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Use of Actual Dimension in the Percolation Model for the Calculation of Breakthrough Time in Oil Recovery: Case of Some Niger Delta Reservoirs

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Abstract

This paper calculates breakthrough time for some Niger Delta reservoirs using a percolation model. The model is less cumbersome and time saving when compared with traditional methods such as the Buckley-Leverett method. By incorporating some reservoir specifics, such as actual reservoir dimension, into the model used in this paper, better results which approximate empirical values closely are obtained. The fact that breakthrough time is reservoir specific is confirmed.

Keywords: Oil recovery, Percolation model, actual reservoir length, Niger Delta, Water Drive Mechanism, Breakthrough time.

1.0 Introduction

Percolation models are often used to make predictions about important parameters in a random medium. This recommends such models for the study of flow in a rock formation (porous medium) containing hydrocarbons. Since it is known that the spatial distribution of rock types is often close to random, the connectivity of sand bodies typifies a percolation cluster.

Many authors such as in [1, 2, 3] have considered percolation models in several ways. In [1] percolation is used to study flow between two sites, [3] discusses travelling time and length in percolation clusters and in [2] percolation model is used to predict breakthrough time (t_{br}), employing a value of 1.33 for the critical exponent α, which is the fractal dimension of the shortest path. The advantage of this percolation model over conventional methods is that t_{br} is obtained more quickly, in a fraction of a second on a spreadsheet. The conventional approach such as the Buckley-Leverett method are computationally very expensive involving a lot of intricacies in obtaining required quantities and measurements used in the formula. This is responsible for the great incentive to produce much simpler models which can predict the uncertainty in performance.

This paper is motivated by earlier work done by the author [4] following the effort of other authors mentioned above on breakthrough time for some Niger Delta reservoir. Good results were obtained, but this paper goes further to use actual values specific to the reservoirs, instead of 'typical' or standard (universal) values. Actual dimensions of the reservoirs obtained from their contour maps, as given by Chevron Nigeria Limited are now used in the model.

As background to the work, we note that in reservoir engineering, one method of oil recovery is the displacement method i.e. water drive mechanism. Water injected at one well (the injector well) is used to "push out" oil at another well – the producer well. The water injected into the well to displace oil "breaks through" at some time. This time of breakthrough is a very important parameter in oil recovery because of its economic implications for the oil industry. Once the water breaks out not much oil is left in that well, breakthrough time is also considered a measure of performance.

For oil to flow out, under water pressure between two wells means that a percolation cluster (well connected open channels) exists. The percolation approach to recovery prediction is a *bond percolation* model where the oil reservoir is modelled as a percolation cluster. The flow is directed from the injector well A to the producer well B, such that we have a *directed percolation* process. The passage of time involved in the flow (before breakthrough time) motivates the study of this oil displacement process as a *directed first passage percolation*. The problem at

hand is not just to predict breakthrough time but to make prediction that will approximate empirical values well enough. This way the oil engineer can benefit from such results for the purpose of his decision making responsibility.

As stated earlier, this paper extends an earlier work done by the author [4] on the use of the classical percolation model for calculation of breakthrough time for some Niger Delta reservoirs. The assumption of uniformity of reservoir length is lifted here, and more accurate results emerge. However it is expected that building more flow physics into the model by calculating the fractal dimension of the boundary of the reservoirs will further improve the model and lead to even more accurate results.
The rest of the paper is organized as follows; in section 2 we provide the basic concepts in percolation theory. Section 3 discusses the origin of the model, the underlying results and findings, in the use of percolation theory in calculation of breakthrough time. The main results of the model and the improved results are obtained in section 4.

2.0 Basic Concepts of Percolation Theory and Preliminary Results

The medium being considered, consists of microscopic pores and channels through which the fluid might pass. Each channel will be open or closed to the passage of the fluid depending on several characteristics of the medium. In the simplest case each channel, independent of others, is open with probability p (the single parameter of the model) and closed with probability $1 - p$.
A basic question is the occurrence or not of percolation i.e. the existence of an infinite path, through open bonds (edges) only, cutting through (spanning) the medium. Let d denote dimension, $d \geq 2$. As in [5] we write $Z = \{...-1,0,1,...\}$ for the set of all integers and Z^d for the set of all vectors $x = (x_1, x_2, ..., x_d)$ of integers.

