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On The Existence Of The Infinite Volume Quantum Stochastic Dynamics

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Abstract

We proof the existence of the infinite volume dynamics for spin systems as the thermodynamic limit $(\Lambda \to \mathbb{Z}^d)$ of the finite volume dynamics within the framework of non commutative L_p –spaces .

Keywords: quasi-local von Neumann algebras, non commutative L_p –spaces, quantum stochastic dynamics.

1.0 Introduction

The Mathematical theory of quantum stochastic dynamics or quantum dynamical semigroup started in the seventies with the celebrated works of Davies[6] and Lindblad [13].Quantum dynamical semigroup can be viewed as a generalization of classical Markov semigroup on the Abelian algebra of functions on some space, they are semigroup of completely positive, identity preserving maps on an operator algebra [17].

The description of infinite quantum spin systems is far less advanced than the one for the classical case, there is a lack of satisfactory description of quantum stochastic dynamics. In the quantum case, it is much harder to construct a Markov semigroup that will satisfy the requirement of a detailed balance condition with respect to a KMS state [14].

However, we will show that having at our disposal a set of families of interpolating non commutative L_p –spaces from which to choose, we can apply the abstract sufficient condition given in[14] to construct an infinite volume translation invariant Markov semigroups of spin flip type as the thermodynamic limit of the corresponding finite volume dynamics, with the desired property for operators of the form $\rho_\Lambda^{\frac{1-t}{2}} x\, \rho_\Lambda^{\frac{1-t}{2}}$.

The extensive literature on the subject and its applications on various areas is cited in references [1,2,3,4,8,10,11,12].

2.0 Non Commutative L_p –Spaces and Quasilocal Algebras

2.1. Quasilocal (spin) Algebras

Let $\mathbb{Z}^d$ be a d-dimensional integer lattice ,and $\mathcal{F}$ denote the family of all its finite subsets. By $\mathcal{F}_0$ we will denote an increasing sequence of finite volumes invading all the lattice $\mathbb{Z}^d$. Given a sequence $\{F_\Lambda\}_{\Lambda\in\mathcal{F}_0}$ we will call its limit as $\Lambda \to \mathbb{Z}^d$ through the sequence $\mathcal{F}_0$ by $\lim_{\mathcal{F}_0} F_\Lambda$.

Let $\mathcal{M}$ be a quasilocal von Neumann algebra with norm $\|.\|$ defined as the inductive limit over a finite dimensional complex matrix algebra A . It is natural to view $\mathcal{M}$as a noncommutative analog of the space of bounded continuous functions.

For a set $X \in \mathcal{F}$, let $\mathcal{M}_X$ denote a subalgebra of operators localised in the set X, i.e the subalgebra in $\mathcal{M}$ isomorphic to A^X.

or arbitrary subset $\Lambda \subset \mathbb{Z}^d$ we define $\mathcal{M}_\Lambda$ to be the smallest (closed) subalgebra of $\mathcal{M}$ containing $\bigcup\{\mathcal{M}_X : X \in \mathcal{F}, X \subset \Lambda\}$ .An operator $f \in \mathcal{M}$ will be called local if there is some $Y \in \mathcal{F}$ such that $f \in \mathcal{M}_Y$. By $\mathcal{M}_0$ we denote the subset of $\mathcal{M}$ consisting of all local operators . We will denote by $\mathcal{M}_0^+, \mathcal{M}^+$ the local nonnegative and nonnegative operators given any finite set $X \in \mathcal{F}$ we have $\mathcal{M} = \mathcal{M}_X \otimes \mathcal{M}_{X^c}$
By $Tr_X, X \in \mathcal{F}$, we denote a normalised partial trace on $\mathcal{M}$, that is a completely positive map $Tr_X : \mathcal{M} \to \mathcal{M}_{X^c}$ which preserves unit and Satisfies the following conditions

$$\forall f \in \mathcal{M}, \quad g, h \in \mathcal{M}_{X^c} \qquad Tr_X(gfh) = g(Tr_X f)h$$

