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On The Dynamic Analysis of Non-Uniform Beams Under Uniformly Distributed Moving Loads

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Abstract

In this paper, the dynamic response of a non-uniform beam subjected to uniformly distributed moving load is investigated. Specifically, the elastic properties of the beam, the flexural rigidity, and the mass density per unit length which are assumed constants are hereby expressed as functions of the spatial variable x. This dynamic response of the beam was analyzed using the finite element technique. Firstly, the non-uniform continuous beam was replaced by a non-continuous (discrete) system made up of beam elements. The modified elemental and overall stiffness, and mass matrices, the elemental and overall centripetal acceleration matrices as well as the load vector were derived. Next, the Newmark's direct integration method was used to obtain the desired response of the beam. The major points of interest in this study were (i) the effect of velocity of the moving load (ii) the effect of load's length, and (iii) the effect of the span length of the beam.

1.0 Introduction

This paper is concerned with the moving load problems. Many loads acting on solids and structures are functions of both time and space and such loads, which in addition, continuously change their positions are called moving loads. Some examples of such loads are cars, trains, trucks and cranes. The moving load problems, on the other hand, deal with the determination of the dynamic effect of the moving loads on elastic structures and particularly on highway and railway bridges. Such a study , is a subject of considerable practical importance. The bridges and other practical structures are usually modeled by elastic structures such as beams, plates, e.t.c.

From historical viewpoints and limiting the problems to the responses of beams ,the moving loads problems were first considered approximately for the case where the mass of the beam was considered negligible compared with mass of the moving load [1] , [2] and [3].The other case, in which the mass of the moving load was negligible compared to the mass of the beam was originally studied by [4], and later by [5] and thereafter by [6]. The more complicated problem involving both cases,(i.e. in which both the mass of the load and that of the beam were taken into consideration) was thoroughly studied by other scholars, such as [7], [8], and [9]. [10] came up with a thorough treatise on the dynamic response of several types of railways bridges traversed by steam locomotives using harmonic analysis. This technique was also used by [11]. Earlier, [12], by using Fourier analysis presented some interesting analyses. The problem of of the dynamic response of bridges under moving loads was reviewed in detail by [13], and later by [14,15,and 16]. One should also make mention of the extended review by [17] in his excellent monograph carried out on this subject. The dynamic response of a simply supported beam traversed by a concentrated moving load was determined by [18]. They developed an interesting technique which, however, cannot easily be applied to various boundary conditions which are of practical interest. [19] presented an analytical numerical method that can be used to determine the dynamic behaviour of beams with different boundary conditions

carrying a concentrated moving mass. The problem of dynamic behaviour of an elastic beam subjected to a moving concentrated mass was also studied by [20]. [21] presented a more versatile technique which can be used to determine the dynamic behaviour of beams having arbitrary end supports. [22], studied the effect of the mass of a moving load on the dynamic response of a simply supported beam. Some interesting results were obtained. A detail analysis of the effect of centripetal and coriolis forces on the dynamic response of light (steel) bridges under moving loads was also carried out by [23]. It is remarked at this juncture, that the elastic parameters of the beams in all the works, discussed, hitherto, are assumed constants. In other words, uniform beams were considered. The reason for this is that by making such an assumption, the various researchers ended up with the governing partial differential equations having constant coefficients only and thereby based the aforementioned investigations, in general, on analytical approaches. Otherwise the researchers could have found it very difficult, if not impossible, to obtain analytical closed-form solution to the problem. However, for practical application, it is useful to consider beams that are not uniform as most of the vibration structural problems involve non-uniform beams. Hence in this paper, beams that are not uniform are considered.

Some of the previous works involving non-uniform beams include that of [24]. They studied the dynamic responses of multi-span non-uniform beams under moving load using the transfer matrix method analysis to solve the moving load problem. [25] also investigated the dynamic behaviour of multi-span non-uniform beams traversed by a moving load at a constant and variable velocities. They used both modal analysis and direct integration methods in their analyses.