Z^d is turned into a graph called the d-dimensional cubic lattice, by adding edges $\langle x, y \rangle$ between all pairs $x, y \in Z^d$ with $\delta(x, y) = 1$. This lattice is denoted by $\mathbf{L}^d = (Z^d, E^d)$, where Z^d is the set of sites of the lattice and $E^d = \{(x, y) \in Z^d : \|x\text{-}y\| = 1\}$ is the set of nearest neighbours if $\delta(x,y) = 1$. Let p and q satisfy $0 \leq p \leq 1$ and $p + q = 1$.

We define a probability space (Ω, F, P_p) with sample space $\Omega = \{0,1\}^{E^d}$. Points of Ω are represented as $\omega = (\omega(e) : e \in E^d)$ and called *configurations*. The value $\omega(e) = 1$ corresponds to e being open and $\omega(e) = 0$ corresponds to e being closed. We have $\mu_e(\omega(e) = 1) = p$ and $\mu_e(\omega(e) = 0) = q$, where μ_e is the Bernoulli measure on (0,1).

2.0.1 Critical Percolation and Critical Exponents

A principal quantity of study in percolation theory is the percolation probability $\theta(p)$ which is the probability that a given vertex belongs to an infinite open cluster. As in [5] this is defined as

$$\theta(p) = p(0 \leftrightarrow \infty)$$

u↔ v denotes the vertex u is connected to the vertex v

Because $\theta(p)$ is the probability that the origin belongs to an infinite cluster, it is more elegantly defined by

$$\theta(p) = p(|c| = \infty) \tag{2.1}$$

where $|c|$ is the cardinality of the open cluster of the origin, $|c|$ is thus a random variable which can take the discrete values $1,2,...,\infty$.

Fundamentally there exists a critical value $p_c = p_c(d)$ of p (called the critical probability) such that

$$\theta(p)=\begin{cases}0 \text{ if } p<p_c & \text{subcritical phase}\\ >0 \text{ if } p>p_c & \text{supercritical phase}\end{cases}$$

We define this critical probability as

$$p_c(d)=\sup\{p:\theta(p)=0\}. \tag{2.2}$$

It is believed that percolation probability has a singularity at $p=p_c$ and that there is a "power law behaviour" at and near this singularity.

The nature of the singularity is supposed to be canonical, it is expected to have certain general features in common with phase transitions in other physical systems. These features are referred to as "*Scaling limits*" and they relate to "*critical exponents*" as in [6, 7]. In what follows, we shall examine the behaviour of the exponents in the neighbourhood of the critical points.

Near the critical point: as *p* approaches p_c from above (or beneath); $\theta(p)$ and $\chi(p)$ are believed to behave as powers of $|p-p_c|$. $\chi(p)$ is the mean size of an open cluster which is interpreted as the mean number of vertices in the open cluster at the origin. According to Kesten [8], the following exists

$$\begin{aligned}&\gamma=-\lim_{p\to p_c}\frac{\log\chi(p)}{\log|p-p_c|}\\ &\beta=-\lim_{p\to p_c}\frac{\log\theta(p)}{\log|p-p_c|}\\ &\delta^{-1}=-\lim_{n\to\infty}\log\frac{P_{pc}(|c|\geq n)}{\log n}.\end{aligned} \tag{2.3}$$

The quantities γ,α,δ are called critical exponents. There are other critical exponents as established in [7,9,10].

Definition: (Power Law)

An interesting concept in percolation theory is the power law relationship. A power law is defined simply as any polynomial relationship that exhibits the property of scale invariance. Most known power laws relate two variables and have the form $f(x)=ax^k$, where *k* is the scaling exponent. For example, $p(k)\sim k^r$ and $p(ak)\sim a^{-r}p(k)$ defines a power law, where $p(k)$ is the probability of occurrence of some event *k*. The relationship does not depend on the scale *k*, but on the coefficient *a*. Next we describe the Buckley-Leverett Method used for calculating breakthrough time.