$$\forall f, g \in \mathcal{M}_X \qquad Tr_X(fg) = Tr_X(gf)$$

Remark: *We recall that a positivity and unit preserving map for which (i) holds is called a conditional expectation.* Let $Tr \equiv lim_{\mathcal{F}_0} Tr_\Lambda$ be the normalised trace on $\mathcal{M}$ then we have

$$Tr(Tr_X(f^*)g) = Tr(f^* Tr_X(g)) \tag{2.1}$$

Let $\Phi \equiv \{\Phi_X \in \mathcal{M}_X\}_{X \in \mathcal{F}}$ be a quantum(Gibbsian) potential of finite range , i.e a family of selfadjoint operators such that

$$\|\Phi\|_1 \equiv \sup_{i \in \mathbb{Z}^d} \sum_{\substack{X \in \mathcal{F} \\ X \ni i}} \|\Phi_X\| < \infty . \tag{2.2}$$

$\Phi \equiv \{\Phi_X \in \mathcal{M}_X\}_{X \in \mathcal{F}}$ is of finite range $R > 0, iff \;\; \Phi_X = 0 \; for \; all \; X \in \mathcal{F}, diam(X) > R$.
We defined a Hamiltonian H_Λ by setting

$$H_\Lambda(\Phi) \equiv \sum_{X \subset \Lambda} \Phi_X \tag{2.3}$$

Hence we have the density matrix given by

$$\rho_\Lambda \equiv \frac{e^{-\beta H_\Lambda}}{Tr e^{-\beta H_\Lambda}} \tag{2.4}$$

with $\beta \in (0, \infty)$. A finite volume Gibbs state φ_Λ is defined as follows

$$\varphi_\Lambda(f) \equiv Tr(\rho_\Lambda f) \tag{2.5}$$

And for sufficiently small $\beta \in (0, \infty)$ the following limit state on $\mathcal{M}$ exist and is given by

$$\varphi \equiv lim_{\mathcal{F}_0} \varphi_\Lambda. \tag{2.6}$$

The finite volume automorphism group associated to potential Φ is denoted by

$$\alpha_t^\Lambda(f) \equiv e^{+itH_\Lambda} f \, e^{-itH_\Lambda} \tag{2.7}$$

One has the following KMS condition for the finite volume state ω_Λ

$$\varphi_\Lambda(f^* g) = \varphi_\Lambda(\alpha_{-i\beta}^\Lambda(g) f^*) \tag{2.8}$$

It is known, [4], that for a class of potentials including the potentials of finite range the following limit exists

$$\alpha_t(f) \equiv \lim_{\Lambda \to \mathbb{Z}^d} \alpha_t^\Lambda(f) \tag{2.9}$$

For every $f \in \mathcal{M}_0$ and defines the automorphism group associated to the infinite volume state φ , in the sense that the following KMS condition is satisfied

$$\varphi \quad (f^{*}g) = \varphi \quad (\alpha_{-i\beta}(g) f^{*}) \tag{2.10}$$

2.2. Non Commutative L_p –Spaces For Quantum Spin Systems On A Lattice

Let $\mathcal{M}$ be a quasilocal von Neumann algebra describe in Section 2.1 and let φ_Λ be a finite volume Gibbs state. Following [20, 22] we define the $L_p(\varphi_\Lambda, t)$, $p \in [1, \infty)$,norms on $\mathcal{M}$.

$$\|x_\wedge\|_p^\varphi = \left(Tr \left| \rho_\wedge^{\frac{1}{2p}} U_{x,t} \left| \rho_\wedge^{\frac{(1-t)}{2}} x \rho_\wedge^{\frac{(1-t)}{2}} \right|^{Pt} \rho_\wedge^{\frac{1}{2p}} \right|^p \right)^{\frac{1}{p}} \tag{2.11}$$

with $x_\Lambda = \rho_\Lambda^{\frac{1-t}{2}} x \, \rho_\Lambda^{\frac{1-t}{2}}$. We have our $L_p(\varphi_\Lambda, t)$, given by

$$L_P(\varphi_\Lambda, t) = \left\{ \rho_\wedge^{\frac{1-t}{2}} x \, \rho_\wedge^{\frac{1-t}{2}} \in M_\wedge : x \in M_+, 0 < t < 1, \|x_\wedge\|_p < \infty \right\} \tag{2.12}$$