Although, the above completed works on both uniform and non-uniform beams are impressive, only concentrated moving loads were considered. However, such loads do not represent the physical reality of the problem formulation. As a matter of fact, concentrated loads do not exist physically. For practical application it is realistic to consider the moving loads as distributed moving loads as opposed to concentrated moving loads. Hence, the present research work deals with the more realistic moving load, namely, distributed moving load. The first work on moving loads, to the best knowledge of the author, to include distributed moving load was that of [26], who carried out an analysis of dynamic behaviour of a beam carrying partially distributed moving masses. They showed that the inertia effect of the moving mass is of importance in the dynamical behaviour of such beam. The work in [26] was extended by the same author [27] by considering the vibration of a Timoshenko beam under partially distributed moving masses. In [28], the vibration analysis of beams traversed by uniform partially distributed moving masses under a simply supported end conditions was studied. The inertia effect of the load was taken into consideration. [29] also investigated the transverse vibration of beams on foundation subjected to distributed masses. He showed that the foundation stiffness and load's distribution have significant effects on the dynamic deflection of the beam. It should, however, be remarked that only uniform beams were considered in all the previous works involving distributed moving loads.

In the context discussed so far, the research work presented in this paper therefore, focuses on determining the dynamic behaviour of a non-uniform Euler-Bernoulli beam subjected to uniform partially distributed moving loads. The solution is obtained by developing a finite element model of the problem which is then solved using Newmark's β method [30]. The solutions are obtained for simply supported and clamped- clamped beams.

2.0 Mathematical Problem Statement.

The moving load problem, such as that of determining the behaviour of a bridge being traversed by a train, can be modeled as non-uniform Euler-Bernoulli beam carrying a load moving at a specified speed. Thus, the corresponding governing equation is[25]:

$$\frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}] + \rho A(x)\frac{\partial^2 y(x,t)}{\partial t^2} = q(x,t)$$

(2.1)

Where y is the transverse displacement of the beam, $EI(x)$,the flexural rigidity, and $A(x)$,the area of the beam are both functions of x coordinate, $q(x,t)$ is the externally applied pressure loading, t is time, and ρ the density which is assumed constant .

The following boundary conditions:

$$y(0,t) = y(l,t) = 0$$
$$\left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_{x=0} = \left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_{x=l} = 0$$

(2.2)

may be considered.

For moving load, $q(x,t)$, which in this work is assumed to be uniformly distributed , we have:

$$q(x,t) = \frac{1}{\varepsilon}[-pg - p(\frac{\partial^2 y}{\partial t^2} + 2v\frac{\partial^2 y}{\partial x \partial t} + v^2\frac{\partial^2 y}{\partial x^2}][H(x-\xi+\frac{\varepsilon}{2}) - H(x-\xi-\frac{\varepsilon}{2})]$$

(2.3)

Where ε is the load's length, ξ is the distance covered by the moving load, V is the moving speed of the load, P is the load, and $H(x)$ is the heavy-side function.

Using (2.3) in (2.1), we have:

$$\frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}] + \rho A(x)\frac{\partial^2 y(x,t)}{\partial t^2} =$$
$$\frac{1}{\varepsilon}[-pg - p(\frac{\partial^2 y}{\partial t^2} + 2v\frac{\partial^2 y}{\partial x \partial t} + v^2\frac{\partial^2 y}{\partial x^2}][H(x-\xi+\frac{\varepsilon}{2}) - H(x-\xi-\frac{\varepsilon}{2})]$$

(2.4)

we define the flexural rigidity $EI(x)$ and the mass density area $A(x)$ respectively as follows:

$$EI(x) = \sum_{r=1}^{nspan} EI_r(x - \sum_{i=1}^{r-1} L_i)[H(x - \sum_{i=1}^{r-1} L_i) - H(x - \sum_{i=1}^{r} L_i)]$$

(2.5)

$$A(x) = \sum_{r=1}^{nspan} A_r(x - \sum_{i=1}^{r-1} L_i)[H(x - \sum_{i=1}^{r-1} L_i) - H(x - \sum_{i=1}^{r} L_i)]$$

(2.6)

We remark at this juncture, that $EI(x)$ and $A(x)$ as defined above are similar to those in [25].

The associated initial conditions are :

$$y(x,0) = \frac{\partial y(x,0)}{\partial t} = 0$$

(2.7)

Thus the initial-boundary value problem describing the behaviour of a non-uniform beam traversed by uniformly distributed moving load is governed by equations (2.2), (2.4), (2.5), (2.6), and (2.7) respectively. The closed-form solution of the above initial-boundary value problem is either impossible or very difficult to obtain using analytical approach, hence, we employ finite element method.

