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Mass Dependence Of The Landau-Migdal Parameter Estimated With Effective Interactions

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Abstract

In this paper, the mass dependence of the Landau-Migdal parameter $g'_{NN}(A)$ is investigated. This investigation requires three stages: In the first stage, we derive an effective interaction using the method of the lowest - order constrained variational (LOCV) approach with two-body correlation functions based on the Nijmegen potential. In the second stage, we have separated the matrix elements of the effective two-body potential into those of the various channels. These are the singlet-even (SE), the singlet-odd (SO), the triplet-even (TE) and the triplet-odd (TO) channels needed to compute the Landau-Migdal parameter. In the final stage we computed the Landau-Migdal parameter, $g'_{NN}(A)$ as a function of the mass number, A.

1.0 Introduction

The determination of the Landau-Migdal Parameter has been the subject of intense investigation in recent years [1,2,3]. It is important to know accurately the value of this parameter because discussions on spin-isospin excitation modes in nuclei depend critically on the value of this parameter. Indeed many previous calculations made use of the universality ansatz for the NN, NΔ and ΔΔ sectors in form of [2,3] :

$$g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} \qquad (1.1)$$

However, new experimental results [1] do suggest the breaking of this universality with smaller values of g'_{NN} in the Δ sector. Also one of the important missing ingredient in the previous calculations is that the mass dependence of this parameter has not been investigated. Such previous estimates included the G-matrix [4] and the phenomenological calculations [5]. In this paper we will first study the mass dependence of this parameter in the NN sector. Our ultimate goal is to estimate this parameter in both the NN, NΔ and ΔΔ sectors but we are not yet ready to do so.
Preliminary results of g'_{NN} for the A = 16 system was presented at the CFIF Fall conference [6] . Here we have extended our investigation to include the results for the A = 40 and A = 90 systems to enable us study its mass dependence.

2.0 Summary of the Method

We shall in this section give a brief summary of the lowest order constrained variational (LOCV) approach discussed in Refs. [7, 8] for the determination of the effective two-body interactions. In the centre of mass rest-frame, the non-relativistic Hamiltonian for an A-fermion system is approximated as [7, 8]:

$$H_o \to H = H_o - T_{CM} = \sum_{i>j} (p_{ij}^2 / M + V_{ij})$$

(2.1)

Here $M \approx m_N A$ is the total mass of the nucleus, T_{CM} is the translational kinetic energy of the centre of mass of the nucleus, $\vec{p}_{ij} = \frac{1}{\sqrt{2}}(\vec{p}_i - \vec{p}_j)$ is the relative momentum of the two interacting pair while V_{ij} is taken in this paper to be the Nijmegen potential [9]. Here H_0 is the original Hamiltonian containing the centre of mass kinetic energy. The difficulty that one experiences with these type of potentials is that, the strength of the repulsive core alone prevents direct application of the Hartree-Fock procedure. In the language of correlated basis functions, this means that the Hartree-Fock trial wavefunction, Φ must be formulated in the form:

$$\Psi = F\Phi,$$

(2.2)

Where Φ is the multi-dimensional product of two-body wavefunctions and F is a symmetric product of two-body correlation functions defined as [10]:

$$F = S \prod_{i>j} f_{ij}$$

(2.3)

The details of the particular choice of correlation functions used in this paper will be discussed below in Eqs. (2.5), (2.6) and (2.7). Owing to its many-body nature, it is not easy to evaluate the matrix elements of the Hamiltonian given in Eq. (2.1), hence we must make approximations. Here we approximate our Hamiltonian to two-body effective interactions. Using Eqs. (2.1), (2.2) and (2.3), we can define an effective two-body interaction in the form [10,11],

$$H_{eff}^{(2)} = \sum_{i>j} f_{ij}^{(2)} (p_{ij}^2 / M + V_{ij}) f_{ij}^{(2)}$$

(2.4)

where $f_{ij}^{(2)}$ are the two-body correlation functions. The two-body correlation functions are generally chosen to have the same form as the inter-nucleon potential. In a more general case, they may be defined as [12]:

$$f_{ij}^{(2)} = \sum_{\lambda} f^{(\lambda)}(r_{ij}) \{\theta_\lambda (ij)\},$$

(2.5)