In particular for $= 2$, the corresponding norm is given by the following scalar product

$$(x_\Lambda, y_\Lambda)_{\omega_\Lambda} \equiv Tr \left(\rho_\Lambda^{\frac{1}{2}} \, (\rho_\Lambda^{\frac{1-t}{2}} x^{*} \rho_\Lambda^{\frac{1-t}{2}}) \, \rho_\Lambda^{\frac{1}{2}} \, (\rho_\Lambda^{\frac{1-t}{2}} y \, \rho_\Lambda^{\frac{1-t}{2}}) \right) \tag{2.13}$$

The properties of the norm are collected by the following theorem

THEOREM 2.1. For $x_\Lambda, y_\Lambda \in \mathcal{M}_0$ and $p, q \in [1, \infty)$ we have

For any $c \in \mathbb{C}$

$$0 \leq \|c x_\Lambda\|_p = |c| . \|x_\Lambda\|_p \tag{2.14}$$

Holder inequalities

$$|(x_\Lambda, y_\Lambda)_{\omega_\Lambda}| \leq \|x_\Lambda\|_p \; \|y_\Lambda\|_q \tag{2.15}$$

With $p, q \in [1, \infty)$ such that $p^{-1} + q^{-1} = 1,$ and if $p \leq q,$ we have

$$\|x_\Lambda\|_p \leq \|x_\Lambda\|_q \leq \|x_\Lambda\| \tag{2.16}$$

Minkowski inequality

$$\|x_\Lambda + y_\Lambda\|_p \leq \|x_\Lambda\|_p + \|y_\Lambda\|_p \tag{2.17}$$

2.3. Generalized Conditional Expectation

For $X \in \mathcal{F},$ Let $E_{X\wedge} : M \to M$ be a map define as follows

$$E_{X\wedge}\left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right) = TR_X\left(\gamma_{x\wedge}^*\left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right)\gamma_{x\wedge}\right) \tag{2.18}$$

where $\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}} \in M_0$ and $\gamma_{X\wedge} \equiv \gamma_{X\wedge}(1/2)$ with $\gamma_{X\wedge}(s) = \rho_\wedge^s (Tr_x \rho_\wedge)^{-S}$

The map $E_{X,\Lambda}$ is completely positive with the following properties,

PROPOSITION 2.1

(i) $E_{X\wedge}\left(\rho^{\frac{(1-t)}{2}} f \rho^{\frac{(1-t)}{2}}\right) \geq 0$ Positive (2.19)

(ii) $E_{X\wedge}(1) = 1$ Unit preserving (2.20)

(iii) $E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right)^* E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right) \leq E_{X\wedge}\left[\left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{1-t}{2}}\right)^*\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right)\right]$

Kadison-Schwarz Inequality (2.21)

(iv) $\left\|E_{X\wedge}\left(\rho^{\frac{(1-t)}{2}} f \rho^{\frac{(1-t)}{2}}\right)\right\| \leq \left\|\rho^{\frac{(1-t)}{2}} f \rho^{\frac{(1-t)}{2}}\right\|$ Bounded (2.22)

(v) $\left[E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right)\right]^* = E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right)^*$ *Invariance (2.23)

(vi) $\left\langle E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right), \left(\rho_\wedge^{\frac{(1-t)}{2}} g \rho_\wedge^{\frac{(1-t)}{2}}\right)\right\rangle = \left\langle\left(\rho_\wedge^{\frac{(1-t)}{2}} f \rho_\wedge^{\frac{(1-t)}{2}}\right), E_{X\wedge}\left(\rho_\wedge^{\frac{(1-t)}{2}} g \rho_\wedge^{\frac{(1-t)}{2}}\right)\right\rangle$

L_2-Symmetry (2.24)

Remark; In general $E_{X,\Lambda}\left(E_{X,\Lambda}(f_\Lambda)\right) \neq E_{X,\Lambda}(f_\Lambda)$.