3.0 The Finite Element Formulation of The Problem

The formulation of non-uniform beam element equation is similar to that of the element with uniform materials [33]. Hence, we applied GWRM to equation (2.4) to obtain;

$$\int_0^l \frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}] + \rho A(x)\frac{\partial^2 y(x,t)}{\partial t^2} =$$

$$1/\varepsilon[-pg - p(\frac{\partial^2 y}{\partial t^2} + 2v\frac{\partial^2 y}{\partial x\partial t} + v^2\frac{\partial^2 y}{\partial x^2}][H(x-\xi+\varepsilon/2) - H(x-\xi-\varepsilon/2)]\}Rdx \qquad (3.1)$$

Where R is the Galerkin's weight or test function.

Rearranging and integrating twice the first term on the left-hand side of (3.1), applying [29], and using the associated boundary condition, we obtain:

$$\int_0^l EI(x)\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 R}{\partial x^2}dx + Q_i^e + \int_0^l \rho A(x)\frac{\partial^2 y}{\partial t^2}Rdx =$$

$$-pg/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}Rdx - p/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial t^2}Rdx - 2pv/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x\partial t}Rdx - pv^2/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x^2}Rdx$$

(3.2)

where $Q = \varphi R - \phi\frac{\partial R}{\partial x}$

$$\varphi = EI(\frac{\partial^3 y}{\partial x^3}), the - shearforce$$

$$\phi = EI(\frac{\partial^2 y}{\partial x^2}), the - bending - moment$$

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Discretization Of The Beam Element Equation

The standard mathematical discretization [32] of beam element into a number of finite elements using equation (3.2) yields;

$$\sum_{i=1}^{n}\{\int_\Omega EI(x)\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 R}{\partial x^2}dx + Q_i^e + \int_\Omega \rho A(x)\frac{\partial^2 y}{\partial t^2}Rdx =$$

$$-pg/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}Rdx - p/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial t^2}Rdx - 2pv/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x\partial t}Rdx - pv^2/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x^2}Rdx\} \qquad (4.1)$$

Where, $\Omega = l_e$,the domain of the beam element.

Finally, the finite element form of (4.1) is:

$$[K]\{y\}+[C]\{\acute{y}\}+[M]\{\ddot{y}\}=\{F\} \qquad (4.2)$$

Where,

$$[K]=\sum_{i=1}^{n}\{\int_\Omega EI(x)\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 R}{\partial x^2}dx + pv^2/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x^2}Rdx\} \qquad (4.3)$$

$$[M]=\sum_{i=1}^{n}\left\{\int_\Omega \rho A(x)\frac{\partial^2 y}{\partial t^2}Rdx + p/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial t^2}Rdx\right\} \qquad (4.4)$$

$$[C]=\sum_{i=1}^{n}\{2pv/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}\frac{\partial^2 y}{\partial x\partial t}Rdx\} \qquad (4.5)$$

$$\{F\}=\sum_{i=1}^{n}\{-pg/\varepsilon\int_{\xi-\varepsilon/2}^{\xi+\varepsilon/2}Rdx + Q^e\} \qquad (4.6)$$

Specification or Introduction of Shape Functions

By using Hermittian interpolation functions [32] to interpolate the transverse displacement, Residual function and their derivatives in the above equations, therefore, from equation (4.3), we have
.

$$[K_{ij}^e]=\begin{bmatrix} K_{11}^e & K_{12}^e & K_{13}^e & K_{14}^e \\ K_{21}^e & K_{22}^e & K_{23}^e & K_{24}^e \\ K_{31}^e & K_{32}^e & K_{33}^e & K_{34}^e \\ K_{41}^e & K_{42}^e & K_{43}^e & K_{44}^e \end{bmatrix} \tag{5.1}$$

Where,

$$K_{ij}^e=K_{(1)ij}^e+K_{(2)ij}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\alpha_{ij}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}dx+K_{(2)ij}^e \tag{5.2}$$

Such that

$$K_{11}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ \frac{36}{l^4}-\frac{144x}{l^5}+\frac{144x^2}{l^6}\ ]dx \tag{5.4}$$
$$+\frac{pv^2}{\varepsilon}[\ (-\frac{6\eta}{l^2}+\frac{6\eta^3}{l^4}-\frac{12\eta^4}{l^5}+\frac{6\eta^2}{l^3}+\frac{24\eta^5}{5l^6})-(-\frac{6\mu}{l^2}+\frac{6\mu^3}{l^4}-\frac{12\mu^4}{l^5}+\frac{6\mu^2}{l^3}+\frac{24\mu^5}{5l^6})\ ]$$

