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Application Of Adomian's Decomposition Method In Solving Nonlinear Partial Differential Equations

****Olanrewaju, P. O. and ** Eegunjobi, A. S. and ***Oke O. A***

***Department of Pure and Applied Mathematics**
Ladoke Akintola University of Technology, Ogbomoso, Nigeria
****Department of Physical Sciences**
BELLS University of Technology, Ota, Nigeria
*****Department of Computer Science and Engineering**
Ladoke Akintola University of Technology, Ogbomoso, Nigeria
[1]Corresponding author: email: Oladapo_anu@yahoo.com : Tel. +2348138485550

Abstract

The Adomian's decomposition method is a powerful method which considers the approximate solution of a nonlinear equation as an infinite series usually converging to the accurate solution. It is shown in literature that Adomian's decomposition method gives better results than any other computational techniques. We use this method to tackle simple heat equation and compare the result with the closed form solution of the giving problem.

Keywords: Adomian decomposition method; accuracy; nonlinear equation; closed form solution.

1.0 Introduction

In recent decades, the Adomian decomposition method has been shown to be extremely efficient and has substantial advantages in solving a wide class of algebraic, differential, and partial differential equations without linearization or smallness assumptions. This method leads to computable, accurate, approximate convergent solutions to linear and nonlinear deterministic and stochastic operator equations (see, [1-2]).

The Adomian decomposition method has been applied to a wide class of stochastic and deterministic problems in many interesting mathematics and physics areas(see, [3-7]). This method has some significant advantages over numerical methods. It provides analytic, verifiable, rapidly convergent approximation which yields insight into the character and the behavior of the solution just as in the closed form solution. Adomian gave a review of the decomposition method in(see,[6,]). Several authors have compared the ADM with some existing techniques in solving different types of problems. In [5], they have compared the Adomian's decomposition method and the perturbation technique are used in solving random non-linear differential equations. Particularly, [8] compared Adomian's decomposition method and Runge–Kutta methods for approximate solutions of some predator prey model equations. [6] proposed a new approach to develop a non-perturbative approximate solution for the Thomas–Fermi equation. He showed that the Adomian's decomposition method minimizes the computational difficulties of the Taylor series in that the components of the solution are determined elegantly by using simple integrals. Moreover, he showed that the Adomian's decomposition method is fast convergent and it attacks non-linear problems in a similar manner as linear problems.

In this paper, we use the Adomian's decomposition method to solve simple heat equation and compare the result with the closed form solution. In section 2 we give general description of Adomian's decomposition method and in section 3 we solve simple heat equation and we consider the result obtained to analytic result of the problem.

2.0 Adomian's decomposition method

According to [1], we consider the nonlinear equation of the form

$$f(y)= 0, \tag{2.1}$$

which can be written as the following canonical form

$$y = b + R(y), \quad (y \in R) \tag{2.2}$$

when R is a nonlinear function and b is a constant.

The Adomian's decomposition method consists of representing the solution of (2.2) as a series

$$y = \sum_{n=0}^{\infty} y_n \,, \tag{2.3}$$

and the nonlinear function as the decomposed form

$$R(y) = \sum_{n=0}^{\infty} B_n \,, \tag{2.4}$$

where B_n (n = 0, 1, 2, …) are the Adomian polynomials of $y_0, y_1, y_2, \ldots, y_n$ given by

$$B_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[R\left(\sum_{i=0}^{\infty}\lambda^i y_i\right)\right]_{\lambda=0}, \quad n=0,1,2,\ldots \tag{2.5}$$

Upon substituting (2.3) and (2.4) into (2.2) yields

$$\sum_{n=0}^{\infty} y_n = b + \sum_{n=0}^{\infty} B_n \tag{2.6}$$

The convergence of the series in (2.6) gives the desired relation

$$\begin{cases} y_0 = b \\ y_{n+1} = B_n, \; n=0,1,2,\ldots \end{cases} \tag{2.7}$$

The polynomials B_n are generated for all kind of nonlinearity by Adomian [3]. The first few polynomials are given by

$$\begin{aligned} B_0 &= R(y_0) \\ B_1 &= y_1 R'(y_0) \\ B_2 &= y_2 R'(y_0) + \frac{1}{2} y_1^2 R''(y_0) \\ &. \\ &. \\ &. \end{aligned} \tag{2.8}$$

It should be pointed out that B_0 depends only on y_0, B_1 depends only on y_0 and y_1, B_2 depends only on y_0, y_1, and y_2, and so on.

Hence we may also write B_n as $B_n(y_0, y_1, y_2, \ldots, y_n)$.

Let $S_m = y_0 + y_1 + y_2 + \ldots + y_n$.

Then $S_m = b + B_0 + B_1 + \ldots + B_{m-1}$ in the $(m+1)$ term approximation of y. Such S_m can serve as a practical solution in each iteration.

3.0 Illustrations with examples

We want to solve the heat equation of the form

$$u_t = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) \text{ with the initial condition } \quad u(x, 0) = x.$$

To obtain the solution, we use the recursive relation in (2.8) by taking $u_0 = x$. The first Adomian polynomial is $B_0 = x$. Therefore, we have $u_1 = t$ and $B_1 = t$. Finally, $u_2 = 0$ which follows that

$u_n (x, t) = 0$ for all n greater or equal to 2. Putting these individual terms in (2.6) or S_m one gets the exact solution.

Similarly,

$$u_t = \frac{\partial}{\partial u}\left(\frac{1}{u}\frac{\partial u}{\partial x}\right) \text{ with the initial condition } u(x,0) = \frac{1}{x}$$

The exact solution for the heat equation is $u(x,t) = \dfrac{1}{x-t}$.

For Adomian's decomposition method, we use the recursive relation again to obtain

$u_0 = \dfrac{1}{x}$, $A_0 = \dfrac{1}{x}$, $u_1 = \dfrac{t}{x^2}$, $A_1 = -\dfrac{t}{x^2}$, $u_2 = \dfrac{t^2}{x^3}$, $A_2 = -\dfrac{t^2}{x^3}$ and so on. In this manner the rest of the terms of the decomposition series have been calculated using MAPLE. Substituting these individual terms we have

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots \qquad , u(x,t) = \frac{1}{x}\left[1 + \frac{t}{x} + \frac{t^2}{x^2} + \frac{t^3}{x^4} + \ldots\right]$$

which gives the exact solution in the closed form.

Note that we used section 2 in order to solve problems in section 3 for better understanding.

Conclusions

We have utilizes the Adomian's decomposition method to heat equations and we established that it has higher accuracy because it gives the same result with the analytic results. Many researchers have utilized this method to several nonlinear equations in Fluid dynamics (see,[7,9,10])

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