3. Stochastic Dynamics

3.1 Lindblad -Type Generator

The generator of a quantum dynamical semi-groups was discussed in Lindblad [13], where in that paper he gave the explicit form of the generator as $L_x(x) = \psi(x) - \frac{1}{2}\{\psi(1), x\} + i[H, x]$, to have a dynamics that describe irreversible processes like dissipation, we will need a generator of the form $L(x) = \psi(x) - \frac{1}{2}\{\psi(1), x\}$ we choose our completely positive map $\psi(x)$ to be a generalized conditional expectation. We begin by defining the generator of our dynamics.

We define the operator $L_{X\wedge} : M \to M$ by

$$L_{X,\wedge}\left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{1-t}{2}}\right) = E_{X,\wedge}\left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{1-t}{2}}\right) - \frac{1}{2}\left\{E_{X,\wedge}(1), \left(\rho_\wedge^{\frac{1-t}{2}} f \rho_\wedge^{\frac{1-t}{2}}\right)\right\} \tag{3.1}$$

Then $L_{X,\wedge}$ has the following properties

PROPOSITION: 3.1

(i) $$L_{X\wedge}\left(\rho_{\wedge}^{\frac{1-t}{2}} 1 \rho_{\wedge}^{\frac{1-t}{2}}\right)=0 \quad (3.2)$$

(ii) $$L_{X,\wedge}\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)^{*}=\left(L_{X,\wedge}\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)\right)^{*} \qquad *-In\,\text{var}\,iance \quad (3.3)$$

(iii) $$L_{X,\wedge}\left(\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)^{*}\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)\right)-L_{X,\wedge}\left(\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)^{*}\right)\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)$$

$$-\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)^{*} L_{X,\wedge}\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right)\geq 0 \qquad Dissipation \quad (3.4)$$

(iv) $$\left\langle L_{X,\wedge}\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right),\left(\rho_{\wedge}^{\frac{1-t}{2}} g \rho_{\wedge}^{\frac{1-t}{2}}\right)\right\rangle=\left\langle\left(\rho_{\wedge}^{\frac{1-t}{2}} f \rho_{\wedge}^{\frac{1-t}{2}}\right),L\left(\rho_{\wedge}^{\frac{1-t}{2}} g \rho_{\wedge}^{\frac{1-t}{2}}\right)\right\rangle \text{ symmetric} \quad (3.5)$$

(v) $$\left\|\mathcal{L}_{X,\Lambda}\left(\rho_{\Lambda}^{\frac{1-t}{2}} f \rho_{\Lambda}^{\frac{1-t}{2}}\right)\right\| \leq 2\left\|\rho_{\Lambda}^{\frac{1-t}{2}} f \rho_{\Lambda}^{\frac{1-t}{2}}\right\|$$

$$\left\|\mathcal{L}_{X,\Lambda}\left(\rho_{\Lambda}^{\frac{1-t}{2}} f \rho_{\Lambda}^{\frac{1-t}{2}}\right)\right\|_{2} \leq \left\|\rho_{\Lambda}^{\frac{1-t}{2}} f \rho_{\Lambda}^{\frac{1-t}{2}}\right\|_{2} \quad (3.6)$$

With this property $L_{X,\wedge}$ is a $*-$invariance, bounded symmetric, pre-markov generator

Let $p_t^{x,\wedge} \equiv e^{tL^{x,\wedge}}$ be the corresponding finite volume dynamics, it has the following properties.

PROPOSITION. 3.2

(i) Positivity preserving
$$p_t^{x,\wedge} f_{\wedge} \geq 0: \quad f_{\wedge} \in M_{\wedge} \quad (3.7)$$

(ii) Unity preserving
$$p_t^{x,\wedge}(I_{\wedge}) = I_{\wedge} \quad (3.8)$$

(iii) L_2 − Symmetry
$$\langle p_t^{x,\wedge} f_{\wedge}, g_{\wedge}\rangle = \langle f_{\wedge}, p_t^{x,\wedge} g_{\wedge}\rangle \quad (3.9)$$

(iv) Contrative
$$\|p_t^{x,\wedge}\| \leq 1 \quad (3.10)$$

(vi) Invariance (3.11)

