Journal of the Nigerian Association of Mathematical Physics
Volume **16 (May, 2010), pp 111 - 118**

Adaptive GFPS of a new four-dimensional hyperchaotic system with uncertain parameters.

Luo Runzi,* Deng Shucheng and Wei Zhengmin
Department of Mathematics, Nanchang University, 330031, P. R. China.
*Corresponding author: *E-mail address: luo rz@163.com.

Abstract

In this paper, we first introduce a new concept of chaotic synchronization: the generalized function projective synchronization(GFPS), and then investigate the GFPS of a new four-dimensional hyperchaotic system in the presence of unknown system parameters. Based on the Lyapunov stability theory a new sufficient condition is proposed to make the states of two identical hyperchaotic systems asymptotically synchronized. Numerical simulations are presented to show the effectiveness of the proposed schemes.

Key words: Generalized function projective synchronization, Lyapunov stability theory, New hyperchaotic system.

1.0 Introduction

Chaos has aroused considerable interests in many areas of science and technology due to its potential application to laser physics, chemical reactor, secure communication and so on [1]. Since Pecora and Carroll [2] introduced a method to synchronize two identical chaotic systems with different initial conditions, several different types of synchronization phenomena including complete synchronization (CS) [2,3], projective synchronization(PjS)[4], phase synchronization (PS) [5], anti-phase synchronization [6], lag synchronization (LS) [7], generalized synchronization (GS) [8], modified projective synchronization (MPS) [9] and so on have been observed and demonstrated in variety of chaotic systems. Recently, Yong Chen et al [10] extended the modified projective synchronization and raised a new projective synchronization, called function projective synchronization, where the responses of the synchronized dynamical states synchronize up to a scaling function factor. It is obvious that modified projective synchronization is the special case of function projective synchronization as $f_i = \alpha_i, i = 1, 2, \cdots, n$.

In Ref. [11], the author has studied the function projective synchronization problem of two Rössler hyperchaotic with unknown system parameters. Based on Lyapunov stability theory an adaptive control law has been proposed to make the states of two identical Rössler hyperchaotic systems asymptotically synchronized.

Since Rössler first introduced the hyperchaotic dynamical system [12], many hyperchaotic systems have been proposed and studied in the last few decades owing to it has more than one positive Lyapunov exponent, and has more complex dynamical characteristics than chaos. For example, a hyperchaotic Chen system was generated by designing a state feedback controller [13]. By using nonlinear hyperbolic function feedback controls, a novel hyperchaotic system was suppressed to an unstable equilibrium [14]. At the same time, by adding a nonlinear quadratic controller to the second equation of the three-dimensional autonomous modified Lorenz chaotic system, a new hyperchaotic Lorenz system was generated from the original three-dimensional Lorenz system [15].

The objective of this paper is to study the generalized function projective synchronization problem of a new four – dimensional hyperchaotic system with unknown parameters. Based on the Lyapunov stability theory a new sufficient condition is proposed to make the states of two new hyperchaotic systems asymptotically synchronized. Numerical simulations are presented to show the effectiveness of the proposed schemes.

This paper has two benefits. The first is that it first introduces the new concept of chaotic synchronization: the generalized function projective synchronization(GFPS) and investigates the GFPS of a new four-dimensional hyperchaotic system with unknown system parameters. The another is that it has the items $\sin y_1, \sin y_2, \cos y_3, \cos y_4$ which don't contain the integral power of state variables in its synchronization schemes, where $y_i, i = 1,2,3,4,$ is the state variables of the slave (or response) system.

The organization of this paper is as follows. In Section 2 the concept of generalized function projective synchronization is defined at first, then the generalized function projective synchronization of two identical new four-dimensional hyperchaotic systems with unknown parameters is introduced. In Section 3, numerical simulation examples are given to verify the effectiveness of our methods. In Section 4, some concluding remarks are given.