The conventional method for calculating breakthrough time (t_{br}) is the Buckley-Leverett method, as given in [11]. By the Buckley-Leverett method the time at which breakthrough occurs is given as

$$t_{bt}=\frac{W_{id_{bt}}}{q_{id}} \tag{2.5}$$

$W_{id_{bt}}$ is the dimensionless number of pore volumes of water injected at time of breakthrough. q_{id} is the dimensionless injection rate. t_{bt} is the time of breakthrough, which is analogous to t_{br} being calculated in this paper. As stated earlier the Buckley-Leverett method is computationally very expensive.

To use the Buckley-Leverett method, the relative permeability curves, the Buckley-Leverett equation and the expression for average water saturation are required.

To utilize each of these components, one needs to have values for some other important quantities. These include velocity of the plane V_{sw}, full differential of water saturation dS_w, fractional flow of water at any point in the reservoir f_w, current value of water saturation S_{we}, flood front saturation S_{wf}, fractional flow at the producing well f_{we}, the cumulative water injected w_i, the injection rate q_i among others. Several values which are required for detailed calculations must be obtained first, before the oil recovery

calculations can be made. Thus a lot of intricacies is involved when using the Buckley-Leverett method. In the following section we discuss some known results in percolation theory.

3.0 Result in Percolation Theory: Use of Critical Exponents and Scaling Laws of Percolation Theory in Predicting the Breakthrough Time for Oil Recovery

According to Andrade et al [1], there is qualitative resemblance between the shortest path and the minimal travelling time of the tracer particle. The shortest path connecting two sites on a percolation cluster is defined as *shortest path* or *chemical distance*. It is denoted by *l* and for some particular case, its value may be denoted l^*. The geometrical distance between the sites on the cluster is denoted *r*. l^* scales with geometric distance *r* as follows:

$$l^* \sim r^{d_{\min}}$$

$$d_{\min} = \begin{cases} 1.13 \pm 0.02 (d = 2) \\ \\ 1.374 \pm 0.005 (d = 3) \end{cases}$$

(3.1)

$d_{\min}$ is the fractal dimension of the shortest path

3.0.1 Minimal Traveling Time (Breakthrough Time) and Fastest Path

Minimal Traveling Time (or breakthrough time) has to do with the dynamics of the flow on the percolation cluster. In the model by King et al [2], the simulation is run for the flow tracer particles starting at the *injection point A* to the *recovery point B*.

The *minimal travelling time* $t_{\min}$ corresponds to the breakthrough time of the liquid (water) that displaces the oil during recovery [3]. Exponents d_x where *x* denotes

$$l_{\min}, t_{\min}, \tilde{l} \text{ or } \tilde{t}$$

are defined by

$$x^* \sim r^{dx}.$$

(3.2)

Here x^* is the characteristic length or time of the corresponding distribution. Andrade et al [1] obtained $t_{\min}$ which scales with l^z, i.e. $t_{\min} \sim l^z$ where $z \approx 1.17$. The expression "scales as" denoted by ~ means "is proportional to, in the limit".

For the particle travelling between two points *A* and *B*, we have $d_{\min} < d_{tm} < d_B$.

d_{tm} is the fractal dimension of a subset of the system

$d_{\min}$ is the fractal dimension of the minimal path and

d_B is the fractal dimension of the entire cluster.

Since *l* scales as $r^{d_{\min}}$, from (3.2), it is proposed that $t_{\min}$ scales as $r^{d_{tm}}$ where

$$d_{tm} = z_{d_{\min}} = 1.33 \text{ and}$$

$$t_{\min} \sim r^{dt_{\min}}$$

(3.3)

$$d_{t_{\min}} = 1.33 \pm 0.05 \text{ (as obtained in [1])}$$

$$t_{\min} \sim r^{zd_{\min}}$$

such that $t_{\min} \sim r^{1.33}$.

(3.4)

3.0.2 Predicting Breakthrough Time using Percolation Model

Andrade et al [1] have shown that the breakthrough time is strongly correlated with the shortest path length (chemical path). There is a power law relationship

$$t \sim r^{\alpha} \tag{3.5}$$

The details about the variables in this expression are given below:

α here is $d_{t_{\min}}$ (see (3.3)).

This is a particular case of the general relationship obtained by Andrade et al [1] and was given in equation (3.2).

