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Solving Variable Coefficient Fourth-Order Parabolic Equation by Modified initial guess Variational Iteration Method

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Abstract

In this paper, a Modified initial guess Variational Iteration Method (MigVIM) is used to solve a non-homogeneous variable coefficient fourth order parabolic partial differential equations. The new method shows rapid convergence to the exact solution.

Keyword: lagrange multiplier, variational iteration method, parabolic partial differential equation.

1.0 Introduction

The variable coefficient fourth order parabolic differential equations arise in the study of the transverse vibration of a uniform flexible beam. Numerical computations of the transverse vibrations have been carried out by a number of authors [11]

The Variational Iteration Method (VIM) was developed and formulated by He J H for solving various problems [3-10)]. The method has been extensively useful for diversified initial and boundary value problems and has potential to cope with the versatility of the complex nature of physical problems.

In [12], the non homogeneous equation with constant coefficient was solved by Adomian Decomposition Method (ADM) . In this paper, we present MigVIM for the fourth order non homogeneous parabolic partial differential equation of the form:

$$\frac{\partial^2 u}{\partial t^2} + \mu(x)\frac{\partial^4 u}{\partial x^4} = f(x,t), \qquad 0 \le x \le 1, t > 0 \tag{1.0}$$

with the initial condition

$$u(x,0), \ = \quad g_0(x)$$

$$\frac{\partial u}{\partial t}(x,0) = g_1(x) \tag{1.2}$$

and the boundary conditions at $x = 0$ and $x = 1$ are of the forms

$$u(0,t), \; = \; f_0(t)\,,\; u(1,t), \; = \; f_1(t)$$
$$\frac{\partial^2 u}{\partial x^2}(0,t) = p_0(t)\,,\; \frac{\partial^2 u}{\partial x^2}(1,t) = p_1(t)$$
(1.3)

2.0 Variational Iteration Method

To illustrate the basic concept of the technique, we consider the following general nonlinear partial differential equation.

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = g(x,t)$$
(2.1)

where L is a linear time derivative operator, R is a linear operator which has partial derivative with respect to x, N is a nonlinear operator and g is an inhomogeneous term. According to VIM, we can construct a correct fractional as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left[Lu_n + R\tilde{u}_n + N\tilde{u}_n - g\right]d\tau$$
(2.2)

where λ is a Lagrange multiplier which can be identified optimally Via variational iteration method. The subscript n denote the nth approximation, $\tilde{u}_n$ is considered as a restricted variation i.e, $\delta\tilde{u}_n = 0$. The successive approximation $u_{n+1}, n \geq 0$ of the solution u will be readily obtained upon using the determined Lagrange multiplier and any selective function u_0, consequently, the solution is given by:

$$u = \lim_{n \to \infty} u_n$$
(2.3)

In MigVIM, equation (2.2) becomes:

$$u_0(x,t) = g_0(x) + tg_1(x) + t^2 g_2(x)$$
(2.4)

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left[Lu_n + R\tilde{u}_n + N\tilde{u}_n - g\right]d\tau$$
(2.5)

where $g_2(x)$ can be found by substituting for $u_0(x,t)$ in (1.0) at $t = 0$.

3.0 Application

Consider the fourth-order problem as considered in [12,13]

$$\frac{\partial^2 u}{\partial t^2} + (1+x)\frac{\partial^4 u}{\partial x^4} = (x^4 + x^3 - \frac{6}{7!}x^7)\cos t, \qquad 0 < x < 1, t > 0 \quad (3.1)$$

$$u(x,0), \; = \; \frac{6}{7!}x^7$$
$$\frac{\partial u}{\partial t}(x,0) = 0$$
(3.2)

$$u(0,t), \;=\; 0\,,\; u(1,t), \;=\; \frac{6}{7!}\cos t$$

$$\frac{\partial^2 u}{\partial x^2}(0,t) = 0\,,\; \frac{\partial^2 u}{\partial x^2}(1,t) = \frac{6}{20}\cos t$$

(3.3)

Applying recursive formula (2.4-2.5) gives

$$u_0(x,t) = \frac{6}{7!}x^7 - \frac{3}{7!}x^7 t^2$$

(3.4)

$$u_1(x,t) = u_0(x,t) + \int_0^t (\tau - t)\begin{bmatrix} \dfrac{\partial^2 u_n(x,\tau)}{\partial \tau^2} + (1+x)\dfrac{\partial^4 u_n(x,\tau)}{\partial x^4} \\ -(x^4 + x^3 - \dfrac{6}{7!}x^7)\cos\tau \end{bmatrix} d\tau$$

(3.5)

this gives;

$$u_1(x,t) = (\frac{t}{840} - \frac{1}{840}\sin t)x^8 + (\frac{1}{840}t\sin t - \frac{1}{840}t^2 + \frac{1}{840}\cos t)x^7$$
$$+(-t + \frac{t^3}{6} + \sin t)x^5 + (\frac{t^2}{2} + \frac{t^3}{6} - \cos t + (-\sin t - 1)t + 1 - \frac{t^4}{8} + \sin t)x^4$$
$$+(\frac{t^2}{2} - \frac{t^4}{8} + 1 - t\sin t - \cos t)x^3$$

(3.6)

Equation (3.6) is the first numerical approximation.

4.0 Conclusion

In this work, the MigVIM was applied to the solution of non-homogeneous variable coefficient fourth-order parabolic partial differential equation. Table.I shows the comparism of fist approximation with the exact solution while Fig. I is the 3-D graphical representation. The ability of the method to give an efficient solution with just one iteration makes it remarkable when compared with [12,13].

This method can be extended to nonlinear variable coefficient parabolic partial differential equation of order greater than four.

