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MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reaction

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Abstract

The study of unsteady magnetohydrodynamic heat and mass transfer in MHD flow past an infinite vertical oscillating plate through porous medium, taking account of the presence of free convection and mass transfer. The energy and chemical species equations are solved in closed form by Laplace-transform technique and then perturbation expansion for the momentum equation. The results are obtained for velocity, temperature, concentration, Sherwood number, Nusselt number and skin-friction. The effects of various material parameters are discussed on flow variables and presented by graphs. A parametric study of all parameters involved was considered, and a representative set of results showing the effect of heat radiation, reaction parameter, Grashof numbers, Hartmann number and permeability factor were illustrated.

Keywords
Free convection, Magnetohydrodynamic flows, Porous medium, mass transfer, Oscillating plate, Viscousity, Oscillatory flow, Permeable surface.

Nomenclature

c : nondimensional concentration
D : is mass diffusivity
T : fluid temperature
u : fluid axial velocity
Sc : Schmidt number
C_f :: skin – friction coefficient
Grc : mass Grashof number
$Gr\tau$: thermal Grashof number
M :Hartmann number
Nu : Nuselt number
Sh : Sherwood number
Pr : Prandtl number
β_τ : coefficient of thermal expansion
β_c: coefficient of concentration expansion
v : fluid transverse velocity
t : time
$i = \sqrt{-1}$: complex identity
c_p: specific heat at constant pressure
y : transverse or horizontal coordinate
α : non - dimensional reaction parameter
C_w: concentration at the wall
T_w : temperature at the wall

Greek Symbols

ω : angular velocity
ϕ : heat generation/absorption coefficient
θ: non - dimensional fluid temperature
ε: epsilon, $0 \leq \varepsilon << 1$

Dimensionless Group

Grt : dimensionless thermal Grashof number
Grc : dimensionless mass Grashof number

Subscripts

W : condition on the wall
∞: ambient condition

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1.0 Introduction

Free convection flows are of great interest in a number of industrial applications such as fibre and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where differences between land and air temperatures can give rise to complicated flow patterns. In industries and nature, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or boundary temperature. Besides unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The phenomenon of heat and mass transfer is observed on buoyancy induced motions in the atmosphere, in bodies of water, quasi – solid bodies, such as earth and so on. Free convection effects on flow past a vertical surface studied by Vedhanayagam *et al.* [1].

Convective heat transfer through porous media has been a subject of great interest for the last three decades. Recently, Magyari *et al* [2] have discussed analytical solutions for unsteady free convection in porous media. Muthukumaraswamy *et al* [3, 4] investigated mass diffusion effects on flow past a vertical surface. Mass diffusion and natural convection flow past a flat plate studied by researchers like Chandrasekhara *et al* [5]. Magnetic effects on such a flow is investigated by Israel *et al* [6]. Sahoo and Sahoo [7] and Chamkha *et al* [8] discussed MHD free convection flow past a vertical plate through porous medium in the presence of foreign mass.

Flows past a vertical plate oscillating in its own plane have many industrial applications. The first exact solution of Navier-Stokes equation was given by Stokes [9] which is concerned with flow of viscous incompressible fluid past an horizontal plate oscillating in its own plane. Natural convection effects on Stokes problem was first studied by Soundalgekar [10]. Soundalgekar *et al* [11] gave an exact solution for magnetic free convection flow past an oscillating plate. Mass transfer effects on flow past a oscillating plate considered by Soundalgekar *et al* [12].

In this paper, we study radiation, magnetic and mass diffusion effects on the free convection flow, when the plate is made to oscillate with a specified velocity.

2.0 Formulation of the problem

We consider unsteady, free convection two-dimensional flow of an incompressible and electrically conducting viscous fluid along an infinite non conducting vertical flat plate through a porous medium. The *x* axis is taken along the plate in the vertically upward direction and *y* axis is taken normal to the plate. A magnetic field of uniform strength B_0 is applied in the direction of flow and the induced magnetic field is neglected. Initially, the plate and the fluid are at same temperature T_∞ in a stationary condition with concentration level C_∞ at all points. At time $t > 0$ the plate starts oscillating in its own plane with a velocity $U_0 \cos \omega t$. Its temperature is raised to T_w and the concentration level at the plate is raised to C_w . Using the Boussinesq approximation, the governing equations for the flow are given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{v}{A} u + \frac{g\beta_\tau}{\rho}\left(T - T_\infty\right) + \frac{g\beta_c}{\rho}\left(C - C_\infty\right) \tag{2.1}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho c_p}\left(T-T_\infty\right) \tag{2.2}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2}-\gamma\left(C-C_\infty\right) \tag{2.3}$$

The initial and boundary conditions are given by

$$\left.\begin{array}{r} u=0,\ T=T_\infty,\ C=C_\infty \ \ for\ all\ \ y,\ t\le 0 \\ u=U_0\cos\omega t,\ T=T_w+\varepsilon e^{i\omega t},\ C=C_w+\varepsilon e^{i\omega t},\ y=0,\ t>0 \\ u=0,\ T=T_\infty,\ C=C_\infty\ \ as\ \ y\to\infty,\ t>0 \end{array}\right\} \tag{2.4}$$

Let us introduce the non-dimensional variables

$$\left.\begin{array}{l} u'=\dfrac{u}{U_0},\ t'=\dfrac{tU_0^2}{v},\ y'=\dfrac{yU_0}{v},\ A'=\dfrac{U_0^2}{v^2} \\ \omega'=\dfrac{\omega v}{U_0^2},\quad \theta=\dfrac{T-T_\infty}{T_w-T_\infty},\quad \phi=\dfrac{C-C_\infty}{C_w-C_\infty} \end{array}\right\} \tag{2.5}$$

where all the physical variables have their usual meanings.

