Journal of the Nigerian Association of Mathematical Physics
Volume **16 (May, 2010), pp 293 – 300**

On the influence of buoyancy and suction/injection In Heat and Mass transfer problems

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Abstract

In this paper, we examined the influence of buoyancy and suction/injection in the problem of unsteady convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture in an optically thin environment is presented. The dimensionless governing equations for this investigation are solved numerically by the fourth-order Runge–Kutta integration scheme along with shooting technique.. Graphical results for velocity, temperature and concentration profiles based on the numerical solutions are presented and discussed.

Keywords: Boundary layer flow; heat and mass transfer; binary mixture; buoyancy; porous plate; suction/injection

Nomenclature

(x, y)	Cartesian coordinates	C_∞	free stream concentration
(u, v)	velocity components	C	concentration of the fluid
T_w	surface temperature	C_w	surface concentration
T_∞	free stream temperature	D_a	Damköhler number
g	gravitational acceleration	k	thermal conductivity
Q	heat generation coefficient	c_p	specific heat at constant pressure
T	fluid temperature	D	diffusion coefficient
R_A	nth order irreversible reaction	U_0	plate uniform velocity
R_G	universal gas constant	$\Pr$	Prandtl number
c	suction parameter	E	activation energy
S_c	Schmidt number	b	heat generation parameter
G_r	thermal Grashof number	G_c	solutal Grashof number

Greek symbols

θ	fluid temperature
φ	fluid concentration
η	similarity variable
γ	activation energy parameter
β	thermal volumetric-expansion coefficient
β_c	concentration volumetric-expansion coefficient
σ	Stefan-Boltzmann constant
α	absorption coefficient
ρ	fluid density
υ	kinematic viscosity

1.0 Introduction

The boundary layer flow of a binary mixture of fluids is always important in view of its applications in various branches of engineering and technology. One of the examples is an emulsion which is the dispersion of one fluid within another fluid. Typical emulsions are oil dispersed within water or water within oil. Another example where the mixture of fluids plays an important role is in multigrade oils. Polymeric type fluids are added to the base oil so as to enhance the lubrication properties of mineral oil [1]. Moreover, all industrial chemical processes are designed to transform cheaper raw materials to high value products through chemical reaction. Several authors have obtained some exact solutions for the boundary layer flow of a binary mixture of incompressible Newtonian fluids [2-4]. Meanwhile, analyses of the transport processes and their interaction with chemical reactions and thermal radiation are quite difficult and are intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Moreover, the study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluid undergoing exothermic or endothermic chemical reaction. The recent advances in understanding physics of flows and computational flow modeling can make tremendous contributions in engineering and industrial processes. [5] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. They found that an increase in Prandtl number might lead to a reduction in the thermal boundary layer thickness. In [6] the transient free convection interaction with thermal radiation of an absorbing-emitting fluid along a moving vertical permeable plate was studied. The problem of unsteady hydromagmetic convection through a porous medium with combined heat and mass transfer with heat source / sink was investigated by [7].

In this article we further investigate the effects of thermal buoyancy, solutal buoyancy and suction/injection on boundary layer problems in the presence of radiative term. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using the fourth-order Runge–Kutta integration scheme along with shooting method. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of buoyancy forces, suction or injection at the plate surface. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2.0 Mathematical formulation

We consider the unsteady one – dimensional convective flow with chemical reaction and radiative heat transfer past a vertical porous plate moving through a binary mixture (Fig. 1). The flow is assumed to be in the *x*-direction, which is taken along the vertical plate.

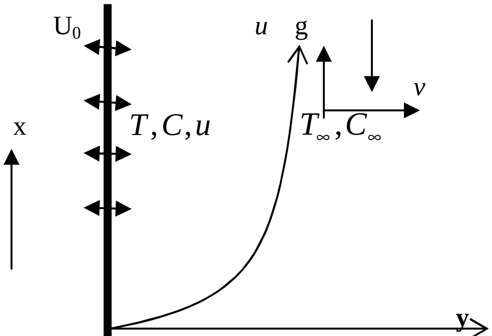

Fig. 1: Flow configuration and coordinate system.

