Journal of the Nigerian Association of Mathematical Physics
Volume **16 (May, 2010), pp 277 – 282**
© J. of NAMP

A Four-Step Block Hybrid Adams-Moulton Methods For The Solution of First Order Initial Value Problems In ODEs

***Yahaya, Y.A and Sokoto, A.M**

***Department of Pure and Applied Mathematics,**
Federal University of Technology, Minna
Department of Mathematics,
Shehu Shagari College of Education, Sokoto
*Corresponding author: E-mail-yusuphyahaya@yahoo.com ; Tel. +2348034519729

Abstract

This paper examines application of the Adam-Moulton's Method and proposes a modified self-starting continuous formula Called hybrid Adams-Moulton methods for the case k=4. It allows evaluation at both grid and off grid points to obtain the discrete schemes used in the block methods. The order, error constant and region of absolute stability of the schemes were analyzed and plotted. Also the derived method was tested on stiff and non stiff problems to ascertain its efficiency.

1.0 Introduction

Even though Adams-moulton methods as an implicit methods have some advantage over many explicit methods, but still need an improvement or modification. [4] observes ''starting values problems and complication of programming''. [2] added that there is always need for search for algorithms. For some problems, no satisfactory algorithms was yet been found, and for others we need several so that we can choose among them, depending on its speed and accuracy.

Since Adams-Bashforth formula which is an explicit class of Adams-moulton methods was modified and improved upon by [6]. In view of the success recorded there is need to improve and modify the implicit class so as to maintain its advantage over the explicit class.

This paper is part of the research effort to provide improvements to the basic algorithm inform of continuous Hybrid Adams-moulton methods and thus allows evaluation at both grid and off grid points to obtain new discrete schemes which can be used to solve stiff and non-stiff initial value problem (IVP) in ordinary differential equations (ODE).

The analysis of the order, error constant and Region of absolute stability of newly constructed block methods will be ascertained. Also the schemes shall be used simultaneously on stiff and non-stiff problem to experiment their efficiency.

2.0 Derivation Of Continuous And Descrete Block Hybrid Methods

Using the general multistep collocation methods (see [1], [3], [5] [6], [7] and [8]) lead to the following D-matrix;

$$D=\begin{pmatrix} 1 & x_n+3h & (x_n+3h)^2 & (x_n+3h)^3 & (x_n+3h)^4 & (x_n+3h)^5 & (x_n+3h)^6 \\ 1 & x_n+\frac{7}{2}h & (x_n+\frac{7}{2}h)^2 & (x_n+\frac{7}{2}h)^3 & (x_n+\frac{7}{2}h)^4 & (x_n+\frac{7}{2}h)^5 & (x_n+\frac{7}{2}h)^6 \\ 0 & 1 & 2x_n & 3x_n & 4x_n & 5x_n & 6x_n \\ 0 & 1 & 2x_n+2h & 3(x_n+h)^2 & 4(x_n+h)^3 & 5(x_n+h)^4 & 6(x_n+h)^5 \\ 0 & 1 & 2x_n+4h & 3(x_n+2h)^2 & 4(x_n+2h)^3 & 5(x_n+2h)^4 & 6(x_n+2h)^5 \\ 0 & 1 & 2x_n+6h & 3(x_n+3h)^2 & 4(x_n+3h)^3 & 5(x_n+3h)^4 & 6(x_n+3h)^5 \\ 0 & 1 & 2x_n+8h & 3(x_n+4h)^2 & 4(x_n+4h)^3 & 5(x_n+4h)^4 & 6(x_n+4h)^5 \end{pmatrix}$$

With the help of MAPLE 11 and above mentioned algorithms given rise to the following continuous scheme

$$y(x)=\frac{1}{h^6}[\frac{64}{325}(x-x_n)^6-\frac{768}{325}h(x-x_n)^5+\frac{672}{65}h^2(x-x_n)^4-\frac{256}{13}h^3(x-x_n)^3$$
$$+\frac{4608}{325}h^4(x-x_n)^2-\frac{539}{325}h^6]y_{n+3}$$

$$-\frac{1}{h^6}[\frac{64}{325}(x-x_n)^6-\frac{768}{325}h(x-x_n)^5+\frac{672}{65}h^2(x-x_n)^4-\frac{256}{13}h^3(x-x_n)^3$$
$$+\frac{4608}{325}h^4(x-x_n)^2-\frac{864}{325}h^6]y_{n+7/2}$$