$$K_{13}^e=K_{31}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ -\frac{36}{l^4}+\frac{144x}{l^5}-\frac{144x^2}{l^6}\ ]dx \tag{5.6}$$
$$+\frac{pv^2}{\varepsilon}[\ (-\frac{6\eta^3}{l^4}+\frac{12\eta^4}{l^5}-\frac{24\eta^5}{5l^6})-(-\frac{6\mu^3}{l^4}+\frac{12\mu^4}{l^5}-\frac{24\mu^5}{5l^6})\ ]$$

$$K_{14}^e=K_{41}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ \frac{12}{l^3}-\frac{6x}{l^4}+\frac{72x^2}{l^5}\ ]dx \tag{5.7}$$
$$+\frac{pv^2}{\varepsilon}[\ (\frac{2\eta^3}{l^3}-\frac{9\eta^4}{2l^4}+\frac{12\eta^5}{5l^5})-(\frac{2\mu^3}{l^3}-\frac{9\mu^4}{2l^4}+\frac{12\mu^5}{5l^5})\ ]$$

$$K_{22}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ \frac{16}{l^2}-\frac{48x}{l^3}+\frac{36x^2}{l^4}\ ]dx \tag{5.8}$$
$$+\frac{pv^2}{\varepsilon}[\ (-\frac{2\eta^2}{l}+\frac{14\eta^3}{3l^2}-\frac{4\eta^4}{l^3}+\frac{6\eta^5}{5l^4})-(-\frac{2\mu^2}{l}+\frac{14\mu^3}{3l^2}-\frac{4\mu^4}{l^3}+\frac{6\mu^5}{5l^4})\ ]$$

$$K_{23}^e=K_{32}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ -\frac{24}{l^3}+\frac{84x}{l^4}-\frac{72x^2}{l^5}\ ]dx \tag{5.9}$$
$$+\frac{pv^2}{\varepsilon}[\ (-\frac{4\eta^3}{l^3}+\frac{13\eta^4}{2l^4}-\frac{12\eta^5}{5l^5})-(-\frac{4\mu^3}{l^3}+\frac{13\mu^4}{2l^4}-\frac{12\mu^5}{5l^5})\ ]$$

$$K_{24}^e=K_{42}^e=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\ \frac{8}{l^2}-\frac{36x}{l^3}+\frac{36x^2}{l^4}\ ]dx \tag{5.10}$$
$$+\frac{pv^2}{\varepsilon}[\ (\frac{4\eta^3}{3l^2}-\frac{5\eta^4}{2l^3}+\frac{6\eta^5}{5l^4})-(\frac{4\mu^3}{3l^2}-\frac{5\mu^4}{2l^3}+\frac{6\mu^5}{5l^4})\ ]$$

$$K_{33}^{e}=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\frac{36}{l^4}-\frac{144x}{l^5}+\frac{144x^2}{l^6}]dx \quad (5.11)$$

$$+\frac{pv^2}{\varepsilon}[(\frac{6\eta^3}{l^4}-\frac{12\eta^4}{l^5}+\frac{24\eta^5}{5l^6})-(\frac{6\mu^3}{l^4}-\frac{12\mu^4}{l^5}+\frac{24\mu^5}{5l^6})]$$

$$K_{34}^{e}=K_{43}^{e}=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[-\frac{12}{l^3}+\frac{60x}{l^4}-\frac{72x^2}{l^5}]dx \quad (5.12)$$

$$+\frac{pv^2}{\varepsilon}[(-\frac{2\eta^3}{l^3}+\frac{9\eta^4}{2l^4}-\frac{12\eta^5}{5l^5})-(-\frac{2\mu^3}{l^3}+\frac{9\mu^4}{2l^4}-\frac{12\mu^5}{5l^5})]$$

$$K_{44}^{e}=\int_{\sum_{e=1}^{r-1}L_e}^{\sum_{e=1}^{r}L_e}\{\sum_{r=1}^{nspan}EI_r(x-\sum_{e=1}^{r-1}L_e)\}[\frac{4}{l^2}-\frac{24x}{l^3}+\frac{36x^2}{l^4}]dx \quad (5.13)$$

$$+\frac{pv^2}{\varepsilon}[(\frac{2\eta^3}{3l^2}-\frac{2\eta^4}{l^3}+\frac{6\eta^5}{5l^4})-(\frac{2\mu^3}{3l^2}-\frac{2\mu^4}{l^3}+\frac{6\mu^5}{5l^4})]$$

Using the same approach, the element mass matrix M_{ij}^{e}, the centripetal acceleration matrix C_{ij}^{e} as well as the element force vector f_i was obtained from equations (4.4), (4.5), and (4.6) respectively. The specification of $Q_{1,}Q_2, Q_3$ and Q_4 depends on the associated boundary conditions for a particular problem [33].