King et al [2] considered a single well pair separated by a Euclidean distance *r*, such that the breakthrough time corresponds to the *first passage time* for transport between the injector and the producer. Basically equation (3.4) provides the relationship applied to real field data. When dealing with more practical aspects of reservoir engineering such as well test analysis, it is conventional to switch to what are called practical or field units [12]. To apply the above results to real field data, units from the dimensionless scaling form of equation (3.4) must be converted to real field units. For example time and distance which are prominent in the scaling law have a non-linear relationship.

In line with this, King et al [2] interpreted equation (3.5) as follows:

$$t \sim r^{\alpha} \text{ means } \frac{t}{t_0} = \left(\frac{r}{r_0}\right)^{\alpha}, \tag{3.6}$$

where r_0 is typical length and is taken as sand body dimension r_s, t_0 is time taken to transit through one sand body.

In a homogenous region of permeability *k* (in Darcies), and for a pair of wells separated by a distance *r* (in cm), the transit time (in seconds) is given by [1] as

$$t_0 = \frac{4\eta U r^2}{3k\Delta P}$$

η is viscosity of the fluid (in centipoises). ΔP is the pressure drop between the wells (in atmospheres). Now

$$U = \ln\left(\frac{r}{r_w}\right) \tag{3.7}$$

where r_w is well-bore radius (in cm). ΔP is linear in the number of sand bodies between the wells such that the drop for each body is $\Delta P/(r/r_s)$. A later simulation [5] gave an expression for t_0 as

$$t_0 = \frac{12\eta U r_s^2}{15k\,\Delta P/(r/r_s)} \tag{3.8}$$

where *U* is the log of the dimensionless sand body size. The results established in [1,2] have been summarized in theorem 3.3 below. The critical exponent which is applied here is α, the fractal dimension of the shortest path.

3.3 Theorem

Let the minimal travelling time $t_{\min}$ correspond to the breakthrough time t_{br} of water that displaces oil. $t_{\min}$ "scales as" $r^{zd_{\min}}$ is denoted $t_{\min} \sim r^{zd_{\min}}$

$zd_{\min} = \alpha = 1.33$ is the fractal dimension of the shortest path. There is a power law relationship namely $t_{\min} \sim r^{\alpha}$

Let $\frac{t}{t_0} = \left(\frac{r}{r_0}\right)^{\alpha}$ denote "$t_{\min} \sim r^{\alpha}$" in real field units. Then $t_{br} = \left(\frac{r}{r_0}\right)^{\alpha} t_0$ where α is the fractal dimension of the shortest path, *r* is the distance between a pair of wells, the injector well and the production well. r_0 is typical length, t_0 is time taken to transit through one sand body and is given as $t_0 = \frac{12\eta \ln(r/r_w) r_s^2}{15k\Delta P/(r/r_s)}$, η is viscosity, *k* is permeability, ΔP is the pressure drop between the wells and r_ω is well bore radius. The following section gives the main results of the paper.

4.0 Application of Model to Real Field Data: Analysis of Results for some Niger Delta Reservoirs

Time of breakthrough t_{br} is now calculated for some specific reservoirs using the results in theorem 3.3. We proceed as follows.

Some known characteristics of the Niger-Delta field/crude such as average field dimensions, viscosity, permeability, gravity values (API), were substituted into the scaling equation (3.6) to obtain t_{br} for the reservoirs of interest. Some other quantities such as well bore radius, change in pressure, well separation etc, were used as given in [2]. An average field in the Niger Delta is of length 8km, width 2km and has excellent sand quality (even though the sands are polygenetic). Porosities and permeabilities are high, up to 40% porosity rate and permeability of between 1 and 2 Darcies [13]. Therefore we use *k* = 1.5D. Also $r_s = 8km$ (typical length) is used for all the reservoirs. The API values from [13] were used to calculate density (g/cm^3) used in the conversion of units of viscosity.

Both the Ewan and the Opolo fields are offshore Niger Delta fields. Data and other information on these fields such as well separation, API values, oil viscosity, permeability, reservoir length etc. as stated were supplied by Chevron Nigeria Limited.

The values of t_{br} were calculated for reservoirs in both fields using a value of 1.33 for α which is the fractal dimension of the shortest path. Given below is the calculation that yielded t_{br} values for the selected reservoirs. The results are shown on table 4.1 below along with empirical values.