2.0 GFPS of new 4D chaotic system with known parameters.

The generalized function projective synchronization is defined like this. Consider the following chaotic systems:

$$\begin{cases} \dfrac{dx}{dt} = F(x,t) \leftarrow \text{drive system}, \\ \dfrac{dy}{dt} = G(x,y,t) \leftarrow \text{response system}, \end{cases} \tag{2.1}$$

where $x = (x_1, x_2, \cdots, x_n)$, $y = (y_1, y_2, \cdots, y_n)$ are state variables. If there exists functions $f_i(x_1, x_2, \cdots, x_n)$, $g_i(y_1, y_2, \cdots, y_n), i = 1,2,\cdots,n$, such that $\lim_{t\to+\infty} \| f_i(x_1, x_2, \cdots, x_n) - g_i(y_1, y_2, \cdots, y_n) \| = 0$, then we call them "generalized function projective synchronization(GFPS)", and we call $f_i(x_1, x_2, \cdots, x_n)$, $g_i(y_1, y_2, \cdots, y_n), i = 1,2,\cdots,n$ "scaling function factors". Obviously, function projective synchronization is the special case of GFPS as $f_i(x_1, x_2, \cdots, x_n) = x_i$ and $g_i(y_1, y_2, \cdots, y_n) = h_i(y_1, y_2, \cdots, y_n) y_i, i = 1,2,\cdots,n$, where $h_i(y_1, y_2, \cdots, y_n)$ is a function.

The nonlinear differential equations that describe the new four-dimensional hyperchaotic chaotic system are

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = dx_1 - x_1 x_3 + cx_2 - x_4, \\ \dot{x}_3 = x_1 x_2 - bx_3, \\ \dot{x}_4 = x_1 + h, \end{cases} \tag{2.2}$$

where a, b, c, d, h are real constants and x_1, x_2, x_3, x_4 are state variables.

When $a = 36, b = 3, c = 28, d = -16$ and $-0.7 \le h \le 0.7$, the system (2.2) is hyperchaotic; and $0.7 < h < 3.5$ or $-3.5 < h < -0.7$, the system (2.2) is chaotic; and $-4.0 < h \le -3.5$ or $3.5 \le h < 4.0$, the system (2.2) displays three kind of periodic orbits, when h varies. The chaoticattractor of system (2.2) with $a = 36, b = 3, c = 28, d = -16, h = 0$ is shown in Fig. 1. For more detailed analysis of the complex dynamics of the system, please see relative reference [16].