$\varphi_\wedge\left(p_t^{x,\wedge} f_\wedge\right)=\varphi_\wedge\left(f_\wedge\right)$

3.2. Jump-Type Stochastic Dynamics For The Infinite Volume (The Spin-Flip Case)

We define the discrete gradient as follows $\partial_j f_\wedge = f_\wedge - Tr_j f_\wedge$, $j\in z^d$ then the Triple bar norm for $f_\wedge \in M_0$ is given by $\left\|\left| f_\wedge \right|\right\| = \sum_{j\in z^d}\left\|\partial_j f_\wedge\right\|$ If the triple bar norm is finite we denote by $M_1 \subset M_0$ the sub algebra of local operators that have a finite triple bar norm. This algebra is dense in M_0. For X a finite set in F. Where F is the family of finite subsets of z^d Let $L_{x+j}\left(f_\wedge\right) = E_{x+j}\left(f_\wedge\right) - f_\wedge$ be a pre-markov elementary generator such that the closure defines an elementary generator. Where E_{x+j} is a 2-positive unit preserving map on $M_\wedge$ such that

$E_{x+j}\left(M_\wedge\right)\subseteq M_{\wedge^c+j}$. Define a finite volume generator $L^{x,\wedge}$ as follows $L^{x,\wedge} \equiv \sum_{j\in\wedge} L_{x+j}$ The generator $L^{x,\wedge}$ is a well define bounded operator on all the algebra M. Define also an infinite volume generator L^x formally by the same formula with $\wedge \equiv Z^d$ that is, $L^x = \sum_{j\in z^d} L_{x+j}$. For this to be define on a large domain, we will require that the elementary generator L_{x+j} satisfy the following regularity property.

Definition: The operator L_{x+j} is called **regular** if and only if there are positive constants b_{jk}^x, with $j,k\in z^d$ such that $\left\|L_{x+j} f_\wedge\right\| \le \sum_{j\in z^d} b_{jk}^x \left\|\partial_k f_\wedge\right\|$

(3.12)

And $\sup_{k\in z^d} \sum b_{jk}^x = b^x < \infty$

Our objective will be to give a condition which allows us to construct an infinite volume dynamics $p_t^x, t>0$ as a limit of a finite volume dynamics in a way that ensure the feller property. i.e $p_t^x M_\wedge \subset M_\wedge$. We will also study the ergodicity of such dynamics p_t^x in our next work.

The CX conditions on the elementary generators L_{x+j}

Definition: The elementary generators L_{x+j}, $j\in z^d$ satisfy CX-condition if and only if there are positive constants a_{kc}^{x+j} for $k,c\in z^d$ such that $a_{k-i,c-i}^{x+j-i} = a_{kc}^{x+j}$ for any $i\in z^d$

and for any $f_\wedge \in M_\wedge$, we have $\left\|\left[\partial_k, L_{x+j}\right]\left(f_\wedge\right)\right\| \le \sum_{c\in z^d} a_{kc}^{x+j}\left\|\partial_c f\right\|$

with

(i) $\frac{1}{|X|}\sum_{k,c\in z^d}\frac{a_{k,c}^{x+j}}{k} < \infty$ $\quad$ $(ii)\frac{1}{|X|}\sum_{k\in x^c+j,c\in z^d} a^{x+j} \le k < 1$

Theorem. 3.2

Suppose the operators L_{x+j} are regular and that the $CX(i)$ condition is satisfied. Then the following limit exist and defines a quantum Markov semigroup on M

$$P_t^x \equiv \lim_{F_0} P_t^{x,\wedge} \tag{3.13}$$

Proof

For $\wedge_i \in F,\ i=1,2$ Let $p_t^i = e^{tL_i} \equiv p_t^{x,\wedge_i}$

Then we have

$$\begin{aligned}\frac{d}{ds}\left(p_s^2 f_\wedge - p_s^1 f_\wedge\right) &= \frac{d}{ds} p_s^2 f_\wedge - \frac{d}{ds} p_s^1 f \\ &= L_2 p_s^2 f_\wedge - L_1 p_s^1 f_\wedge \\ &= L_2 p_s^2 f_\wedge - L_2 p_s^1 f_\wedge + L_2 p_s^1 f_\wedge - L_1 p_s^1 f_\wedge \\ &= L_2\left(p_s^2 f_\wedge - p_s^1 f_\wedge\right) + (L_2 - L_1) p_s^1 f_\wedge \end{aligned} \tag{3.14}$$