Ewan D-01

D-01/EW-01

$$t_0(D-01/EW-01) = \frac{12\eta U r_s^2}{15k\left[\Delta P/\left(\frac{r}{r_s}\right)\right]}$$

Well separation *r* = 0.91 = 91000cm

Reservoir length r_s = r_0 = 8km = 800000cm

Viscosity $\eta = 2.04$

Well bore radius = 20.32cm

$$U = \ln\left(\frac{r}{r_\omega}\right) = \ln\frac{91000}{20.32} = 3.65$$

$K = 2D$

$$t_0(D-01/EW-01) = \frac{12 \text{ x } 2.04 \text{ x } 3.65 \text{ x } (800000)^2}{15 \text{ x } 2 \text{ x } \left[150/\frac{0.91}{8}\right]}$$

$$= \frac{57185280000000}{39560.44}$$

$$t_0[\text{in secs}] = 1445516784 \sec s$$

$$t_0[\text{in years}] = \frac{1445516784}{315360000} = 45.84 \text{years}$$

$$t_{br} = \left(\frac{r}{r_s}\right)^{1.33\pm0.05} = 2.51 \text{ years or } 2.8 \text{ years}$$

Ewan C-02
C-02/EW-02

Well separation r = 1.15km = 115000cm
Reservoir length r_0 = 8km = 800000cm
Viscosity $\eta = 3.49$

ΔP = 500psi
Well bore radius = 20.32cm

$$U = \ln\frac{115000}{20.32} = \ln\left(\frac{r}{r_\omega}\right) = \ln(5659.45) = 3.75$$

$$K = 2D$$

$$t_0(C-02/EW-02) = \frac{12 \text{ x } 3.49 \text{ x } 3.75 \text{ x } (800000)^2}{15 \text{ x } 2 \text{ x } \left[500\text{psi}/\frac{1.15}{8}\right]}$$

$$= \frac{1005120000000000}{104166.67} = 964915169.1\text{secs}$$

$$t_0[\text{in years}] = \frac{964915159.1}{31536000} = 30.59\text{years}$$

$$t_{br} = \left(\frac{1.15}{8}\right)^{1.33\pm0.05} = 2.32\text{years or } 2.56 \text{ years}$$

Opolo Field

$$t_0(Opolo) = \frac{12\eta U r_s^2}{15K\left[\Delta P/\left(\frac{r}{r_s}\right)\right]}$$

Well separation r = 0.72km = 72000cm
Reservoir length = r_0 = 8km = 800000cm
Viscosity $\eta = 0.2$
ΔP = 10psi
Well bore radius r_ω = 8 inches = 20.32cm

$$U = \ln\frac{72000}{20.32} = \ln 3543.30 = 3.55$$

$$K = 2D$$

$$t_0(Opolo) = \frac{12 \text{ x } 0.2 \text{ x } 3.55 \text{ x } (800000)^2}{15 \text{ x } 2 \text{ x } \left[10\text{psi}/\frac{0.72}{8}\right]}$$

$$t_0[\text{in seconds}] = \frac{6816000}{3333.33} = 2044.82\text{secs}$$

$$t_0[\text{in years}] = \frac{2044.82}{31536000} = 0.0001\text{years}$$

$$t_{br}(Opolo) = (0.09)^{1.33\pm0.05} \text{ x } 0.0001$$

$$= 4.1 \text{ x } 10^{-6} \text{ or } 4.6 \text{ x } 10^{-6}$$

t_{br} values for Ewan C-03 and Ewan B-12 reservoirs were obtained by the same process. Data used and all results are displayed on table 4.1.

Table 4.1

Field Reservoir	Well Separation Acres km	API Values	Viscosity (CP)	Permeability (md)	Breakthrough Time Range (in years)	
					Theoretical Value with (S.E.)	Empirical Value with (S.E.)
Ewan						
B-12/EW-02 (One well)	2018 2.90	17.3	8.9	530-1500	52.5 (one well)	1.5 (One well)
C-02/EW-02	326 1.15	21.5	3.49	530-2000	2.56(0.12)	5.0(1.70)
C-03/EW-01	394 1.30	20.1	5.32	505-2673	4.03(0.18)	5.6(1.74)
D-01/EW-01	204 0.91	25.4	2.04	100-2000	2.80(0.16)	4.5(1.08)
Opolo						
D-01/OP-02	130 0.72	40	0.2	500-2000	4.1×10^{-6} (2.30×10^{-7})	1.8(1.10)

In table 4.1 standard error for the theoretical and empirical values of breakthrough time is given in bracket beside the values.