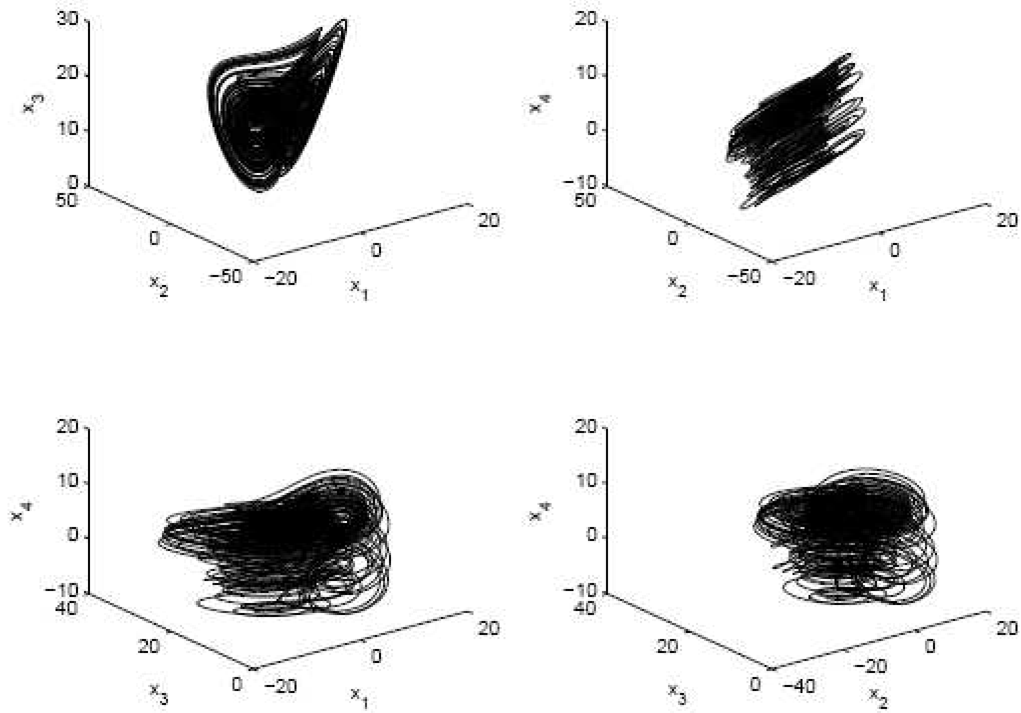

Fig.1. The chaotic attractor of system (2) with $a = 36, b = 3, c = 28, d = -16, h = 0$

We assume system (2.2) is the master (or drive) system, then the controlled slave (or response) system can be defined below

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + u_1, \\ \dot{y}_2 = d_1 y_1 - y_1 y_3 + c_1 y_2 - y_4 + u_2, \\ \dot{y}_3 = y_1 y_2 - b_1 y_3 + u_3, \\ \dot{y}_4 = y_1 + h_1 + u_4, \end{cases} \tag{2.3}$$

where a_1, b_1, c_1, d_1 and h_1 are parameters of the slave system which needs to be estimated, and u_1, u_2, u_3 and u_4 are the nonlinear controller such that two hyperchaotic systems can be synchronized in the sense that

$$\begin{cases} \lim_{t\to\infty} \| f_1(x_1, x_2, x_3, x_4) - g_1(y_1, y_2, y_3, y_4) \| = 0, \\ \lim_{t\to\infty} \| f_2(x_1, x_2, x_3, x_4) - g_2(y_1, y_2, y_3, y_4) \| = 0, \\ \lim_{t\to\infty} \| f_3(x_1, x_2, x_3, x_4) - g_3(y_1, y_2, y_3, y_4) \| = 0, \\ \lim_{t\to\infty} \| f_4(x_1, x_2, x_3, x_4) - g_4(y_1, y_2, y_3, y_4) \| = 0. \end{cases} \tag{2.4}$$

In our synchronization scheme we assume $f_i(x_1, x_2, x_3, x_4) = (\alpha_{i1} x_i + \alpha_{i2}) x_i, i = 1, 2, 3, 4$, where

$$
\begin{aligned}
&g_1(y_1,y_2,y_3,y_4)=\beta_{11}\sin y_1+\beta_{12}y_1,\ g_2(y_1,y_2,y_3,y_4)=\beta_{21}\sin y_2+\beta_{22}y_2,\\
&g_3(y_1,y_2,y_3,y_4)=\beta_{31}\cos y_3+\beta_{32}y_3,\ g_4(y_1,y_2,y_3,y_4)=\beta_{41}\cos y_4+\beta_{42}y_4,\\
&\alpha_{i1}^2+\alpha_{i2}^2\neq 0,\beta_{i1}^2+\beta_{i2}^2\neq 0(i=1,2,3,4).
\end{aligned}
$$