Having obtained the element stiffness, mass, dynamic matrices, and the associated load vectors, the final solution of equation (4.2) was obtained using Newmark's method with the assistance of a computer program written in Visual Basic.

6. Numerical Examples:

In this research work, a two- node simply supported structural beam element was modeled(fig.2a} in order to illustrate the above procedure. The total length of the beam, L=10m,the mass density per beam length $\rho = 7.04gm^3$,the beam's element area $A = 20m^2$, and the load's length $\varepsilon = 0.5m$.

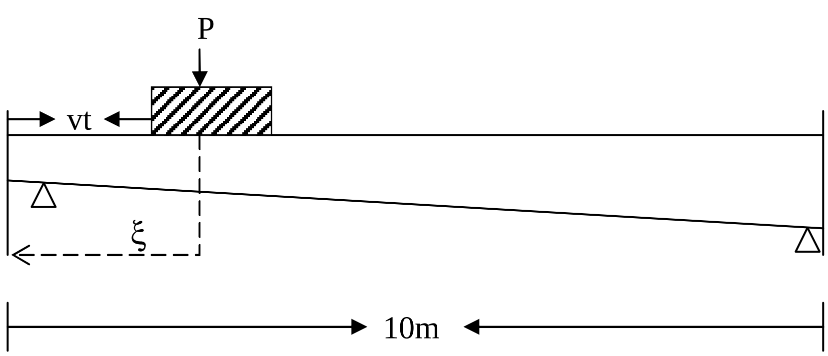

Fig (2a) A non-uniform beam under moving load.

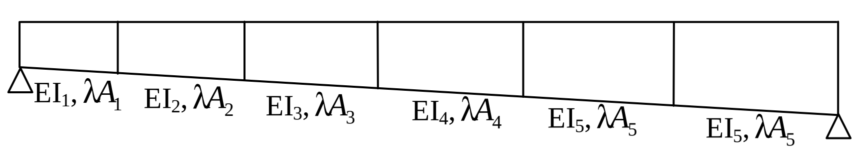

Fig (2b) A discretized non- uniform beam under moving load.

The beam element is discretized into 6 non-uniform element model (fig.2b) with the length of each element given as $L_1 = 1m$, $L_2 = 1.4m$, $L_3 = 1.5m$, $L_4 = 1.6m$, $L_5 = 2m$, $L_6 = 2.5m$, and the flexural rigidities $EI_1 = 2.7728\times10^5 Nm$, $EI_2 = 3.9947\times10^5 Nm$, $EI_3 = 8.2858\times10^5 Nm$, $EI_4 = 2.6179\times10^6 Nm$, $EI_5 = 6.3936\times10^6 Nm$, $EI_6 = 9.3936\times10^6 Nm$, while $A_{11} = 2m^2$, $A_{12} = 2.8m^2$, $A_{13} = 3m^2$, $A_{14} = 3.2m^2$, $A_{15} = 4m^2$, $A_{61} = 5m^2$. For the secondary variables, the bending moment at both ends are equal to zero, the shear force at both ends is 30Nm. To obtain the effect of the velocity on the dynamic response of non-uniform beam elements to moving loads, the velocity is varied from 3m/s to 9m/s. Similarly, the responses at different sizes of the load's length , and span- length of the beam element were also presented, while a comparison between the responses non-uniform simply supported and cantilever beams is also discussed.

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The solutions of problems of non-uniform beams under moving loads formed the basis for this comparison, which led to the following additional conclusions:

(a) Effects of velocity on the dynamic response of the non-uniform beam: The effect of increasing in velocity on the dynamic response of non-uniform simply supported beam under distributed moving load is shown in figure3. It shows that for the initial velocity V_0 smaller than a certain value**,** denoted by $V_0^/$, the value of the deflections(y) increases with increasing in velocity. However, for $V_0 > V_0^/$, the foregoing trend just reverses, the critical value of the initial velocity for this problem is $V_0^/ = 5m/s$, while the reverse case is shown in figure4. the implication is that after exceeding the critical value of the velocity, the deflections decreases as the velocity increases.