4.0.1 Analysis/Remark

From the information displayed on table 4.1, we make the following remarks:

(i) Generally, in the literature and also on the field, once η which is viscosity for oil is much bigger than 1, which is the viscosity for water, water will likely finger through, and one will not do a 'good' job with the water drive mechanism. An alternative recovery method may be applied. According to Dake [12] consideration could be given to the application of thermal recovery methods with the aim of reducing the viscosity ratio. However in some cases (as is observed for some Niger Delta fields) where water flooding seems a feasible option despite the high viscosity of the oil, very high pressure is used. Indeed ΔP is as high as 500psi. This way a good percentage of recovery is still made. ΔP is a reflection of the reservoir force i.e. the force behind the drive.

(ii) We observe that we have more accurate values when 1.33 – 0.05 = 1.28 is used for α. This is because some further work by Ogbogbo forthcoming in [14] shows that actual fractal dimensions of the Ewan reservoirs are all less than 1.33. Therefore values of α greater than 1.33 lead to less accurate results for these reservoirs. This explains why t_{br} values displayed on table 4.1 are those obtained by using α = 1.28.

(iii) Ewan B-12 is a one well reservoir. In Ewan B-12, rock is like shale, oil is glued to the sand, and has a high viscosity of 8.9cp. In this reservoir/well unless pressure is radically increased recovery will not be meaningful. Field Engineers confirm that this well is not doing well as a result of low API and high viscosity. It is not surprising therefore that the model does not yield any meaningful result for t_{br} even with ΔP of 1000. The viscosity is quite high, the reservoir being a one well reservoir has value 1 for r/r_s. The geology of the field is what could provide explanation for the poor performance of Ewan B-12.

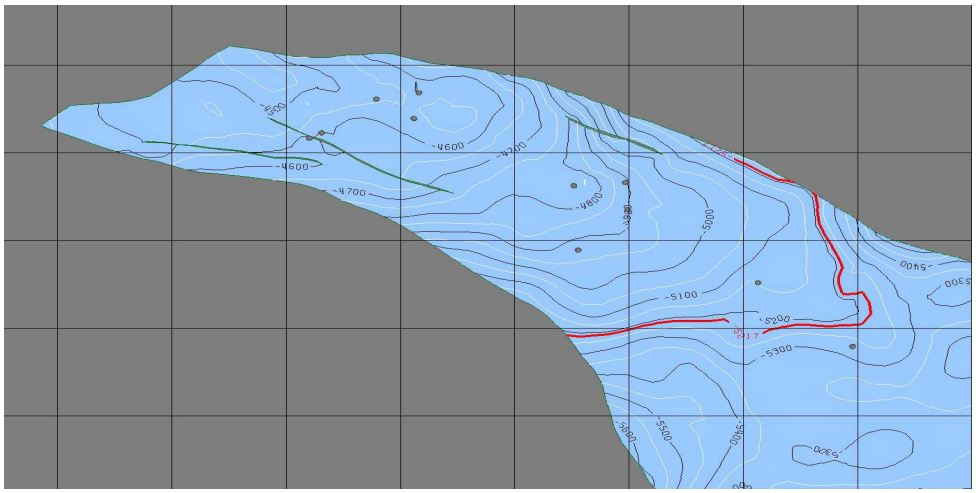
(iv) The Opolo field has oil of very low viscosity of 0.2cp, thus the pressure required here need not be high, hence ΔP = 10psi is used. Despite the very low value of ΔP used the result of Opolo field is very low. This is largely because of the value of well separation which is quite low and the low viscosity of crude in this field. ΔP = 150psi is also used for D-01/Ew-01 since its viscosity is not very high.

(v) A comparison of the theoretical values of t_{br} (as calculated from the model) with the empirical values obtained from the field (Chevron Nigeria Limited), reveals that the theoretical values are close to the average value of t_{br} (empirical value). These values could have approximated better had some of the flow physics been taken into serious cognizance. By some of the assumptions of the model, most of the flow physics is lost.