By subtracting Eq. (2.2) from Eq. (2.3) we have

$$
\begin{aligned}
\dot{e}_1 &= (2\alpha_{11}x_1+\alpha_{12})\dot{x}_1-(\beta_{11}\cos y_1+\beta_{12})\dot{y}_1\\
&=-k_1e_1+(2\alpha_{11}x_1+\alpha_{12})(x_2-x_1)(a-a_1)+[(2\alpha_{11}x_1+\alpha_{12})(x_2-x_1)a_1-\\
&\quad(\beta_{11}\cos y_1+\beta_{12})(y_2-y_1)a_1+k_1e_1-(\beta_{11}\cos y_1+\beta_{12})u_1],\\
\dot{e}_2 &= (2\alpha_{21}x_2+\alpha_{22})\dot{x}_2-(\beta_{21}\cos y_2+\beta_{22})\dot{y}_2\\
&=-k_2e_2+(2\alpha_{21}x_2+\alpha_{22})x_1(d-d_1)+(2\alpha_{21}x_2+\alpha_{22})x_2(c-c_1)+[(2\alpha_{21}x_2+\alpha_{22})\\
&\quad(d_1x_1-x_1x_3+c_1x_2-x_4)-(\beta_{21}\cos y_2+\beta_{22})(d_1y_1-y_1y_3+c_1y_2-y_4)+k_2e_2-\\
&\quad(\beta_{21}\cos y_2+\beta_{22})u_2],\\
\dot{e}_3 &= (2\alpha_{31}x_3+\alpha_{32})\dot{x}_3-(-\beta_{31}\sin y_3+\beta_{32})\dot{y}_3\\
&=-k_3e_3-(2\alpha_{31}x_3+\alpha_{32})x_3(b-b_1)+[(2\alpha_{31}x_3+\alpha_{32})(x_1x_2-b_1x_3)-(-\beta_{31}\sin y_3+\beta_{32})\\
&\quad(y_1y_2-b_1y_3)+k_3e_3-(-\beta_{31}\sin y_3+\beta_{32})u_3],\\
\dot{e}_4 &= (2\alpha_{41}x_4+\alpha_{42})\dot{x}_4-(-\beta_{41}\sin y_4+\beta_{42})\dot{y}_4\\
&=-k_4e_4+(2\alpha_{41}x_4+\alpha_{42})(h-h_1)+[(2\alpha_{41}x_4+\alpha_{42})(x_1+h_1)-(-\beta_{41}\sin y_4+\beta_{42})\\
&\quad(y_1+h_1)+k_4e_4-(-\beta_{41}\sin y_4+\beta_{42})u_4],
\end{aligned}
$$

where

$$
\begin{cases}
e_1=(\alpha_{11}x_1+\alpha_{12})\,x_1-(\beta_{11}\sin y_1+\beta_{12}y_1)\ ,\\
e_2=(\alpha_{21}x_2+\alpha_{22})x_2-(\beta_{21}\sin y_2+\beta_{22}y_2),\\
e_3=(\alpha_{31}x_3+\alpha_{32})x_3-(\beta_{31}\cos y_3+\beta_{32}y_3),\\
e_4=(\alpha_{41}x_4+\alpha_{42})x_4-(\beta_{41}\cos y_4+\beta_{42}y_4).
\end{cases}
\tag{2.5}
$$

Thus we have the error dynamical system between Eqs. (2.2) and (2.3)

$$
\begin{cases}
\dot{e}_1=-k_1e_1+(2\alpha_{11}x_1+\alpha_{12})(x_2-x_1)(a-a_1)+[(2\alpha_{11}x_1+\alpha_{12})(x_2-x_1)a_1-\\
\quad(\beta_{11}\cos y_1+\beta_{12})(y_2-y_1)a_1+k_1e_1-(\beta_{11}\cos y_1+\beta_{12})u_1],\\
\dot{e}_2=-k_2e_2+(2\alpha_{21}x_2+\alpha_{22})x_1(d-d_1)+(2\alpha_{21}x_2+\alpha_{22})x_2(c-c_1)+\\
\quad[(2\alpha_{21}x_2+\alpha_{22})(d_1x_1-x_1x_3+c_1x_2-x_4)-(\beta_{21}\cos y_2+\beta_{22})\\
\quad(d_1y_1-y_1y_3+c_1y_2-y_4)+k_2e_2-(\beta_{21}\cos y_2+\beta_{22})u_2],\\
\dot{e}_3=-k_3e_3-(2\alpha_{31}x_3+\alpha_{32})x_3(b-b_1)+[(2\alpha_{31}x_3+\alpha_{32})(x_1x_2-b_1x_3)-\\
\quad(-\beta_{31}\sin y_3+\beta_{32})(y_1y_2-b_1y_3)+k_3e_3-(-\beta_{31}\sin y_3+\beta_{32})u_3],\\
\dot{e}_4=-k_4e_4+(2\alpha_{41}x_4+\alpha_{42})(h-h_1)+[(2\alpha_{41}x_4+\alpha_{42})(x_1+h_1)-(-\beta_{41}\sin y_4+\beta_{42})\\
\quad(y_1+h_1)+k_4e_4-(-\beta_{41}\sin y_4+\beta_{42})u_4].
\end{cases}
\tag{2.6}
$$

