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Properties of steady solutions of a reacting non-Newtonian viscous MHD poiseuille flow

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Abstract

We revisit an Eyring-powell reacting fluid whose viscosity depends on temperature and the vertical distance, we further assume that the MHD flow satisfies the poiseuille boundary conditions. We show that the velocity field has two solutions corresponding to each solution of the temperature. In particular we show that the upper solution coincides with the lower solution of the velocity and vice-versa. Moreover the two solutions never cross each other in the interior layer.

1.0 Introduction

Recently Attia [3] examined the influence of temperature dependent viscosity on the hydro magnetic Couette flow of dusty fluid with heat transfer between parallel plates. The pressure gradient was assumed constant and an external uniform magnetic field is applied perpendicular to the plates. The paper shows that the steady temperature and velocity have maximum values between the plates.

Earlier, dusty fluids were examined by [1]. and [7]. In 2006, [4] investigated the existence of secondary flows for a reacting Poiseuille flow when the viscosity depends exponentially on temperature. The paper shows that the two flows never merge between the plates.

In a more recent paper, [2] revisited the non-Newtonian MHD model of Eldabe et al (2003) and the pressure gradient is a function of y and the numerical solution revealed the existence of two velocity solutions. And also [9] proved the existence, uniqueness and stability of strong solutions for the planar magnetohydrodynamic equations for isentropic compressible fluids.

In this paper, we shall still assume that the pressure gradient is a function of y but not as in [4]. We investigated the properties of solutions and the effect of the magnetic field on the flow.

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2.0 Mathematical model

2.1 Dimensional equations

We assume that the pressure gradient

$$-\frac{\partial P}{\partial x} = c\left[1+\left(1-\left(\frac{y'}{h}\right)^2\right)\right] \tag{2.1}$$

Also we assume that the dynamic viscosity

$$\mu = \mu_0 \exp\left[\gamma\left(1-\left(\frac{y'}{h}\right)^2 + \beta\frac{E(T-T_0)}{RT_0^2}\right)\right] \tag{2.2}$$

Hence the steady momentum and energy equations are

$$c\left(1-\left(\frac{y'}{h}\right)^2\right) + \frac{d}{dy'}\left\{\mu_0 \exp\left[\gamma + \left(1-\left(\frac{y'}{h}\right)^2 + \alpha\frac{E(T-T_0)}{RT_0^2}\right)\right]\frac{du}{dy'}\right\} + \frac{d}{dy'}\left(\frac{1}{\pi c}\frac{du}{dy'}\right) + \rho g\beta(T-T_0) - \sigma B_0^2 u - \frac{\mu}{D}u = 0 \tag{2.3}$$

and

$$\frac{d}{dy'}\left(k\frac{dT}{dy'}\right) + QAe^{-\frac{E}{RT}} = 0 \tag{2.4}$$

where

c is a constant

ρ is density of fluid

g is acceleration due to gravity

y' is the vertical coordinate

x is the horizontal coordinate

u is the horizontal velocity component

μ_0 is viscosity at $y' = h$

h is the gap between parallel plates

π is the Eyring-Powell constants

β is the coefficient of thermal expansion

Q is the heat released per unit mass

A is the pre-exponential factor

E is the activation energy

R is the universal gas constant

T_0 is the wall temperature

D is the permeability constant

α, γ, α are constants

K is the thermal conductivity

$U_0 = \frac{\mu_0}{\rho h}$ is the constant horizontal velocity

2.2 Non-dimensioalization

Let $y = {y'}/{h}$, $\phi = {u}/{U_0}$, $\theta = \frac{E(T - T_0)}{RT_0^2}$ We obtain

$$\frac{d}{dy}\left((e^{[\gamma+1-y^2+\beta\theta]} + B)\frac{d\phi}{dy}\right) + A\theta + F(1-y^2) - (Ha^2 + Gf(\theta, y))\phi = 0, -1 < y < 1 \qquad (2.5)$$

$$\frac{d^2\theta}{dy^2} + \delta g(\theta) = 0, \quad \delta > 0, \ -1 < y < 1 \qquad (2.6)$$

where $$\phi(-1) = \phi(1) = \theta(-1) = \theta(1) = 0 \qquad (2.7)$$

The functions $g(\theta)$,f(θ, y) are continuous and have continuous first order derivative, and are bounded in -1<y<1.