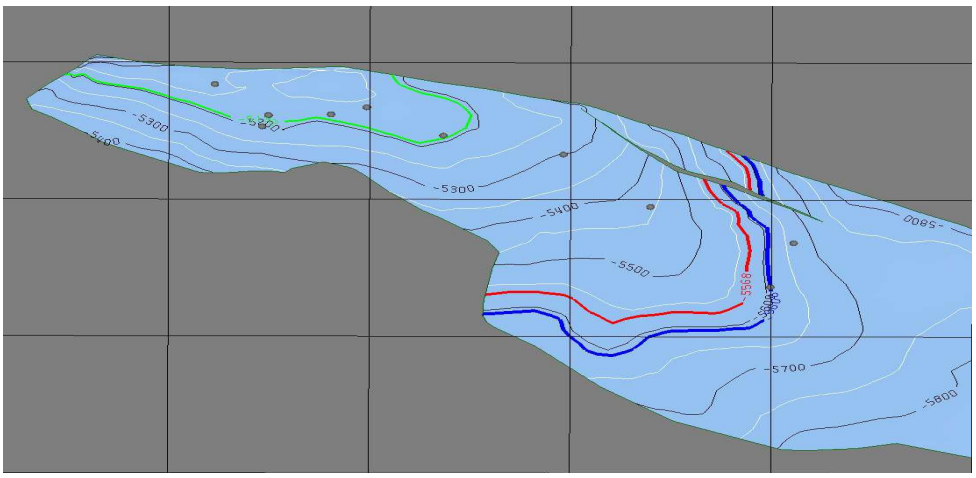
(vi) Moreover, a value of 500psi for ΔP is an average value, this means that ΔP could be much higher for reservoirs with oil viscosity of higher than 5cp.

The contour maps of some of the reservoirs are shown below. Each square represents 1km.

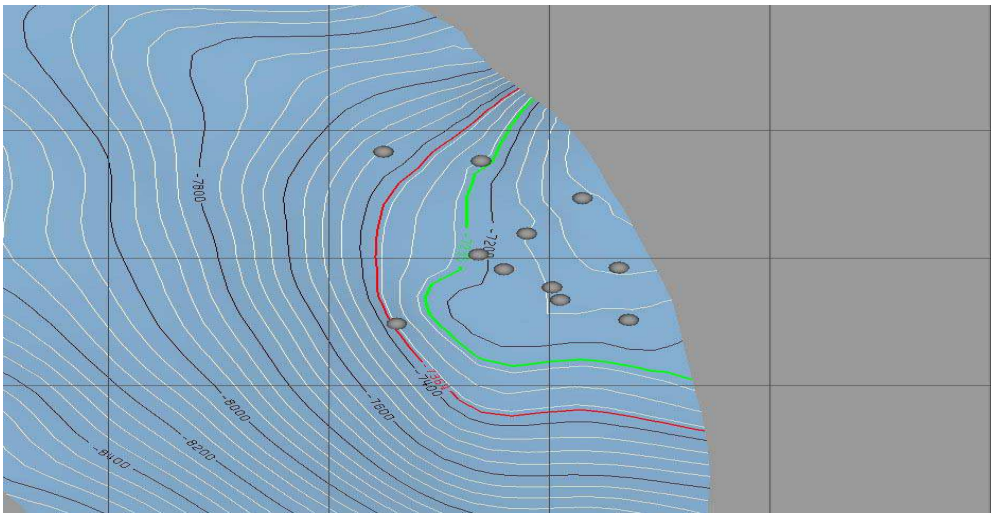
EWAN C – 02



EWAN D – 01



OPOLO D – 01



From the maps (Ewan C-03 is not displayed) we observe that actual reservoir length for Ewan D-01, Ewan C-03, Ewan C-02 and Opolo D-01 are 5km, 8km, 8km and 5km respectively. The typical length of 8km used for all reservoirs is true for Ewan C-02 and Ewan C-03 but led to less accurate result for Ewan D-01 and Opolo D-01.