with $\{\theta_\lambda(ij)\} = 1, P_\sigma, P_\tau, P_{\sigma\tau}, S_{ij}, P_{ls}$, etc, depending on the form of the chosen potential. In our case the chosen potential is the Nijmegen potential [9] which is expanded in terms of the central, spin-orbit and tensor components. It seems reasonable to allow the correlation operators the same degrees of freedom, where in this case

$$\{\Theta_\lambda\} = \{1, S_{ij}, P_{ls}\}.$$

Recent and previous studies [13, 14] regarding nuclear matter and finite nuclei have revealed three main features of the two-body correlation functions. These are: (i) the 'wound' induced in the two-body wave function by the repulsive core of the N-N interaction, (ii) the tensor correlations especially in the $^3S_1 - {}^3D_1$ channel and, (iii) the meson exchange corrections.

Of these three features, the most significant feature has been found to be the effect of tensor correlations. This has led to a parametrization of the two-body correlation functions in the form [13,14]:

$$r_{ij} < r_c \quad : \quad f^{(\lambda)}(r_{ij})\Theta_\lambda(ij) = 0.$$

$$r_{ij} \geq r_c \quad : \quad f^{(\lambda)}(r_{ij})\Theta_\lambda(ij) = f(r_{ij})(1+\alpha^\lambda(A)S_{ij}),$$

(2.6)

where,

$$f(r_{ij}) = 1 - e^{-\beta^2 (r_{ij} - r_c)^2}$$

with r_c = 0.25fm and β=25fm^{-2}. The parameter α^{λ}(A) represents the strength of the tensor correlations and is non-zero only in the lowest coupled $^3S_1$ - $^3D_1$ channel. In maintaining the choice of our simple form of the two-body correlation functions, we are aware of other more sophisticated forms of two-body correlation functions that are highly density-dependent and differ in different tensor channels such as the $^3P_2$ - $^3F_2$ channel. However we have argued [7] that tensor correlations in the coupled $^3S_1$ - $^3D_1$ channel are the most dominant and their inclusion should be enough to obtain high quality values of two-body matrix elements. In the present paper we are not interested in
the full Hamiltonian. We are interested only in the effective two-body potential energy term of Eq. (2.4), i.e.

$$h_{eff}^{(2)} = \sum_{i>j} f_{ij}^{(2)} V_{ij} f_{ij}^{(2)} \qquad (2.7)$$

In the more general case of $h_{eff}^{(2)}$ one may include the NΔ contributions. In that case we may write the potential schematically as:

$$V_{ij} = V_{NN} + V_{N\Delta} + V_{\Delta\Delta} \qquad (2.8)$$

and the new $f_{ij}^{(2)}$ will take the form:

$$f_{ij}^{(2)} \rightarrow f_{NN} + f_{N\Delta} + f_{\Delta\Delta} \qquad (2.9)$$

This inclusion is very desirable and requires further study but we are not yet ready to do so at this stage. We shall in this paper investigate the form of Eq. (2.7) in the NN sector before extending our approach to the NΔ sector in the next paper.

3.0 Channel Separation of the Matrix Elements of the Effective Two-Body Potential

In this section we first separate the relative potential two-body matrix elements into those of the various channels: These are the SO, SE, TE and the TO which stand for the singlet-odd, the singlet-even, the triplet-even and the triplet-odd channels respectively.

We next fit the oscillator SO, TO, SE, TE, two-body potential matrix elements to those of the oscillator sum of Yukawa functions with different ranges. For the central Yukawa V_c, potential functions discussed here these are given by:

$$V_c = \sum_p D_p M(r_{ij}/R_P) \qquad (3.1)$$

Here M(x) is defined as M(x) = $\frac{e^{-x}}{x}$. The R_p are the ranges while the D_p are the strengths of the interaction which are determined by fitting the oscillator matrix elements of Eq. (3.1) to the oscillator matrix elements of Eq. (2.7).