Our aim is to find control laws $u_i (i = 1,2,3,4)$ for stabilizing the error variables of system (2.6) at the origin. For this end, we propose the following control law and update rule for system (2.3):

$$\begin{cases} u_1 = [(2\alpha_{11}x_1 + \alpha_{12})(x_2 - x_1)a_1 - (\beta_{11}\cos y_1 + \beta_{12})(y_2 - y_1)a_1 + k_1e_1]/(\beta_{11}\cos y_1 + \beta_{12}), \\ u_2 = [(2\alpha_{21}x_2 + \alpha_{22})(d_1x_1 - x_1x_3 + c_1x_2 - x_4) - (\beta_{21}\cos y_2 + \beta_{22})(d_1y_1 - y_1y_3 + c_1y_2 - y_4) \\ \qquad + k_2e_2]/(\beta_{21}\cos y_2 + \beta_{22}), \\ u_3 = [(2\alpha_{31}x_3 + \alpha_{32})(x_1x_2 - b_1x_3) - (-\beta_{31}\sin y_3 + \beta_{32})(y_1y_2 - b_1y_3) + k_3e_3]/(-\beta_{31}\sin y_3 + \beta_{32}), \\ u_4 = [(2\alpha_{41}x_4 + \alpha_{42})(x_1 + h_1) - (-\beta_{41}\sin y_4 + \beta_{42})(y_1 + h_1) + k_4e_4]/(-\beta_{41}\sin y_4 + \beta_{42}), \end{cases} \tag{2.7}$$

where $k_i > 0 (i = 1, \dots, 4)$, and the update rule for five unknown parameters a_1, b_1, c_1, d_1, h_1 is

$$\begin{cases} \dot{a}_1 = (2\alpha_{11}x_1 + \alpha_{12})(x_2 - x_1)e_1, \\ \dot{b}_1 = -(2\alpha_{31}x_3 + \alpha_{32})x_3e_3, \\ \dot{c}_1 = (2\alpha_{21}x_2 + \alpha_{22})x_2e_2, \\ \dot{d}_1 = (2\alpha_{21}x_2 + \alpha_{22})x_1e_2, \\ \dot{h}_1 = (2\alpha_{41}x_4 + \alpha_{42})e_4. \end{cases} \tag{2.8}$$

Thus, we can establish the following theorem.

Theorem. For given nonzero scaling function factors $f_i, g_i (i = 1,2,3,4)$, the generalized function projective synchronization between drive systems (2.2) and response system (2.3) will occur by the control law (2.7) and the update rule (2.8).

Proof. Choose the following Lyapunov function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_h^2),$$

where $e_a = a - a_1, e_b = b - b_1, e_c = c - c_1, e_d = d - d_1, e_h = h - h_1$. The time derivative of V along the trajectory of error system (2.6) is