(b)Effects of load's length: In order to investigate the influence of the load's length on the dynamic response of non-uniform beam having the same properties as those of the one in figure3, but with $\varepsilon = 0.5$, $\varepsilon = 0.7$, $\varepsilon = 0.9$ respectively were studied. This shows that the deflections(y) increases with increasing in load's length as described in figure5.

(c) Effects of the span-length of the beam element :Furthermore, the span- lengths of $L = 10m$, $16m$ and $22m$ of the beam elements were used to study the influence of the span-length on the dynamic response of non-uniform beam having the same physical properties as those in figure3. It was observed that the deflections increases with increasing in the span- length of the beam, this is shown in figure6.

(d)*Effects of changing in boundary conditions*: However, if the boundary condition is changed from simply supported type to a cantilever one, the behavioural pattern of the responses is in other way round(figure7). That is, the deflections(y) decreases with increasing in velocity after exceeding the critical value of the velocity $V_0^/ = 5m/s$ (figure (8). In figure 9,it shows that for cantilever beam, the amplitude increases with increases in the load's length which is in conformity with that of simply supported beam. Furthermore, if the span length of the beam is increased, the amplitude also increases just like in the simply supported case, this is shown in figure 10.

7. CONCLUSION

A detail analysis of the dynamic response of a non-uniform beam subjected to uniformly distributed moving load has been studied. The finite element model of the problem was obtained by applying the Galerkin's weighted Residual approach, while the displacement and residual functions were interpolated using Hermittian polynomial. Finally, the Newmark's β numerical technique was employed for the evaluation of the resulted equations. The results obtained, for the effects of velocity of the moving loads, the load's length, the effect of the span length of the beam, apart from being extensions of the works done in [26], [27] and [28], they are also in agreement with those in [22], [23] and [24].

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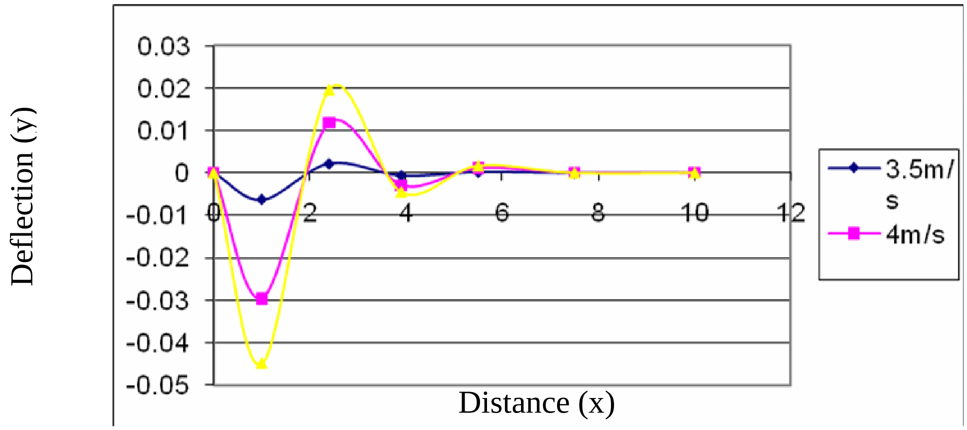

Figure 3: *Effect of increasing in velocity on the dynamic response of a non-uniform simply supported beam.*

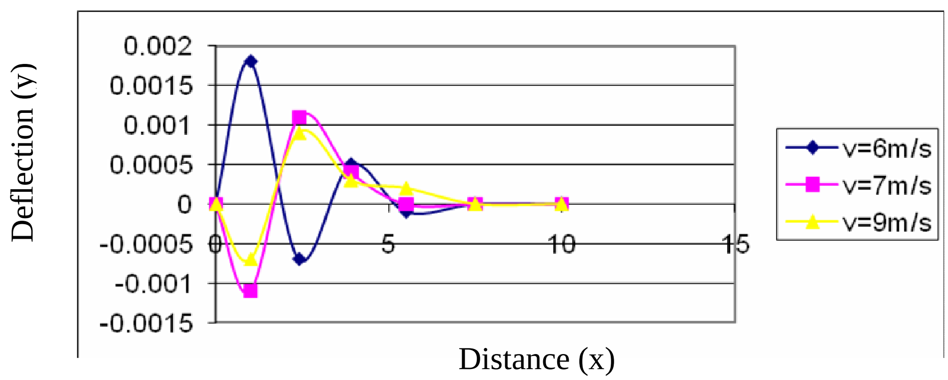

Figure4: *Effect of exceeding the critical value of the velocity on the dynamic response of non-uniform simply supported beam under moving load*