In this way we obtain a potential in each of these channels that will fit approximately the Nijmegen potential relative two-body matrix elements of Eq. (2.7). We chose the ranges with p $\leq$ 4 to be 0.25, 0.4 and 1.414 fm, which are consistent with the one-boson exchanges. The shorter ranges correspond to heavier meson exchanges such as σ, ρ and ω mesons, while the longest range of 1.414 fm corresponds to the one-pion exchange. We next use the method of Hosaka *et al.* [4] to determine a value for the Landau-Migdal parameter. We cast our potential model in the spin-isospin formalism as:

$$V_{\sigma\tau} = \frac{1}{16}\{V_{SO}(r) + V_{TO}(r) - V_{SE}(r) - V_{TE}(r)\} \qquad (3.2)$$

It is advantageous to make a convolution of the potential function in momentum space such that the direct and the exchange terms of the Landau-Migdal parameter, g'_{NN} may be written as [4]:

$$g'_{NN\,(dir)} = \lim_{q \to 0} \left(\frac{m_\pi}{f_\pi}\right)^2 \left\{\frac{1}{16}\left(V_{SO}(q) + V_{\tau o}(q) - V_{SE}(q) - V_{TE}(q)\right)\right\}$$

(3.3)

and

$$g'_{NN\,(exch)} = -\left(\frac{m_\pi}{f_\pi}\right)^2 \left\{\frac{1}{16}\left(V_{SO}(q) + V_{\tau o}(q) + V_{SE}(q) + V_{TE}(q)\right)\right\}$$

(3.4)

where in Eq. (3.4), q = $2p_F$ as discussed in Ref. [4] with p_F being the Fermi momentum of the nucleons.

4.0 The Results

The present paper is intended to evaluate the Landau - Migdal parameter as a function of the mass number A, with the Nijmegen potential [9]. Here, we extend the work of Ref. [6] which was designed for the A = 16 system to now include the A = 40 and the A = 90 systems in order to study the mass dependence of $g'_{NN}(A)$.

In Table 1, the set of our potential two-body matrix elements are calculated for the A = 40 system for the SE, SO, TE and TO channels. It should be noted that only the TE channel is affected by the strength of the tensor correlations and so we set $\alpha^\lambda(40) = 0.04$ and $\hbar\omega$ = 11.0 MeV appropriate for this system. In all the other channels we set $\alpha^\lambda(40) = 0$.

We notice the very good agreement between our matrix elements and the G-matrix calculations of [4] in all channels.

Finally, in Table 2 we used the set ($\hbar\omega$, $\alpha^\lambda(90)$) = (8.8 MeV, 0.03) appropriate for the A = 90 system to repeat the procedure of Table 1 for the relative potential two-body matrix elements. Here too we notice that in all channels our results are in excellent agreement with the G-matrix results of [4].

4.1 Strength of Interactions and the Landau-Migdal parameter

Tables 3 and 4 give the calculated strengths of our interaction based on a least squares fit of our calculated two-body potential matrix elements in the various channels to those corresponding to the sum of Yukawa potentials based on eq. (3.1).

In Table 5, we have used our fitted potential to obtain estimates for the value of the Landau-Migdal parameter, $g'_{NN}(A)$, corresponding to the A = 40 and A = 90 systems, which we have found to be 0.58 and 0.55 respectively. For completeness we have reproduced our results for the A = 16 [6] for comparison.

These results indicate that the value of the parameter is approximately constant over the given mass range A = 16 - 90 and that g'_{NN} is therefore mass independent contrary to what we had expected. The results also show that our calculated estimates are in very good agreement with other workers as shown in Table 5 and that our model is thus a reasonable model.

5.0 Conclusion

In this study we have examined the mass dependence of the Laudau-Midgal parameter $g'_{NN}(A)$ based on the LOCV calculation with the Nijmegen potential [9] folded with two-body correlation functions. For the A = 40 and A = 90 systems, our estimates gave $g'_{NN}(A)$ to be 0.58 and 0.55 respectively. On the other hand, our previous estimates for the A = 16 system gave a value of $g'_{NN}(A) = 0.65$ [6] These estimates indicate that the value of this parameter is approximately constant over the mass range A = 16 to A = 90, contrary to our expectation.

These results also show that our calculated estimates are in good agreement with those of other workers and that our model is a reasonable one. In our subsequent work, we hope to include the effects of NΔ and the ΔΔ contributions into our calculations.