Hence

$$\begin{aligned}\frac{d}{ds} p_{t-s}^2\left(p_s^2 f_\wedge - p_s^1 f_\wedge\right) &= \frac{d}{ds} p_{t-s}^2 p_s^2 f_\wedge - \frac{d}{ds} p_{t-s}^2 p_s^1 f_\wedge \\ &= -L_2 p_{t-s}^2 p_s^2 f_\wedge + p_{t-s}^2 L_2 p_s^2 f_\wedge + L_2 p_{t-s}^2 p_s^1 f_\wedge - p_{t-s}^2 L_1 p_s^1 f_\wedge \\ &= p_{t-s}^2 (L_2 - L_1) p_s^1 f_\wedge \end{aligned} \tag{3.15}$$

Integrating this equation from 0 to t, we have

$$\int_0^t \frac{d}{ds} p_{t-s}^2\left(p_s^2 f_\wedge - p_s^1 f_\wedge\right) = \int_0^t ds\ p_{t-s}(L_2 - L_1) p_s^1 f_\wedge \tag{3.16}$$

and by the contractivity property of the semigroup on the left hand side we have

$$\left\|p_s^2 f_\wedge - p_s^1 f_\wedge\right\| \le \left\|\int_0^t ds\ p_{t-s}^2 (L_2 - L_1) p_s^1 f_\wedge\right\|$$

Using contractivity on the right hand side we have

$$\left\|p_s^2 f_\wedge - p_s^1 f_\wedge\right\| \le \left\|\int_0^t ds\,(L_2 - L_1) p_s^1 f_\wedge\right\| \tag{3.17}$$

We study carefully the expression $(L_2 - L_1) p_s^1 f_\wedge$. The difference of two elementary Markov generators equals to an

elementary generator L_{x+j}. It is sufficient to study for $j \in z^d$. By regularity assumption we have

$$\left\|L_{x+j} p_s^1 f_\wedge\right\| \le \sum_{k \in z^d} b_{jk}^x \left\|\partial_k p_s^1 f_\wedge\right\|, \quad j \in \wedge_2/\wedge_1 \tag{3.18}$$

we study the term $\partial_k p_s^{\wedge_1} f_\wedge$

using the differential equation in [14]

$$\frac{d}{d\hat{s}}\left(\partial_k p_{\hat{s}}^{\wedge_1} f_\wedge\right) = \partial_k \frac{d}{d\hat{s}} p_{\hat{s}}^{\wedge_1} f_\wedge = \partial_k L_1 p_{\hat{s}}^{\wedge_1} f_\wedge$$

We have the following

$$\frac{d}{d\hat{s}} p^{\wedge_1}_{s-\hat{s}}\left(\partial_k p^{\wedge_1}_{\hat{s}} f_\wedge\right) = -L_1 p^{\wedge_1}_{s-\hat{s}}\left(\partial_k p^{\wedge_1}_{\hat{s}} f_\wedge\right) + p^{\wedge_1}_{s-\hat{s}}\partial_k L_1 p^{\wedge_1}_{\hat{s}} f_\wedge \tag{3.19}$$

$$= p^{\wedge_1}_{s-\hat{s}}\partial_k L_1 p^{\wedge_1}_{\hat{s}} f_\wedge - p^{\wedge_1}_{s-\hat{s}} L_1 \partial_k p^{\wedge_1}_{\hat{s}} f_\wedge$$

$$= p^{\wedge_1}_{s-\hat{s}}\left(\partial_k L_1 - L_1\partial_k\right) p^{\wedge_1}_{\hat{s}} f_\wedge \qquad = p^{\wedge_1}_{s-\hat{s}}\left[\partial_k, L_1\right] p^{\wedge_1}_{\hat{s}} f_\wedge \tag{3.20}$$