$$\begin{aligned} \frac{dV}{dt} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \dot{e}_4e_4 + \dot{e}_ae_a + \dot{e}_be_b + \dot{e}_ce_c + \dot{e}_de_d + \dot{e}_he_h \\ &= e_1[-k_1e_1 + (2\alpha_{11}x_1 + \alpha_{12})(x_2 - x_1)(a - a_1) + (2\alpha_{11}x_1 + \alpha_{12})(x_2 - x_1)a_1 - \\ &(\beta_{11}\cos y_1 + \beta_{12})(y_2 - y_1)a_1 + k_1e_1 - (\beta_{11}\cos y_1 + \beta_{12})u_1] + e_2[-k_2e_2 + (2\alpha_{21}x_2 + \alpha_{22}) \\ &x_1(d - d_1) + (2\alpha_{21}x_2 + \alpha_{22})x_2(c - c_1) + (2\alpha_{21}x_2 + \alpha_{22})(d_1x_1 - x_1x_3 + c_1x_2 - x_4) - \\ &(\beta_{21}\cos y_2 + \beta_{22})(d_1y_1 - y_1y_3 + c_1y_2 - y_4) + k_2e_2 - (\beta_{21}\cos y_2 + \beta_{22})u_2] + e_3[-k_3e_3 - \\ &\quad (2\alpha_{31}x_3 + \alpha_{32})x_3(b - b_1) + (2\alpha_{31}x_3 + \alpha_{32})(x_1x_2 - b_1x_3) - (-\beta_{31}\sin y_3 + \beta_{32})(y_1y_2 - b_1y_3) \\ &+ k_3e_3 - (-\beta_{31}\sin y_3 + \beta_{32})u_3] + e_4[-k_4e_4 + (2\alpha_{41}x_4 + \alpha_{42})(h - h_1) + (2\alpha_{41}x_4 + \alpha_{42}) \\ &(x_1 + h_1) - (-\beta_{41}\sin y_4 + \beta_{42})(y_1 + h_1) + k_4e_4 - (-\beta_{41}\sin y_4 + \beta_{42})u_4] + \dot{a}_1(a_1 - a) + \\ &\dot{b}_1(b_1 - b) + \dot{c}_1(c_1 - c) + \dot{d}_1(d_1 - d) + \dot{h}_1(h_1 - h). \end{aligned} \tag{2.9}$$

By substituting the control input (2.7) and the update rule (2.8) into Eq. (2.9), we have

$$\frac{dV}{dt} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 = -e^T K e, \qquad (2.10)$$

where $e = (e_1, e_2, e_3, e_4)^T$, and $K = diag(k_1, k_2, k_3, k_4)^T$.

Since $\dot{V} \le 0,$ we have $e_1, e_2, e_3, e_4 \to 0$ as $t \to \infty,$ i.e., $\lim_{t\to\infty} \|e\| = 0.$ This completes the proof.

3.0 Numerical simulation

Numerical simulations results are presented to demonstrate the effectiveness of the proposed synchronization methods. Fourth-order Runge-Kutta method is used to solve the systems of differential equations (2.2) and (2.3). In addition, a time step of size 0.001 is employed. The parameters are chosen to be $a = 36, b = 3, c = 28, d = -16$ and $h = 0$ in all simulations so that the new hyperchaotic system exhibits a hyperchaotic behavior if no control is applied. The initial states of the drive system are $x_1(0) = 11, x_2(0) = 12, x_3(0) = 13$ and $x_4(0) = 14$ and initial states of the response system are $y_1(0) = 5, y_2(0) = 6, y_3(0) = 7$ and $y_4(0) = 8.$ Suppose the function factors are $f_i(x_1, x_2, x_3, x_4) = (x_i + 2)x_i, i = 1, 2, 3, 4$, and $g_1(y_1, y_2, y_3, y_4) = \sin y_1 + 2y_1, g_2(y_1, y_2, y_3, y_4) = \sin y_2 + 2y_2,$ $g_3(y_1, y_2, y_3, y_4) = \cos y_3 + 2y_3, g_4(y_1, y_2, y_3, y_4) = \cos y_4 + 2y_4,$ so that $\cos y_1 + 2 \ne 0, \cos y_2 + 2 \ne 0, -\sin y_3 + 2 \ne 0, -\sin y_4 + 2 \ne 0.$ Furthermore, the initial values of estimated parameters are chosen as $a_1(0) = b_1(0) = c_1(0) = d_1(0) = 0, h_1(0) = 1$ and the control gains are $(k_1, k_2, k_3, k_4) = (100, 100, 100, 100)$. Synchronization of systems (2.2) and (2.3) via adaptive control law (2.7) and (2.8) are shown in Figs. 2 and 3. Fig. 2 displays the synchronization errors between systems (2.2) and (2.3). Obviously, they tend to zero after a sufficiently long time. Fig. 3 indicates that the identified parameters $a_1(t), b_1(t), c_1(t), d_1(t), h_1(t)$ approach the desired values: $a = 36, b = 3, c = 28, d = -16, h = 0$ as $t \to \infty$.