Table 1. Calculated relative matrix elements for A = 40 with $\hbar\omega$ = 11.0MeV and $\alpha^{\lambda} = 0.03$. The first entry

for a column is the present (LOCV) calculation with the Nijmegen potential [9]. The second entry in parentheses is the G-matrix calculations of Hosaka *et al.* [4] with the Paris potential [15]. Here only the TE channels are affected by the tensor correlations.

SE (S/S)	N=0	1	2	TE (S/S)	N=0	1	2
n' = 0	-5.8763	-5.4918	-4.7471	n' = 0	-7.0766	-6.1312	-4.9283
	(-5.2187)	(-4.5601)	(-3.6117)		(-8.040)	(-7.4460)	(-6.3224)
1		-5.5753	-5.0009	1		6.0623	-5.1743
		(-4.2845)	(-3.4698)			(-7.3175)	(-6.3477)
2			-4.6566	2			4.7256
			(-2.8761)				(-5.6668)
SO (P/P)	n' = 0	1	2	TO (P/P)	n' = 0	1	2

	1.7052	1.8021	1.7809			0.2057	0.3017
	(1.7260)	(1.7184)	(1.5952)			(-0.0970)	(-0.0939)
1		2.4056	2.5654	1		0.2057	0.3017
		(2.2136)	(2.2640)			(-0.0970)	(-0.0939)
2			2.9772	2			0.4395
			(2.5781)				(-0.0701)

Table 2. Calculated relative matrix elements for A = 90 with $\hbar\omega$ = 8.8MeV and α^{λ} = 0.03. The first entry

for a column is the present (LOCV) calculation with the Nijmegen potential [9]. The second entry in parentheses of a column is the G-matrix calculations of Hosaka *et al.* [4] with the Paris potential [15]. Here again only the TE channels are affected by the tensor correlations.

SE (S/S)	n=0	1	2	TE (S/S)	N=0	1	2
n' = 0	-4.5258	-4.4143	-3.9905	n' = 0	-4.6907	-4.2442	-3.5646
	(-4.1471)	(-3.8535)	(-3.2648)		(-6.4335)	(-6.2354)	(-5.5745)
1		-4.6028	-4.2987	1		-4.2679	(-5.7768)
		(-3.7805)	(-3.2868)			(-6.3463)	(-1.2154)
2			-4.1380	2			-3.5051
			(-2.9243)				(-5.3810)

SO (P/P)	n' = 0	1	2	TO (P/P)	n' = 0	1	2
n' = 0	1.1714	1.2558	1.2449	n' = 0	0.0456	0.0543	0.0758
	(1.2020)	(1.2299)	(1.1562)		(-0.0256)	(-0.0591)	(-0.0745)
1		1.6660	1.7921	1		0.1039	0.1512
		(1.5768)	(1.6314)			(0.0751)	(-0.0843)
2			2.0875	2			0.2256
			(1.8566)				(0.0813)

Table 3. Best-fit interaction strength (MeV) for A = 40, $\hbar\omega$ = 11.0MeV and α^{λ} = 0.040.

S/No.	Channel	$R_1 = 0.25fm$	$R_2 = 0.040fm$	$R_3 = 0.70fm$	$R_4 = 1.1414fm$
1	SE	10016.0	-3316.0		-10.463
2	TE	15266.9	-4873.3		-10.463
3	SO	8004.0	28.3		31.389
4	TO	9165.3	-1094.6		3.488

Table 4 Calculated value of $g'_{NN\,(A)}$ compared with those of other workers.

S/No.	Channel	$R_1 = 0.25fm$	$R_2 = 0.040fm$	$R_3 = 0.70fm$	$R_4 = 1.1414fm$
1	SE	10022.8	-3304.6		-10.463
2	TE	13442.8	-4156.4		-10.463
3	SO	6708.7	326.7		31.389
4	TO	12452.9	-1542.8		3.488

Table 5. Calculated value of $g'_{NN\,(A)}$ compared with those of other workers.

Source		g'_{dir}	g'_{erch}	g'_{total}
Present	A = 16	0.81	-0.17	0.65*
	A = 40	0.97	-0.39	0.58
	A = 90	0.98	-0.43	0.55

G-matrix[4]		0.94	-0.43	0.56
$\pi + \sigma$ [4]		0.97	-.11	0.86
Gamow-Teller systematics [16]		-	-	0.7-0.8

*taken from Ref. [6]

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