We choose Cartesian axes (x, y) parallel and perpendicular to the plate, respectively and velocity components are (u, v), respectively. Then, the governing equations for such a flow are [9-12],

$$\frac{\partial v}{\partial y} = 0 , \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) + g\beta_c\left(C - C_\infty\right), \tag{2.2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q + 4\sigma\alpha T^4 , \tag{2.3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + R_A , \tag{2.4}$$

where u and v are velocity components in x and y directions respectively, T is the temperature, t is the time, g is the acceleration due to gravity, β is the thermal expansion coefficient, β_c is the concentration expansion coefficient, υ is the kinematic viscosity, D is the chemical molecular diffusivity, k is the thermal conductivity, ρ is the density, T_w is the wall temperature, T_∞ is the free stream temperature, C_w is the species concentration at the plate surface, C_∞ is the free stream concentration, $Q = \left(-\Delta H\right)R_A$ is the heat of chemical reaction and $\Delta\mathrm{H}$ is the activation enthalpy. We employed Arrhenius type of the n^{th} order irreversible reaction given by,

$$R_A = k_r e^{-E/R_G T} C^n , \tag{2.5}$$

where k_r is the chemical reaction rate, R_G is the universal gas constant and E is the activation energy parameter. The appropriate initial and boundary conditions are

$$u\left(y,0\right) = 0 ,\ T\left(y,0\right) = T_w ,\ C\left(y,0\right) = C_w , \tag{2.6}$$

$$u\left(0,t\right) = U_0 ,\ T\left(0,t\right) = T_w ,\ C\left(0,t\right) = C_w ,\ t > 0 , \tag{2.7}$$

$$u \to 0 ,\ T \to T_\infty ,\ C \to C_\infty \ \text{as}\ y \to \infty , t > 0, \tag{2.8}$$

where U_0 is the plate characteristic velocity. We introduce the following dimensionless quantities and parameters,

$$\begin{gathered} u = U_0 F(\eta),\ (\theta, \theta_w) = \frac{\left(T, T_w\right)}{T_\infty}, (\varphi, \varphi_w) = \frac{(C, C_w)}{C_\infty}, G_r = \frac{4\upsilon t g\beta T_\infty}{Y_0}, \\ G_c = \frac{4\upsilon t g\beta_c C_\infty}{U_0},\ P_r = \frac{\upsilon}{\lambda},\ \lambda = \frac{k}{\rho c_p}, S_c = \frac{\upsilon}{D},\ \gamma = \frac{E}{R_G T_\infty}, \eta = \frac{y}{2\sqrt{\upsilon t}}, \\ k_0 = k_r e^{-\frac{E}{R_G T_\infty}}, b = \frac{(-\Delta H)C_\infty}{\rho c_p T_\infty}, Ra = \frac{16\sigma\alpha t T_\infty^3}{\rho c_p}, Da = 4tk_0 C_\infty^{n-1}. \end{gathered} \tag{2.9}$$

From Eq. (2.1), v is either constant or a function of time. Following [6], we choose

$$v = -c\left(\frac{\upsilon}{t}\right)^{\frac{1}{2}}, \tag{2.10}$$

where c > 0 is the suction parameter and c < 0 is the injection parameter. Eqs. (2.2) – (2.4) then become

$$F'' + 2(\eta + c)F' = -G_r(\theta - 1) - G_c(\varphi - 1), \tag{2.11}$$

$$\frac{1}{p_r}\theta'' + 2(\eta + c)\theta' = bD_a\varphi^n \exp\left(\gamma\left(1 - \frac{1}{\theta}\right)\right) - Ra\,\theta^4, \tag{2.12}$$

$$\frac{1}{S_c}\varphi'' + 2(\eta + c)\varphi' = -D_a\varphi^n \exp\left(\gamma\left(1 - \frac{1}{\theta}\right)\right), \tag{2.13}$$

with the boundary conditions

$$F(0) = 1,\ \theta(0) = \theta_w,\ \varphi(0) = \varphi_w, \qquad F(\infty) = 0,\ \theta(\infty) = 1,\ \varphi(\infty) = 1, \tag{2.14}$$

where prime symbol represents derivatives with respect to η, *b* is the heat generation parameter, *Da* is the Damköhler number, *Ra* is the radiation parameter, γ is the activation energy parameter, G_r is the thermal Grashof number and G_c solutal Grashof number.