The calculation for t_{br} is now repeated for Ewan D-01 and Opolo D-01 with $r_0 = 5km$. The new values are displayed on table 4.2. The calculation is shown below:

Ewan D-01

D-01/EW-01

$$t_0(D-01/EW-01) = \frac{12\eta U r_s^2}{15k\left[\Delta P/\left(\frac{r}{r_s}\right)\right]}$$

Well separation r = 0.91 = 91000cm

Reservoir length r_s = r_0 = 5km = 500000cm

Viscosity $\eta = 2.04$

Well bore radius = 20.32cm

$$U = \ln\left(\frac{r}{r_\omega}\right) = \ln\frac{91000}{20.32} = \ln 4478.35 = 3.65$$

$$K = 2D$$

$$t_0(D-01/EW-01) = \frac{12 \text{ x } 2.04 \text{ x } 3.65 \text{ x } (500000)^2}{15 \text{ x } 2 \text{ x } \left[150/\frac{0.91}{5}\right]}$$

$$= \frac{22338000000000}{24725.27} = 903448000 \sec s$$

$$t_0[\text{in years}] = \frac{903448000}{315360000} = 28.65 \text{years}$$

$$t_{br} = \left(\frac{r}{r_s}\right)^{1.33\pm0.05} = 2.98 \text{ years or } 3.2 \text{ years}$$

Opolo Field

$$t_0(Opolo) = \frac{12\eta U r_s^2}{15K\left[\Delta P/\left(\frac{r}{r_s}\right)\right]}$$

Well separation r = 0.72km = 72000cm

Reservoir length = r_0 = 5km = 500000cm

Viscosity $\eta = 0.2$

ΔP = 10psi

Well bore radius r_ω = 8 inches = 20.32cm

$$U = \ln \frac{72000}{20.32} = \ln 3543.30 = 3.55$$

$$K = 2D$$

$$t_0(Opolo) = \frac{12 \text{ x } 0.2 \text{ x } 3.55 \text{ x } (500000)^2}{15 \text{ x } 2 \text{ x } \left[10\text{psi}/\frac{0.72}{5}\right]}$$

$$t_0[\text{in seconds}] = \frac{2.13 \text{ x } 10^{12}}{2083.33} = 1022400000\text{secs}$$

$$t_0[\text{in years}] = \frac{1022400000}{31536000 \text{ years}} = 32.42\text{years}$$

$$t_{br}(Opolo) = (0.144)^{1.33} \text{ x } 32.42 = 2.46 \text{ years}$$

$$= 2.46 \text{ years or}$$

$$= (0.144)^{1.28} \text{ x } 32.42 = 2.71 \text{ years}$$

Table 4.2

Field Reservoir	Well Separation Acres / km	API Values	Viscosity (CP)	Permeability (md)	Breakthrough Time Range (in years)	
					Theoretical Value with (S.E.)	Empirical Value with (S.E.)
C-02/EW-02	326 / 1.15	21.5	3.49	530-2000	2.56(0.12)	5.0(1.70)
C-03/EW-01	394 / 1.30	20.1	5.32	505-2673	4.03(0.18)	5.6(1.74)
D-01/EW-01	204 / 0.91	25.4	2.04	100-2000	3.2(0.15)	4.5(1.08)
Opolo D-01/OP-02	130 / 0.72	40	0.2	500-2000	2.71(0.17)	1.8(1.10)

Remark about Results

Comparing table 4.1 with table 4.2, we observe that t_{br} values for Ewan D-01 and Opolo D-01 (3.2 and 2.71 respectively) are closer to the empirical values of 4.5 and 1.8. When typical length was used for these reservoirs the values obtained were not that close to empirical value. In fact the Opolo reservoirs yielded a very small t_{br} value which could not be reckoned with. This indicates an improvement on earlier results for prediction of time of breakthrough.

5.0 Conclusion/Recommendation

The paper has demonstrated that using actual value of reservoir dimension in the percolation model leads to an improvement in the prediction of breakthrough time in oil recovery.
Generally, the percolation model is used to obtain breakthrough time more quickly than by the use of conventional methods. In the absence of other geological exceptions the results obtained are quite useful.

The model made a number of simplifying assumptions, such as considering oil reservoirs as square boxes. It also assumes uniformity of flow pattern for all reservoirs. This is not the case, considering the varying shapes of the contour maps of the reservoirs. Thus calculating the actual fractal dimensions of the boundaries of the reservoirs, will capture more of the flow physics leading to better results. Further work on incorporating more of the flow physics in the model is recommended.

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