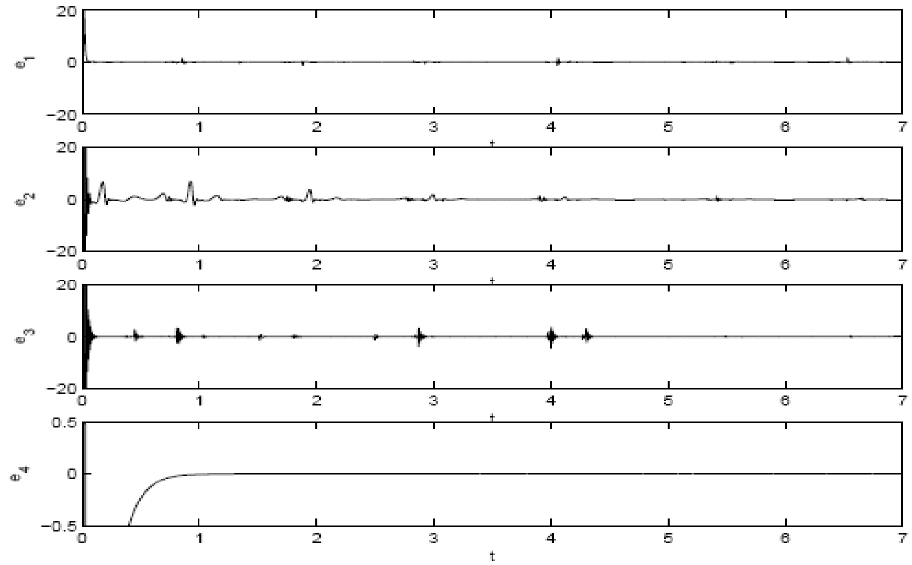

Figure 2: Error signals between drive and response systems.