3.0 Method of solution

The set of non-linear ordinary differential equations (2.11)–(2.13) with boundary conditions in (2.14) have been solved numerically by using the Runge–Kutta integration scheme with a modified version of the Newton–Raphson shooting method with G_c, G_r *and* c as prescribed parameters. The computations were done by a program which uses a symbolic and computational computer language MAPLE [8]. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-7} in nearly all cases. The value of y_∞ was found to each iteration loop by the assignment statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞, to each group of parameters G_c, G_r ,*and* c is determined when the values of unknown boundary conditions at $\eta = 0$ not change to successful loop with error less than 10^{-7}

4.0 Results and discussion

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters encountered in the problem. To be realistic, the values of Schmidt number (Sc) are chosen for hydrogen (Sc = 0.22), water vapour (Sc = 0.62), ammonia (Sc = 0.78) and Propyl Benzene (Sc = 2.62) at temperature 25^{o}C and one atmospheric pressure. The values of Prandtl number is chosen to be Pr = 0.71 which represents air at temperature 25^{o}C and one atmospheric pressure. Attention is focused on positive values of the buoyancy parameters i.e. Grashof number $G_r > 0$ (which corresponds to the cooling problem) and solutal Grashof number $G_c > 0$ (which indicates that the chemical species concentration in the free stream region is less than the concentration at the boundary surface). The cooling problem is often encountered in engineering applications; for example in the cooling of electronic components and nuclear reactors. It is noteworthy that the heat transfer rate at the plate surface increases with fluid suction, while it decreases with increasing parameter values of fluid injection.

Fig. 2 shows the effect of the thermal buoyancy force parameter G_r on the horizontal velocity in the momentum boundary layer. It is clearly seen that fluid velocity is highest at the moving plate surface and decreases to the free stream zero value far away from the plate satisfying the boundary condition. As the thermal buoyancy force parameter increases in the presence of uniform suction, the flow rate retard and thereby giving rise to a decrease in the velocity profiles. The momentum boundary layer thickness generally decreases with increasing values of G_r. Meanwhile, it is interesting to note that a reverse flow occurs within the boundary layer as the intensity of thermal buoyancy force increases. The effect of solutal buoyancy force parameter G_c is depicted in Fig. 3. As seen from this figure, the fluid velocity decreases from its maximum value at the moving plate to the free stream zero value. Moreover, a local peak velocity occurs within the boundary layer as solutal Grashof number increases. Fig. 4 illustrates the effect of wall suction and injection on the horizontal velocity in the momentum boundary layer. The trend of the velocity profiles in this figure is same as those shown in Fig. 2. It is observed that the momentum boundary layer thickness decreases with increasing wall suction (c > 0) and increases with increasing wall injection (c < 0).

Fig. 5 depicts the variation of temperature profile against span wise coordinate η for varying values of wall temperature parameter θ_w and fixed values of other physical parameter in the presence of uniform suction. It is interesting to note from this figure that the fluid temperature increases (or decreases) toward the free steam temperature whenever the plate temperature is lower (or higher) than the free steam temperature. The effect of suction / injection parameter on the fluid temperature is highlighted in Fig. 6. The trend of the temperature profiles in this figure is same as those shown in Fig. 7. It is observed that the fluid temperature increases with suction and decreases with injection.

The effect of suction / injection parameter on the chemical species concentration in the boundary layer is depicted in Fig.7. From this figure, it is seen that the species concentration within the boundary layer is higher for suction and lower for injection.