$$-\frac{1}{h^5}[\frac{61}{29250}(x-x_n)^6-\frac{1301}{39000}h(x-x_n)^5+\frac{1111}{5200}h^2(x-x_n)^4-\frac{3251}{4680}h^3(x-x_n)^3$$
$$+\frac{46481}{39000}h^4(x-x_n)^2-h^5(x-x_n)+\frac{8043}{26000}h^6]f_n$$

$$+\frac{1}{h^5}[\frac{349}{29250}(x-x_n)^6-\frac{1721}{9750}h(x-x_n)^5+\frac{7811}{7800}h^2(x-x_n)^4-\frac{1543}{585}h^3(x-x_n)^3$$
$$+\frac{4646}{1625}h^4(x-x_n)^2-\frac{18669}{13000}h^6]f_{n+1}$$

$$-\frac{1}{h^5}[\frac{307}{9750}(x-x_n)^6-\frac{2781}{6500}h(x-x_n)^5+\frac{2799}{1300}h^2(x-x_n)^4-\frac{3691}{780}h^3(x-x_n)^3$$
$$+\frac{12243}{3250}h^4(x-x_n)^2+\frac{3087}{6500}h^6]f_{n+2}$$

$$+\frac{1}{h^5}[\frac{3199}{29250}(x-x_n)^6-\frac{13121}{9750}h(x-x_n)^5+\frac{15687}{650}h^2(x-x_n)^4-\frac{6853}{585}h^3(x-x_n)^3$$
$$+\frac{41638}{4875}h^4(x-x_n)^2-\frac{26019}{13000}h^6]f_{n+3}$$

$$+\frac{1}{h^5}[\frac{157}{14625}(x-x_n)^6-\frac{4699}{39000}h(x-x_n)^5+\frac{7817}{15600}h^2(x-x_n)^4-\frac{4309}{4680}h^3(x-x_n)^3$$
$$+\frac{8423}{13000}h^4(x-x_n)^2-\frac{2793}{26000}h^6]f_{n+4}$$

(2.1)

Evaluating continuous formula in equation (2.1) above at $x = x_n$ $x = x_{n+1}$, $x = x_{n+2}$ and $x = x_{n+4}$ we obtain the following four discrete schemes that form the block methods:

$$\frac{1}{161280}h\left(83f_n-672f_{n+1}+3038f_{n+2}+44072f_{n+3}+41600f_{n+7/2}+2583f_{n+4}\right)$$

(2.2)

$$Y_{n+1}-y_{n+3}=\frac{1}{90}h\left(f_n-34f_{n+1}-114f_{n+2}-34f_{n+3}+f_{n+4}\right)$$

(2.3)

$$Y_{n+2}+\frac{27}{325}y_{n+3}-\frac{352}{325}y_{n+7/2}=$$
$$-\frac{3}{26000}h\left(33f_n-322f_{n+1}+3988f_{n+2}+9378f_{n+3}+283f_{n+4}\right)$$

(2.4)

$$Y_{n+4}+\frac{539}{325}y_{n+3}-\frac{864}{325}y_{n+7/2}=$$

$$\frac{1}{234000}h\left(413f_n-3242f_{n+1}+13668f_{n+2}+135542f_{n+3}+47663f_{n+4}\right)$$

(2.5)

Differentiating continuous formula (2.1) once and evaluate at $x = x_{n+7//2}$ we have

$$y_{n+3}-y_{n+7/2}=$$

$$\frac{1}{161280}h\left(83f_n-672f_{n+1}+3038f_{n+2}+44072f_{n+3}+41600f_{n+7/2}+2583f_{n+4}\right)$$

(2.6)

Each of the above schemes has order 6 and error constant [$\frac{161}{20800}$, $\frac{-1}{756}$, $\frac{151}{145600}$, $\frac{-2851}{3931200}$ and $\frac{-15}{774144}$]^T respectively. It is thus convergent and simultaneously provides values for y_1 , y_2, y_3, $y_{\frac{7}{2}}$ and y_4 without looking for any other method to provides y_1, y_2 and y_3

or to predict $y_{\frac{7}{2}}$ for the computation of y_4 which was the case before now (see [5]). Hence, this is a remarkable improvement.