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Table 1:Absolute errors for first approximate solutions, U_1 (x,t) and Exact Solution.

x	t	U_1 (x,t)	Exact Solution	absolute error
0.1	0.01	1.1904175259E-10	1.1904166672E-10	8.5870059398E-17
0.2		1.5237335356E-08	1.5237333340E-08	2.0162870612E-15
0.3		2.6034414622E-07	2.6034412511E-07	2.1116398338E-14
0.4		1.9503788235E-06	1.9503786675E-06	1.5602750923E-13
0.5		9.3001310486E-06	9.3001302122E-06	8.3637251753E-13
0.6		3.3324051461E-05	3.3324048014E-05	3.4474194468E-12
0.7		9.8035942943E-05	9.8035931333E-05	1.1610741746E-11
0.8		2.4964850292E-04	2.4964846944E-04	3.3479873388E-11
0.9		5.6937268710E-04	5.6937260161E-04	8.5489957019E-11
0.1	0.02	1.1902626758E-10	1.1902381032E-10	2.4572659708E-15
0.2		1.5235098312E-08	1.5235047721E-08	5.0591270683E-14
0.3		2.6030543556E-07	2.6030507316E-07	3.6240044419E-13
0.4		1.9500880124E-06	1.9500861082E-06	1.9041366714E-12
0.5		9.2987435500E-06	9.2987351811E-06	8.3689327170E-12
0.6		3.3319080512E-05	3.3319049365E-05	3.1147369826E-11
0.7		9.8021325334E-05	9.8021225820E-05	9.9513940965E-11
0.8		2.4961130075E-04	2.4961102185E-04	2.7889335253E-10
0.9		5.6928789572E-04	5.6928719501E-04	7.0071059437E-10
0.1	0.03	1.1901079492E-10	1.1899405164E-10	1.6743285448E-14
0.2		1.5231590458E-08	1.5231238610E-08	3.5184856624E-13
0.3		2.6024226184E-07	2.6023999093E-07	2.2709067476E-12
0.4		1.9496085933E-06	1.9495985420E-06	1.0051246063E-11
0.5		9.2964479888E-06	9.2964102841E-06	3.7704635273E-11
0.6		3.3310844556E-05	3.3310718839E-05	1.2571653611E-10
0.7		9.7997093548E-05	9.7996718267E-05	3.7528056843E-10
0.8		2.4954962304E-04	2.4954861338E-04	1.0096667543E-09
0.9		5.6914733469E-04	5.6914486016E-04	2.4745220102E-09
0.1	0.04	1.1901504535E-10	1.1895239365E-10	6.2651698527E-14
0.2		1.5227297674E-08	1.5225906387E-08	1.3912866093E-12
0.3		2.6015759381E-07	2.6014888491E-07	8.7088987430E-12
0.4		1.9489515406E-06	1.9489160176E-06	3.5523014941E-11
0.5		9.2932760490E-06	9.2931557539E-06	1.2029511084E-10
0.6		3.3299423429E-05	3.3299057269E-05	3.6616014277E-10
0.7		9.7963432983E-05	9.7962411124E-05	1.0218587825E-09
0.8		2.4946387691E-04	2.4946125025E-04	2.6266645320E-09
0.9		5.6895185851E-04	5.6894561130E-04	6.2472074868E-09
0.1	0.05	1.1906605558E-10	1.1889884052E-10	1.6721505773E-13
0.2		1.5223057301E-08	1.5219051587E-08	4.0057136762E-12
0.3		2.6005673197E-07	2.6003176422E-07	2.4967743745E-11
0.4		1.9481364297E-06	1.9480386031E-06	9.7826606899E-11
0.5		9.2892828804E-06	9.2889719159E-06	3.1096449356E-10
0.6		3.3284950100E-05	3.3284065821E-05	8.8427961520E-10
0.7		9.7920634904E-05	9.7918307821E-05	2.3270833372E-09
0.8		2.4935466156E-04	2.4934894120E-04	5.7203567656E-09

0.9	5.6870264609E-04	5.6868946836E-04	1.3177734992E-08

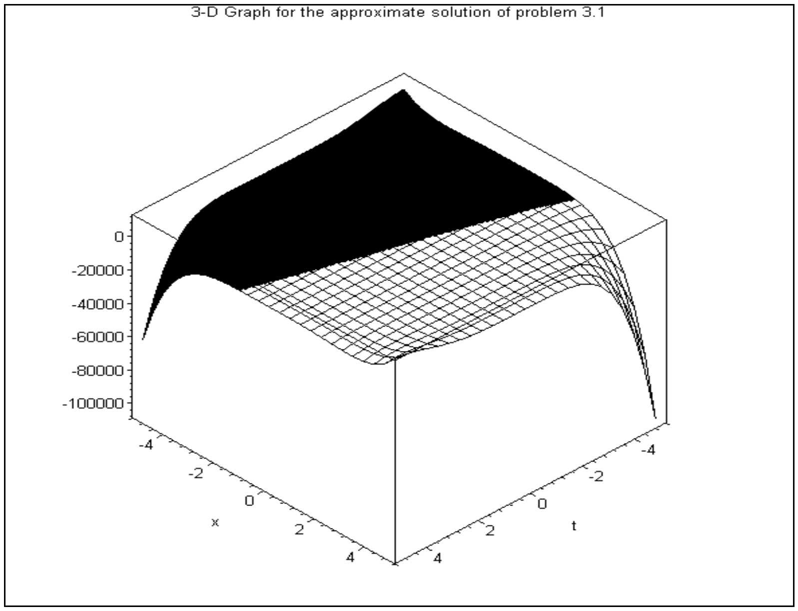

Fig. I

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