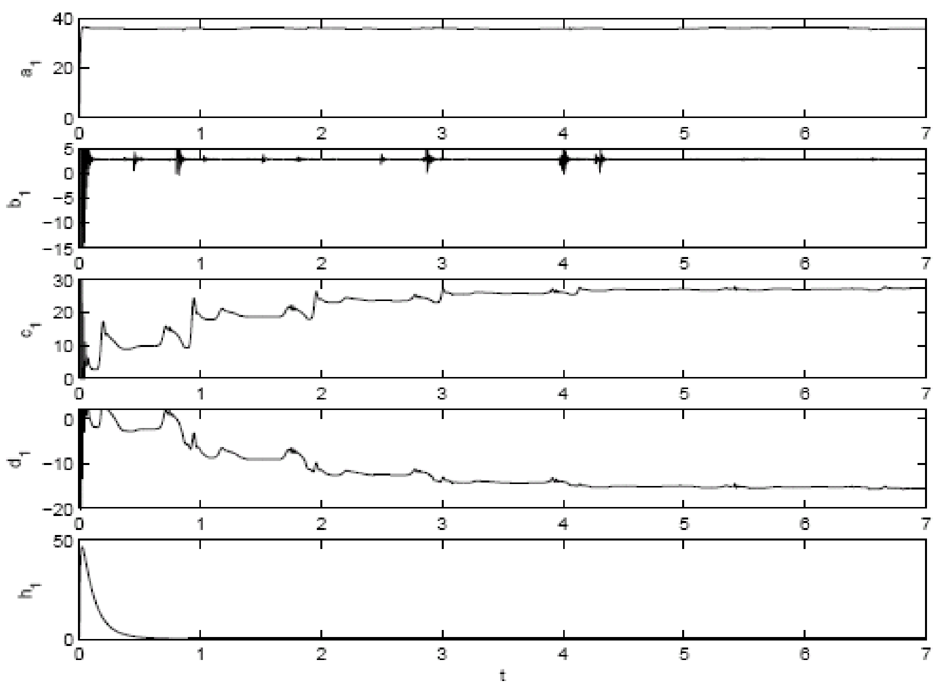

Figure 3: Estimated values for unknown parameters.

4.0 Conclusions

This paper has investigated the generalized function projective synchronization problem of a new hyperchaotic system. A novel synchronization method has been proposed for this new hyperchaotic system with all the system parameters unknown based upon adaptive control. By this method, one can achieve hyperchaotic synchronization and identify the unknown parameters simultaneously. Numerical simulation are used to verify the effectiveness of the proposed chaos synchronization scheme.

References

[1] G. Chen, X. Dong, From chaos to order: methodologies, perspectives and applications. Singapore: World Scientific; 1998.

[2] L.M. Pecora , T.L. Carroll, Synchronization in chaotic systems, Phys Rev Lett 64 (1990) 821-824.

[3] K. Kemih, S. Filali, M. Benslama , and M. Kimouche, Passivity-based control of chaotic Lü system, International Journal of Innovative Computing, Information and Control 2(2006)331-337.

[4] R. Mainieri, J. Rehacek, Projective synchronization in three-dimensional chaotic systems. Phys Rev Lett 82(1999)3042-3046.

[5] M. Rosenblum, A. Pikovsky, J. Kurtz, Phase synchronization of chaotic oscillators, Phys. Rev. Lett. 76(1996) 1804-1807.

[6] B.A.Idowu,U.E.Vincent,A.O.Obawole,Anti-synchronization between two new different chaotic systems via active control, Journal of the Nigerian Association of Mathematical Physics 11 (2007)15-20.

[7] C. Cailian, F. Gang, G. Xinping, An adaptive lag-synchronization method for time-delay chaotic systems, in: Proceedings of the American Control Conference (2005) 4277-4382.

[8] G.H. Li, Generalized projective synchronization between Lorenz system and Chen's system. Chaos, Solitons & Fractals 32 (2007) 1454-1458.

[9] G.H. Li, Modified projective synchronization of chaotic system. Chaos, Solitons & Fractals 32 (2007) 1786-1790.

[10] Y. Chen, H.L. An, Z. B. Li, The function cascade synchronization approach with uncertain parameters or not for hyperchaotic systems, Applied Mathematics and Computation 197 (2008) 96-110.

[11] R. Z. Luo, Adaptive function project synchronization of Rössler hyperchaotic system with uncertain parameters, Physics Letters A 372 (2008) 3667-3671.

[12] O.E. Rössler, An equation for hyperchaos, Phys. Lett. A 71 (1979) 155-157.

[13] Y. Li, W.K.S. Tang, G.R. Chen, Generating hyperchaos via state feedback control, Internat. J. Bifur. Chaos 15 (2005) 3367-3375.

[14] Z. Yan, Controlling hyperchaos in the new hyperchaotic Chen system, Appl. Math. Comput. 168 (2005) 1239-1250.

[15] T. Gao, G. Chen, Z. Chen, S. Cang, The generation and circuit implementation of a new hyper-chaos based upon Lorenz system, Phys. Lett. A 361 (2007) 78-86.

[16] T. G. Gao, Q. L. Gu, Z. Q. Chen, Analysis of the hyper-chaos generated from Chen's system, Chaos, Solitons and Fractals 39 (2009) 1849-1855.