Hence we get

$$\frac{d}{d\hat{s}} p^{\wedge_1}_{s-\hat{s}}\left(\partial_k p^{\wedge_1}_{\hat{s}} f_\wedge\right) = \sum_{i\in\wedge_1} p^{\wedge_1}_{s-\hat{s}}\left(\left[\partial_k, L_{x+i}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right)$$

Integration of this equation and using contractivity property of the Markov semi-group we have the following.

$$\int_0^s \frac{d}{ds} p^{\wedge_1}_{s-\hat{s}}\left(\partial_k p^{\wedge_1}_{\hat{s}} f_\wedge\right) = \int_0^s ds \sum_{i=\wedge_1} p^{\wedge_1}_{s-\hat{s}}\left(\left[\partial_k, L_{x+i}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right)$$

$$\int_0^s \frac{d}{ds} p^{\wedge_1}_{s-\hat{s}}\left(\partial_k p^{1}_{\hat{s}} f_\wedge\right) = \sum_{i=\wedge_1}\int_0^s ds\; p^{\wedge_1}_{s-\hat{s}}\left(\left[\partial_k, L_{x+i}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right)$$

$$p^{\wedge_1}_{s}\left(\partial_k p^{\wedge_1}_{\hat{s}} f_\wedge\right) - p^{\wedge_1}_{0}\left(\partial_k p^{\wedge_1}_{s} f_\wedge\right) = \sum_{i=\wedge_1}\int_0^s ds\; p^{\wedge_1}_{s-\hat{s}}\left(\left[\partial_k, L_{x+i}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right)$$

$$p^{\wedge_1}_{s}\left(\partial_k p^{\wedge_1}_{s} f_\wedge\right) = \left(\partial_k p^{\wedge_1}_{s} f_\wedge\right) + \sum_{i\in\wedge_1}\int_0^s ds\; p^{\wedge_1}_{s-\hat{s}}\left(\left[\partial_k, L_{x+i}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right)$$

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \left\|\partial_k f_\wedge\right\| + \sum_{i\in\wedge_1}\int_0^s ds \left\|\left[\partial_k, L_{x+j}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right\|$$

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \left\|\partial_k f_\wedge\right\| + \sum_{i\in\wedge_1}\int_0^s d\hat{s} \left\|\left[\partial_k, L_{x+j}\right] p^{\wedge_1}_{\hat{s}} f_\wedge\right\| \tag{3.21}$$

If the Condition $CX(i)$ is satisfied, the right hand side become bounded by

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \left\|\partial_k f_\wedge\right\| + \int_0^s ds \sum_i \left(\sum_{c\in z^d} a^{x+i}_{kc}\left\|\partial_c p^{\wedge_1}_{s} f_\wedge\right\|\right)$$

Where $\sum a^{x+i}_{kc}$ is a translation invariant matrix

Let $\quad a_x(k) = \sum a^{x+i}_{kc} = \sum_{k_1 c\in z^d} a^{x}_{kc} \le k|x| < \infty$

Hence

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \left\|\partial_k f_\wedge\right\| + \int_o^s ds \sum_{i\in\wedge_1} a_x(k)\left\|\partial_c p^{\wedge_1}_{s} f_\wedge\right\|$$

Therefore

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \left\|\partial_k f_\wedge\right\| + s\sum_i a_x\left\|\partial_c p^{\wedge_1}_{s} f_\wedge\right\|$$

$$\le \left\|\partial_k f_\wedge\right\| + \sum_i s a_x\left\|\partial_c p^{\wedge_1}_{s} f_\wedge\right\|$$

$$\le \left\|\partial_k f_\wedge\right\| + \sum_c \left(e^{sa_k}\right)_{k,c}\left\|\partial_c p^{\wedge_1}_{s} f_\wedge\right\|$$

$$\left\|\partial_k p^{\wedge_1}_{s} f_\wedge\right\| \le \sum_c \left(e^{sa_x}\right)\left\|\partial_c f_\wedge\right\| \tag{3.22}$$

Using the previous relations (3.17), (3.18), (3.22), we have

$$\left\|p^{\wedge_2}_{s} f_\wedge - p^{\wedge_1}_{s} f_\wedge\right\| \le t\sum_{k,c\in z^d} b^{x}_{jk}\left(e^{sa_x}\right)_{k,c}\left\|\partial_c f_\wedge\right\|$$