3.0 Region Of Absolute Stability

Using MATLAB software gives the following stability polynomials:

$$
\begin{aligned}
f(z) = \frac{1}{637000} \Big(\eta^2 \big(&149058000000\eta\, z^4 - 562435965000\eta\, z^3 \\
&- 78260546000\eta\, z^2 - 1006224947000\eta\, z - 546546000000\eta \\
&+ 52907838969z^2 + 54709430010z^3 - 10836189000z^4 \\
&- 354517371000z\big)\Big) \Big/ \big(234000z^4 - 882945z^3 - 122858z^2 \\
&- 1579631z - 858000\big)
\end{aligned}
$$

$$
\begin{aligned}
f(z,p) := -\frac{3}{637000} \Big(\eta^2 \big(&7366081207056000z^5 \\
&- 113405318455697375z^4 + 253896767633147540z^3 \\
&+ 89317410196832813z^2 + 30263283890268000z \\
&+ 1078082575245000z^6 - 101391968106000000\big)\Big) \Big/ \\
&\big(234000z^4 - 882945z^3 - 122858z^2 - 1579631z - 858000\big)^2
\end{aligned}
\tag{3.1}
$$

Putting f (z) and f (z, p) above in MATLAB software shows that the block method for k=4 have the region of

absolute stability below:

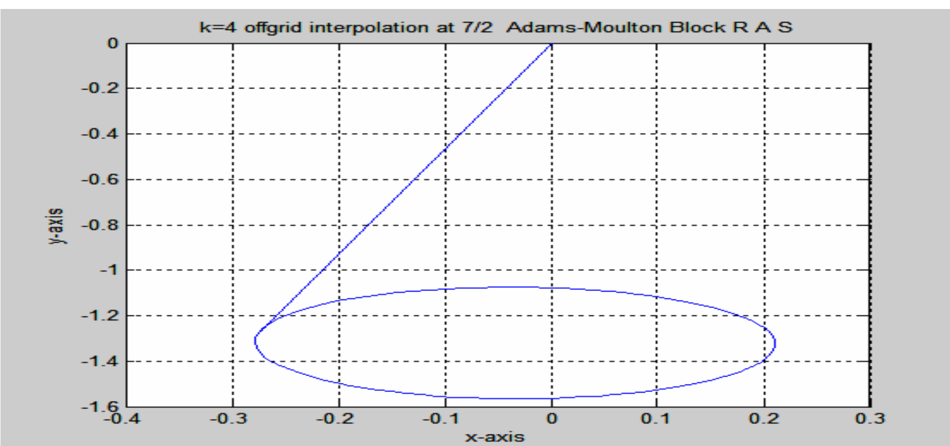

Figure (3.1)

4.0 Numerical Experiments

We use newly constructed block Hybrid methods and standard Adams-Moulton methods for k=4 to solve non-stiff and stiff initial value problems (IVP), in order to test for efficiency of the schemes derived.

Consider equations bellow:

$$y'(x) = -\lambda y, \; (0) = 1, \; 0 \le x \le 1, \; h = 0.1 \quad \text{Exact solution: } y(x) = e^{-\lambda x} \tag{4.1}$$

$$y'(t) = -9y, \; y(0) = e, \; 0 \le t \le 1, \; h = 0.1 \quad \text{Exact solution: } y(t) = e^{1-9t}, \; 0 \le t \le 1 \tag{4.2}$$

Solving (4.1) and (4.2) analytically, by standard Adams-Moulton methods and the new Hybrid Block methods respectively, we obtain the following results:

Numerical result for example 4.1

Table 4.1

X	Exact solutions y(x)	Block Hybrid methods for k=4, off-grid at $x_{n+7/2}$	Standard Adams-Moulton methods for k=4
0.1	0.9084837418	0.9078374173	0.8948291667
0.2	0.8187307530	0.8187307526	0.8097669331
0.3	0.7408182200	0.7408182202	0.7326611970
0.4	0.6703200460	0.6703200456	0.6629402813
0.5	0.6065306590	0.6065306588	0.5998527760
0.6	0.5488116360	0.5488116354	0.5427690912
0.7	0.4965853030	0.4965853031	0.4911176416
0.8	0.4493289640	0.4493289635	0.4443814918
0.9	0.4065696590	0.4065696588	0.4021300991
1.0	0.3678794410	0.3678794404	0.3638639333

Table of comparison of absolute errors for example 4.1(non-stiff)