Also $$f(\theta(-y), -y) = f(\theta(y), y), \ \theta(-y) = \theta(y), \ \phi(-y) = \phi(y) \qquad (2.8)$$

It has been shown by [5] that (2.6) which satisfies (2.8) has two solutions

$$\theta_1(y) = 2\ln\left[\exp\left(\frac{1}{2}\theta_{m1}\right)Sech\, c_1 y\right] \qquad (2.9)$$

$$\theta_2(y) = 2\ln\left[\exp\left(\frac{1}{2}\theta_{m2}\right)Sech\, c_2 y\right] \qquad (2.10)$$

where $$\sqrt{\frac{\delta}{2}} = \exp\left(-\frac{1}{2}\theta_m\right)\cosh^{-1}\exp\left(\frac{1}{2}\theta_m\right), \ \theta(0) = \theta_m \qquad (2.11)$$

Also , $\theta_1(y) < \theta_2(y), \quad -1 < y < 1$ and

$$c_i^2 = \frac{1}{2}\delta\exp(\theta_{mi}), \quad i = 1,2 \qquad (2.12)$$

***Theorem* 2.1**

There exists a solution of (2.5) and (2.6) which satisfies (2.7) and (2.8)

Proof

Clearly (2.6) has solutions as listed in (2.9) and (2.10). Let $x_1 = y, x_2 = \phi, x_3 = \phi'$ then the derivatives with respect to y,

$$x_1' = 1 = f_1(x_1, x_2, x_3) \qquad (2.13)$$

$$x_2' = x_3 = f_2(x_1, x_2, x_3) \qquad (2.14)$$

$$x_3' = \frac{[Ha^2 + Gf(\theta, x_1)]x_2 - A\theta - F(1 - x_1^2) - \frac{\partial f}{\partial x_1}x_3}{f(\theta, x_1) + B} = f_3(x_1, x_2, x_3) \qquad (2.15)$$

Clearly $\frac{\partial f_i}{\partial x_j}, i = 1,2,3$ are bounded. Hence by theorem 11.2 of [6] the problem has a solution.

***Theorem* 2:2**

Let θ and ϕ satisfy (2.25) - (2.28) then

(i) $\phi(-y) = \phi(y)$

(ii) Maxima ϕ occurs at $y = 0$

(iii) Max ϕ decreases as Ha increases

Proof

(i) We replace *y* by *–y* and differentiate. We obtain equations (2.5) and (2.7) for ϕ since $\theta(-y) = \theta(y)$. Hence (i) holds.

(ii) Since $\phi(y) \geq 0, \phi(-1) = \phi(1) = 0$ and $\phi(-y) = \phi(y)$, then $\phi_{\max}$ occurs at $y = 0$

(iii) It suffices to focus on the region close to y = 0. In this region (2.5) reduces to

$$\exp(\gamma + 1 + \beta\theta_{\max})\frac{d^2\phi}{dy^2} + A\theta_{\max} + F - Ha^2 + G\exp(\gamma + 1 + \beta\theta_{\max})\phi = 0$$

Then $$\frac{d^2\phi}{dy^2} + \frac{A\theta_{\max} + F}{\exp(\lambda + 1 + \beta\theta_{\max})} - \frac{\left[Ha^2 + G\exp(\lambda + 1 + \beta\theta_{\max})\right]\phi}{\exp(\gamma + 1 + \beta\theta_{\max})} = 0 \qquad (2.16)$$

Therefore $$\phi = ae^{\alpha y} + be^{-\alpha y} + \frac{A\theta_{\max} + F}{Ha^2 + G\exp(\gamma + 1 + \beta\theta_{\max})} \qquad (2.17)$$

where $$\alpha = \sqrt{\frac{Ha^2 + G\exp(\gamma + 1 + \beta\theta_{\max})}{\exp(\gamma + 1 + \beta\theta_{\max})}} \qquad (2.18)$$

Thus $\frac{d\alpha}{d\theta_{\max}} \leq 0$. Hence α decreases as $\theta_{\max}$ increases. Therefore $\phi(\theta_1) > \phi(\theta_2)$ when $\theta_1 < \theta_2$.

(iv) $\alpha^2 = \frac{Ha^2 + G\exp(\gamma + 1 + \beta\theta_{\max})}{\exp(\gamma + 1 + \beta\theta_{\max})}$. Hence $2\alpha\frac{d\alpha}{d(Ha)} = \frac{2Ha}{\exp(\gamma + 1 + \beta\theta_{\max})}$.

Therefore $\frac{d\alpha}{d(Ha)} \geq 0$. and ϕ increases as Ha increases. This completes the proof. ■

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