With the help of (2.5), on dropping primes ($'$) the governing equations with the boundary conditions reduce to

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\tau\theta+Grc\phi-\left(M+\frac{1}{K}\right)u \tag{2.6}$$

$$\Pr\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}+B\theta \tag{2.7}$$

$$Sc\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}-\alpha Sc\phi \tag{2.8}$$

$$\left.\begin{array}{l} u=0,\quad \theta=0,\quad \phi=0,\ \ for\,all\ \ y,\ \ t\le 0 \\ u=0,\ \theta=1+\varepsilon e^{i\omega t},\quad \phi=1+\varepsilon e^{i\omega t},\ y=0,\ t>0 \\ u\to 0,\quad \theta\to 0,\quad \phi\to 0,\ as\ y\to\infty,\ \ t>0 \end{array}\right\} \tag{2.9}$$

Where the parameters are as defined below:

$$\left.\begin{array}{l} Gr\tau=\dfrac{g\beta_\tau\left(T_w-T_\infty\right)v}{U_0^3},\ \ Grc=\dfrac{g\beta_c\left(C_w-C_\infty\right)v}{U_0^3},\ \ M=\dfrac{\sigma B_0^2 v}{\rho U_0^2} \\ K=\dfrac{U_0^2 A}{v^2},\ \ \Pr=\dfrac{\mu c_p}{k},\quad B=\dfrac{Q\mu v}{kU_0^2\rho},\quad Sc=\dfrac{v}{D},\quad \alpha=\dfrac{\gamma v}{U_0^2} \end{array}\right\}$$

3.0 Method of solution

We solve the governing equations (2.7) and (2.8) in an exact form by using Laplace transforms. Equation (2.6) is then solve using perturbative series expansion in the limit of ε. The Laplace transforms of the equations (2.7), (2.8) and the boundary conditions (2.9) are given by

$$\frac{d^2\bar{\phi}}{dy^2} - Sc(s+\alpha)\bar{\phi} = 0 \quad (3.1)$$

$$\frac{d^2\bar{\theta}}{dy^2} + (B - s\,\Pr)\bar{\theta} = 0 \quad (3.2)$$

where s is the Laplace transformation parameter.

$$\left.\begin{array}{l} \bar{\theta} = \bar{\phi} = \dfrac{1}{s} \qquad as\ \ y = 0,\ t > 0 \\ \bar{\theta} = 0,\ \ \bar{\phi} = 0\, as\ \ y \to \infty,\ t > 0 \end{array}\right\} \quad (3.3)$$

Solving equations (3.1), (3.2) with the help of equation (3.3), we get

$$\bar{\theta}(y,s) = \frac{1}{s}e^{-\sqrt{Sc(\alpha+s)}y} \quad (3.4)$$

$$\bar{\phi}(y,s) = \frac{1}{s}e^{-\sqrt{s\Pr - B}y} \quad (3.5)$$

Inverting equations (13) and (14), we get

$$\theta(y,\ t) = erfc\left(\frac{y\Pr}{2\sqrt{\Pr t - B}}\right) \quad (3.6)$$

$$\phi(y,\ t) = erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{\alpha+t}}\right) \quad (3.7)$$

Now having obtained the solutions to equations (2.7) and (2.8), we now substitute for $\theta(y,\ t)$ and $\phi(y,\ t)$ in equation (2.6) using (3.4) and (3.5), which gives

$$\frac{d^2u}{dy^2} + \frac{du}{dy} - (M^2 + \frac{1}{K})u = -Gr\tau \cdot erfc\left(\frac{y\Pr}{2\sqrt{\Pr t - B}}\right) - Grc \cdot erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{\alpha+t}}\right) \quad (3.8)$$

$$u(y) = \cos\omega t + \varepsilon e^{i\omega t} \quad at\ y = 0,\quad u(y) \to 0 \quad as\ y \to \infty$$

Now, we seek a perturbative series expansion about ε for our dependent variable. This is justified since ε is small, thus we write

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t}u_1(y) + o(\varepsilon^2) + ... \quad (3.9)$$