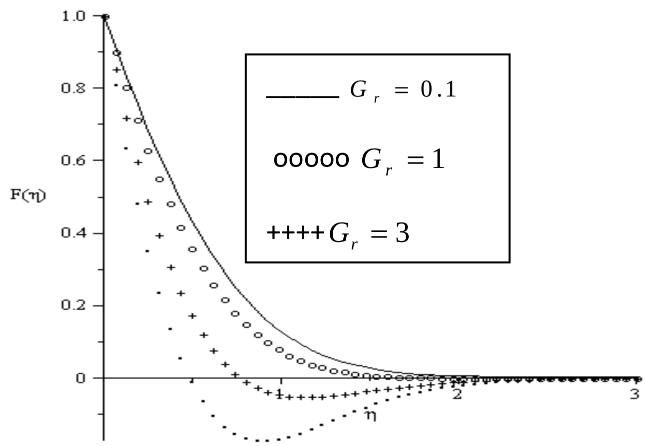

Fig. 2. Variation of the boundary layer velocity profiles with increasing values of thermal Grashof number when $G_c = \gamma = Ra =$ Da$= \theta_w = \varphi_w =$ c =0.1, $b = n = 1$, $Sc = 0.62$.

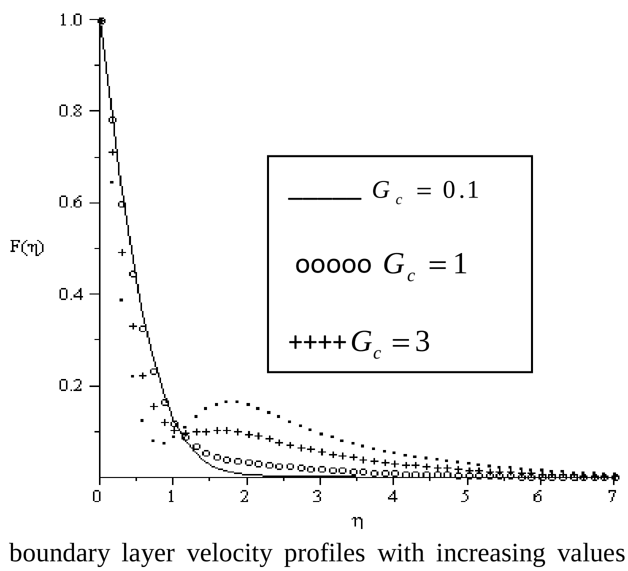

Fig. 3. Variation of the boundary layer velocity profiles with increasing values of the solutal Grashof number when $G_r = \gamma = Ra =$ Da$= \theta_w = \varphi_w =$ c =0.1, $b = n = 1$, $Sc = 0.62$.

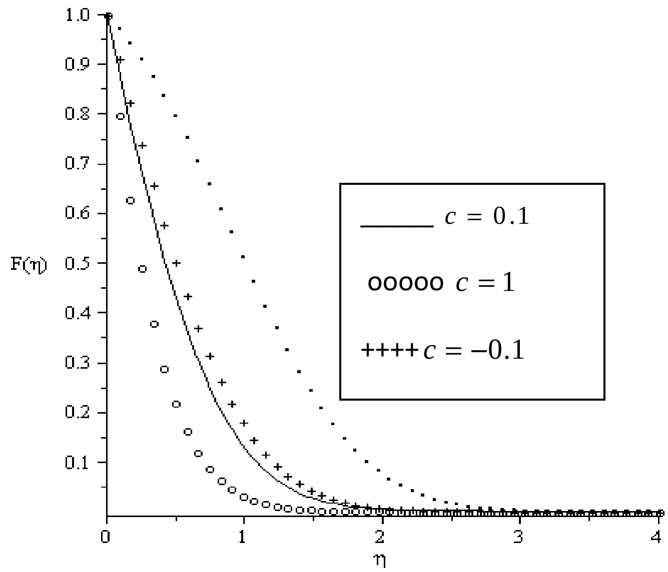

Fig. 4. Variation of the boundary layer velocity profiles with increasing values of suction/injection parameter when G_c =G_r = γ= Ra= Da=$\theta_w = \varphi_w$ =0.1, b = n= 1, Sc = 0.62.