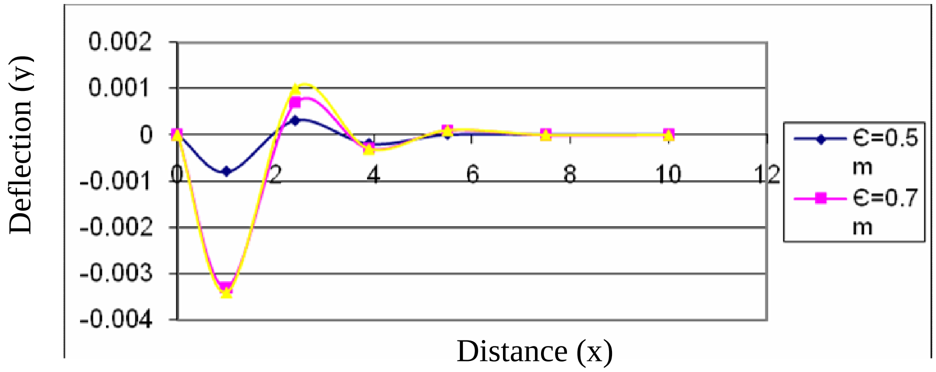

Figure5: *Effect of increasing in load's length on the dynamic response of non-uniform simply supported beam under moving load*

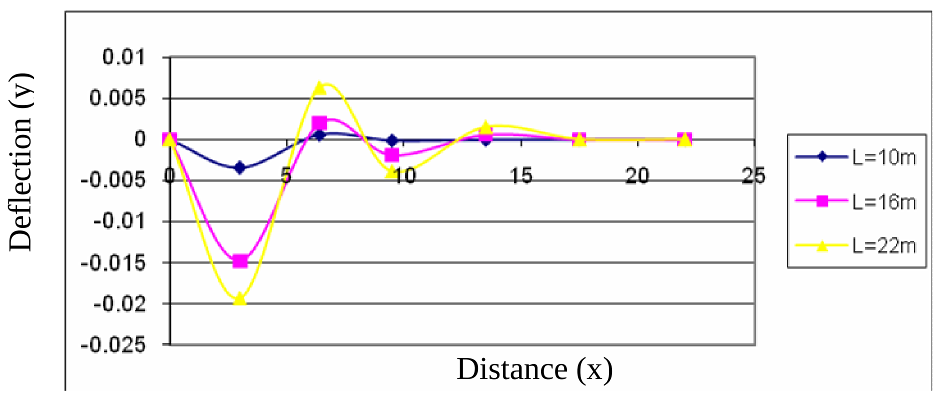

Figure6: *Effect of increases in span-length on the dynamic response of non-uniform simply supported beam under moving load*

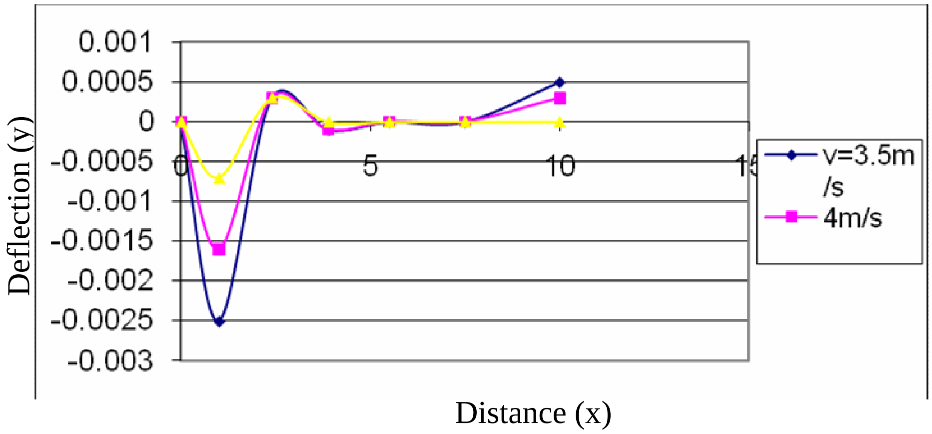

Figure7: *Effect of increasing in velocity on the dynamic response of non-uniform cantilever beam under moving load*

