir are plotted on the graph of Fig 12. The discharge Q is shown both as a function of H and of $H^{3/2}$. This later graph is seen to be a straight line through the origin, so providing experimental confirmation of the theoretical prediction of Equation (2.8).

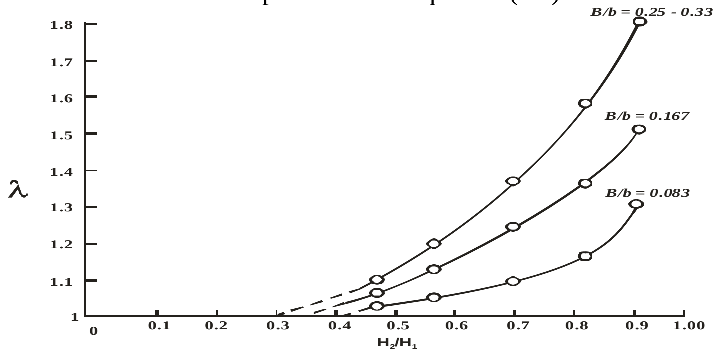



**Fig. 11. $\lambda$ against $H_2/H_1$ for narrow crested weirs**

***Example Problems with the developed equation***

(a) To find Q for sharp crested weir with P=15.14cm, $H_1$=9.24cm and $H_2$=3.71cm one may proceed as follows. For H1/P from Equation (2.7) C = 0.6598

Q unit width = $2/3(0.6598)(2g)^{½}$ $(g.24)^{3/2}$ = 0.5473 l/s/cm

$H_2/H_1 = 0.401$

And from equation (2.5) $\psi = 0.929$

Then using equation (2.4) Q = 0.5473 x 0.929 = 0.5081/s/cm

(b) In the presence of a sloping face of $45^O$, the discharge can be calculated as

$H_1$ = 8.45cm

P = 15.39cm

H2 = 4.4cm

$H_1/P = 0.55$

So using fog 2 for the $45^O$ slope C = 0.712

Q1/unit width = 2/3(0.712)1/2 (8.45)3/2 = 0.5165 l/s per cm

For $H_2/H_1$ = 0.529, from fig 5, $\psi$ = 0.87 and from fid 6a = 1.044.

Q = 0.87 x 44.04 x 0.5165 = 0.4671 l/s per cm

Q measured = 0.46 l/s per cm.

## 5.0 Conclusion

A flow equation has been developed for the discharge over a sharp rectangular weir in such a way that it can be used for both free and submerged flows. The equation been verified experimentally. The presence of an upstream shaping face whose height is less than the full height of the weir do not have any effect in the discharge case. The presence of a slope of height P increases the discharge and the increase depends on the ratio $H_2/H_1$ and on the angle of slope $\theta$ but increase as the angle increases. For a slope of $30^O$ to the vertical, there is no change in the value for the same $H_2/H_1$ and the experimental points followed the curve obtained for vertical weir. This increase is caused by the new component of the velocity in the direction of the flow resulting from the presence of the slope. This component gives rise to additional discharge.
For narrow crested weirs with rounded upstream corner, the discharge is greater than that resulting from sharp edged weirs. The increase of the width of the weir in the direction of flow increases the value of the discharge until the broad crested weir condition is reached. The value of the discharge in the free overfall condition and in submerged condition can be obtained as shown in the example below:

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