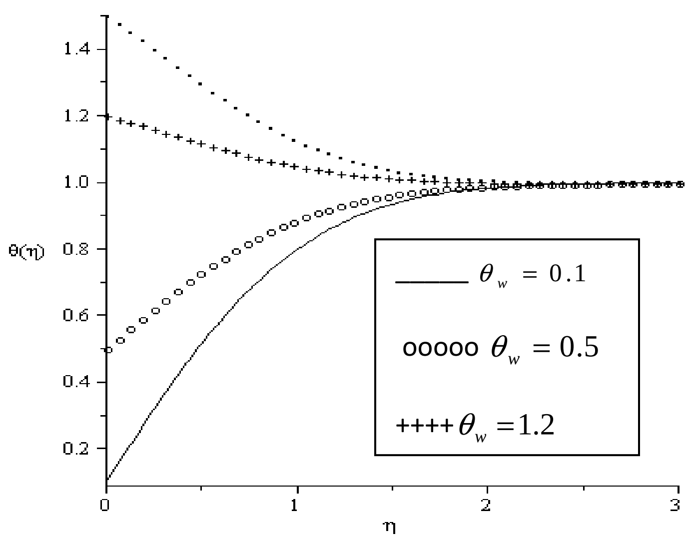

Fig. 5. Variation of the boundary layer temperature profiles with increasing values of the wall temperature when G_r = G_c = Ra=γ= Da= φ_w = c =0.1, b = n=1, Sc = 0.62.

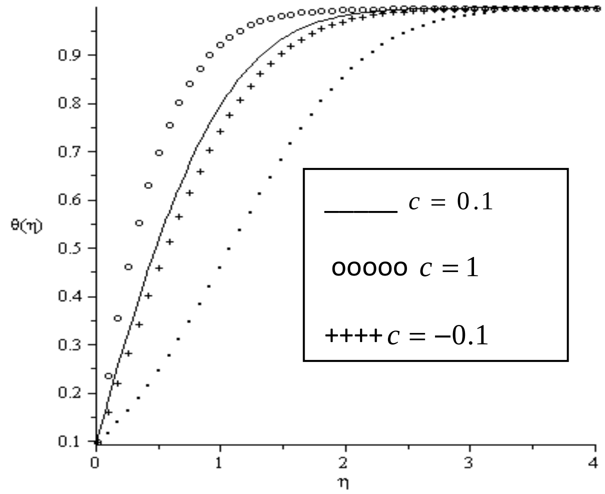

Fig. 6. Variation of the boundary layer temperature profiles with increasing values of suction/injection parameter when G_c =G_r = γ= Ra= Da= $\theta_w = \varphi_w$ =0.1, b = n= 1, Sc = 0.62.

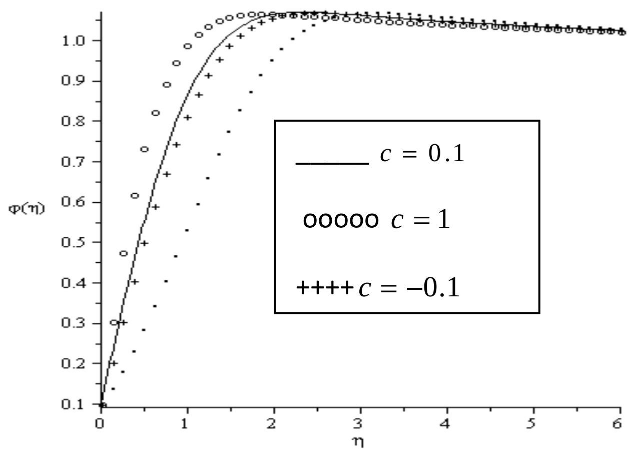

Fig. 7. Variation of the boundary layer concentration profiles with increasing values of suction/injection parameter when G_c = G_r = γ= Ra= Da= $\theta_w = \varphi_w$ =0.1, b = n= 1, Sc = 0.62.

5.0 Conclusion

The effects of wall suction / injection and buoyancy forces on unsteady convection of a viscous incompressible fluid past a vertical porous plate is studied. A set of non-linear coupled differential equations governing the fluid velocity, temperature and chemical species concentration is solved numerically for various material parameters. A comprehensive set of graphical results for the velocity, temperature and concentration is presented and discussed. Our results reveal among others, that the fluid velocity within the boundary decreases with increasing values of buoyancy forces and wall suction, and increases with wall injection.

Acknowledgements
POO want to thank Professor Makinde for his useful suggestions and support at Cape Peninsula University of Technology, Cape Town, South Africa.

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