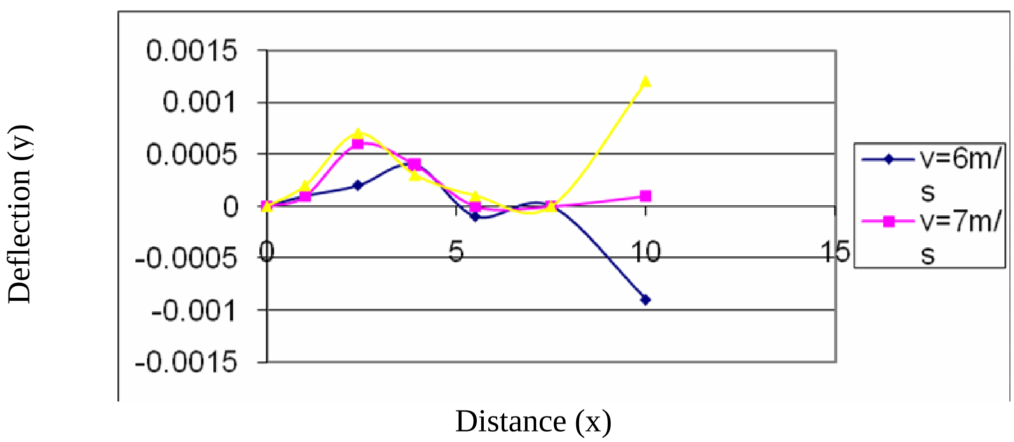

Figure8: *Effect of exceeding critical value of the velocity on the response of cantilever beam*

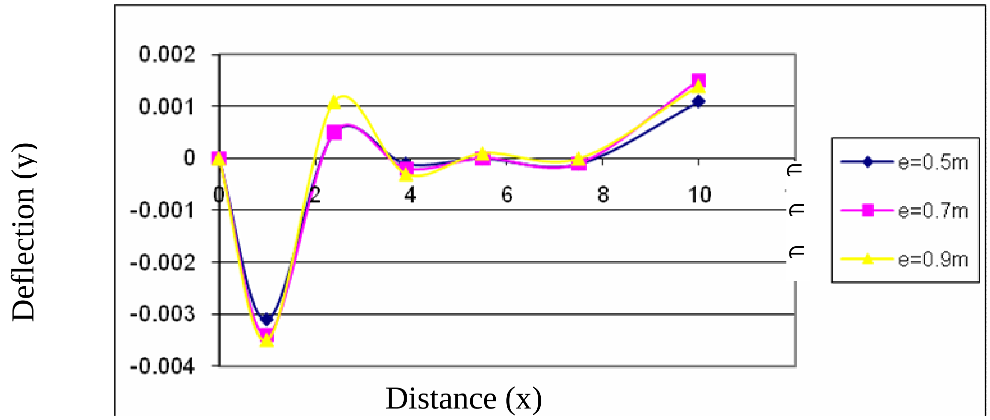

Figure 9: *Effects of increases in load's length on the dynamic response of the cantilever beam*

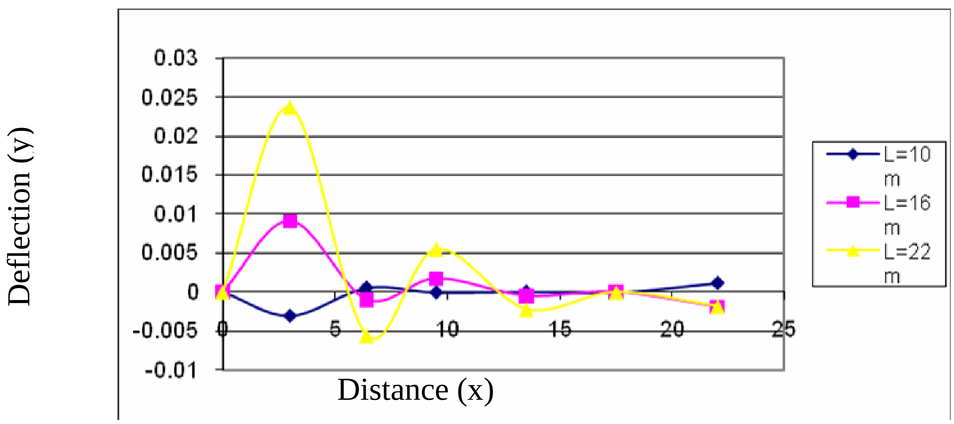

Figure 10: *Effects of increases in the length of the beam on the dynamic response of cantilever beam.*

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