Hence for any $\wedge_2 \subseteq z^d$ containing a set $\wedge_1$ we have

$$\left\| p_s^{\wedge_2} f_\wedge - p_s^{\wedge_1} f_\wedge \right\| \leq t \sum_{j\in\wedge_1^c} \sum_{k,c} b_{jk}^x \left(e^{sa_x}\right)_{k,c} \left\| \partial_c f_\wedge \right\| \tag{3.23}$$

The summability properties of the matrices b_{jk}^x on the right hand lead us to conclude that the limit $p_t^x f_\wedge = \lim_F p_t^{x,\wedge} f_\wedge$ .Exist for all local elements $f_\wedge \in M_0$.Hence by continuity in the norm $\|\bullet\|$, it exist also for any $f \in M$.

Conclusion:

We have been able to establish the existence of an infinite volume stochastic dynamics , in our next work we intend to show that it has exponential decay to equilibrium and is strongly ergodic

REFERENCES

[1] Ahmed, N.U., (1991), Semigroup theory with applications to systems and control, Longman group UK.

[2] Averson, W., (2002), The Domain Algebra of a CP-semi group, Pacific J. math, 203

[3] Bratteli, O. and Kishimoto, A., (1977), Generation of Semigroup and Two-dimensional Quantum lattice systems. J. funct. Anal. 35, 344- 368 .

[4] Bratteli, O. and Robinson, D.W. (1979), Operator algebra and Quantum statistical Mechanics I , Springer-verlag ,Berlin.

[5] Bratteli, O.(preprints), On Dynamical Semigroups and compact group actions,..

[6] Davies, E.B. (1974), Markovian master equations, Comm. math. Phys. **39,** 91

[7] Evan, D.E. and Hanchen-Olsen, H., (1979), the Generators of positive semigroups, J. Funct. Anal. 32, 207-212.

[8] Evan, D.E., (1977), Irreducible Quantum Dynamical Semigroups; Comm. Math. Phys. 54, 293-297.

[9] Evans D.E., (1976), Positive Linear maps on Operator Algebra, Comm. Math. Phys. 48, 15-22.

[10] Frigerio, A., (1978), Stationary states of Quantum Dynamical semigroups, Comm.. math. Phys. 63, 269-276.

[11] Knezevic,I.,(2007),Decoherence and Quantum transport in Nanodevices, J. comp. Theor. Nanosci., 4, 749-760.

[12] Lindblad, G., (1976), On the generators of Quantum Dynamical semigroups, Comm. Math. Phys. 48, 119-130.

[13] Lindblad,G.,(1975), Completely positive maps and entropy inequalities, Comm. Math. Phys. **40**,147.

[14] Majewski, A.W. and Zegarlinski, B., (1995), Quantum Stochastic dynamics I: Spin Systems on a lattice, MPEJ 1.

[15] Olkiewicz, R. and Zegarlinski, B., (1999), Hypercontractivity in Noncommutative L_p Space, J. funct. Anal. 161, 246-285.

[16] Robinson, D.W., (1982), Basic theory of one parameter semigroups, Center for Mathematical Analysis, Australian National University.

[17] Stephane, A. and Rebolledo, R.(2003), Quantum stationary states and classical markov semigroups, preprints.

[18] Stephane,A., Joyce,A.and Pillet,C.A.(Eds),(2006),Open quantum systems I , springer-verlag Berlin Heidelberg.

[19] Takesaki, M.,(1979), Theory of operator Algebra I, springer-verlag berlin Heidelberg.Trunov, N. V. (1979) , On a non-Commutative analogue of the L_P Space. Izvestiya Vuz, Soviet Mathematics, **23, 69-77**.

[20] Zegarlinski, B., (2000), Analysis of classical and quantum interacting particle systems. (personal lect. notes from the author).

[21] Zolotarev, A.A. (1982), L_p- Spaces with respect to state on a von Neumann algebra, Izvestiya Vuz , Soviet Mathematics, **(26), 36-43**.