Table 4.2

X	Exact solutions y(x)	Standard Adams-Moulton methods for k=4	Block Hybrid methods for k=4, off-grid at $x_{n+7/2}$
0.1	0.9084837418	$1.37X10^{-2}$	$7.36x10^{-10}$
0.2	0.8187307530	$8.96X10^{-3}$	$4.78x10^{-10}$
0.3	0.7408182200	$8.16X10^{-3}$	$4.82x10^{-10}$
0.4	0.6703200460	$7.38X10^{-3}$	$4.36x10^{-10}$
0.5	0.6065306590	$6.68X10^{-3}$	$9.13x10^{-10}$
0.6	0.5488116360	$6.04X10^{-3}$	$6.94x10^{-10}$
0.7	0.4965853030	$5.47X10^{-3}$	$6.91x10^{-10}$
0.8	0.4493289640	$4.95X10^{-3}$	$6.17x10^{-10}$
0.9	0.4065696590	$4.48X10^{-3}$	$9.41x10^{-10}$
1.0	0.3678794410	$4.05X10^{-3}$	$7.71x10^{-10}$

Numerical result for example 4.2(stiff-problem)

Table 4.3

T	Exact solutions y(t)	Block Hybrid methods for k=4, off-grid at $x_{n+7/2}$	Standard Adams-Moulton methods for k=4
0.1	1.105170918	1.103994442	1.116169023
0.2	0.449338964	0.449177460	0.469773633
0.3	0.182683524	0.182461349	0.179696107
0.4	0.074273578	0.074136190	0.073496335
0.5	0.030197383	0.030109439	0.027818097
0.6	0.012773390	0.012250498	0.011244607
0.7	0.004991593	0.004976301	0.004256153

0.8	0.002020294	0.002021930	0.001711274
0.9	0.000825104	0.000821180	0.000649232
1.0	0.000335462	0.000334110	0.000226012

Table of comparison of absolute errors for example 4.2(stiff-problem)

Table 4.4

T	Exact solutions y(t)	Standard Adams-Moulton methods for k=4	Block Hybrid methods for k=4, off-grid at $x_{n+7/2}$
0.1	1.105170918	1.10E-02	1.18E-03
0.2	0.449338964	2.04E-02	1.62E-04
0.3	0.182683524	2.99E-03	2.22E-04
0.4	0.074273578	7.77E-04	1.37E-04
0.5	0.030197383	2.38E-03	8.79E-05
0.6	0.012773390	1.53E-03	5.23E-04
0.7	0.004991593	7.35E-04	1.53E-05
0.8	0.002020294	3.09E-04	1.64E-06
0.9	0.000825104	1.76E-04	3.92E-06
1.0	0.000335462	1.09E-04	1.35E-06

5.0 Analysis Of The Result

A close observation of the above tables shows that the discrete schemes of the newly constructed block Hybrid methods with one off-grid interpolation point performs far-better than the standard Adams-Moulton methods when applies to non-stiff equations and performs a little better than the standard Adams-Moulton methods when applies to stiff differential equations.

6.0 Conclusion

It is however noteworthy, that Conventionally, Adams Bash-forth are usually used as the predictor before Applying Adams-Moulton as the Corrector for an accurate results in Adams-Code. A Continuous Formulation of a modified self-starting Adams Moulton herein referred to as Hybrid Method is Proposed. It can be seen from the tables of results that the newly constructed schemes performs far-better than the standard Adams on Non-stiff Differential Equations because the present approach eliminates its use in predictor-corrector mode which hitherto is a non-uniform order implementation, since the present block methods are self stating its uniform order is retained. Even though its Application to Stiff differential Equations yielded poor errors but still performs better than the conventional Adams.

References

1) Folasade, O.G (2002), Application of Simpson's class of methods and its hybrid form for the solution of initial value problems.MSc thesis unpublished, University Jos, Nigeria.
2) Francis. S (1988). Schuam's outline of theory and problems of Numerical Analysis. Second Edition.mc Graw Hill. 471p
3) Garba .G .M (2008). On a continuous formulation of some high order parallel methods for second order ordinary differential equations. MSc thesis unpublished, Nigerian Defence Academy Kaduna Nigeria.
4) Harley, P.J and Atkinson L.V(1983). An Introduction to Numerical Methods with Pascal. Addison Wesley Publisher Ltd.
5) Lambert, J.D (1973). Computational Methods in Ordinary Differential equations. London,John Wiley and Son Ltd. 278P

6) Oluwale,O.I (2004). Block hybrid Adams-Bashforth methods for the solution of ordinary differential equations. M.Sc thesis unpublished, University of Jos Nigeria.

Yahaya,Y.A(2004). Some Theories and Applications of Continuous linear Multistep Methods for Ordinary Differential Equations. Ph.D thesis unpublished, University Jos Nigeria.