The equation (3.9) becomes

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} - (M^2 + \frac{1}{K})u_0 = -Gr\tau \cdot erfc\left(\frac{y\Pr}{2\sqrt{\Pr t - B}}\right) - Grc \cdot erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{\alpha+t}}\right) \quad (3.10)$$

$$u_0(y) = \cos\omega t \quad at\ y = 0,\quad u_0(y) \to 0 \quad as\ y \to \infty$$

$$\begin{array}{l} \dfrac{d^2u_1}{dy^2} + \dfrac{du_1}{dy} - (M^2 + \dfrac{1}{k} + i\omega)u_1 = 0 \\ u_1(y) = e^{i\omega t} \quad at\ y = 0,\quad u_1(y) \to 0 \quad as\ y \to \infty \end{array} \quad (3.11)$$

The solutions to the equations (3.10), and (3.11) respectively are;

$$u_0(y) = a_1 e^{-ny} + a_2 erfc\left(\frac{y\,\mathrm{Pr}}{2\sqrt{\mathrm{Pr}\,t - B}}\right) + a_3 erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{\alpha + t}}\right)$$

$$u_1(y) = e^{i\omega t} e^{-my}$$

Hence by the perturbation expansion defined in (19) above, we have equation (3.12) the solution to the velocity profile.

$$u(y,\, t) = a_1 e^{-ny} + a_2 erfc\left(\frac{y\,\mathrm{Pr}}{2\sqrt{\mathrm{Pr}\,t - B}}\right) + a_3 erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{\alpha + t}}\right) + \varepsilon e^{2i\omega t} e^{-my} \qquad (3.12)$$

where

$$a_2 = \frac{Gr\tau}{B + M + \frac{1}{K}}\left(\frac{\mathrm{Pr}}{\sqrt{\pi(\mathrm{Pr}\,t - B)}}\right)^3,\ a_3 = \frac{Grc}{\alpha Sc - M - \frac{1}{K}}\left(\sqrt{\frac{Sc}{\pi(\alpha + t)}}\right)^3$$

$$a_1 = \cos\omega t - a_2 - a_3,\ n = \sqrt{M + \frac{1}{K}},\ m = \sqrt{M + \frac{1}{K} + i\omega}$$

3.1 Skin-Friction

We now study skin-friction from velocity field. It is given by

$$c_f = \frac{T_f}{\rho u_\omega v_\omega} = \frac{d^2}{dy^2} u(y,t)\bigg|_{y=0},\ \tau_f = \mu \frac{du}{dy}\Big|_{y=0}$$

which reduces to

$$c_f = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Therefore

$$c_f = a_1 n - \frac{a_2\,\mathrm{Pr}}{\sqrt{\pi}\sqrt{\mathrm{Pr}\,t - B}} - \frac{a_3}{\sqrt{\pi}}\sqrt{\frac{Sc}{\alpha + t}} - \varepsilon m e^{2i\omega t}$$

3.2 Nusselt Number

In non-dimensional form, the rate of heat transfer at the wall is computed from Fourier's law and is given by

$$Nu = \frac{q_\omega v}{(T_\omega - T_\infty) K v_\omega} = \frac{-d}{dy}\theta(y\,t)\bigg|_{y=0},\ q_\omega = -K\frac{dT}{dy}\Big|_{y=0},$$

Therefore, $Nu = \frac{\mathrm{Pr}}{\sqrt{\pi}\sqrt{\mathrm{Pr}\,t - B}}$.

3.3 Sherwood Number

The rate of mass transfer at the wall which is the ratio of length scale to the diffusive boundary layer thickness is given by

$$Sh = \frac{J_\omega v}{(c_\omega - c_\infty)D v_\omega} = -\frac{d}{dy}\phi(y,t)\bigg|_{y=0}, \quad J_\omega = -D\frac{d\phi}{dy}\bigg|_{y=0}$$

which implies

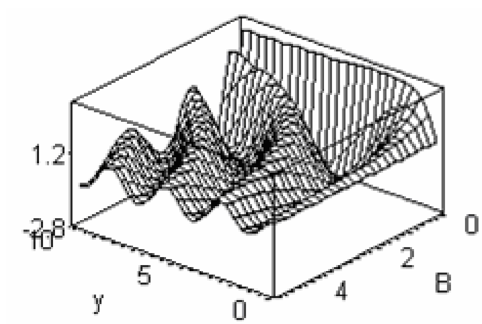
$$Sh = \sqrt{\pi}\sqrt{\frac{Sc}{\alpha + t}}$$

4.0 Discussion

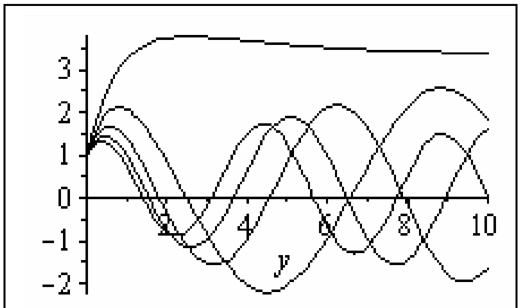
In order to point out the effects of various parameters on flow characteristic, the following discussion is set out. The values of the Prandtl number is chosen Pr = 0.71 (plasma).

The values of the Schmidt number is chosen to represent the presence of species by water vapour (0.60). All other parameters are primarily chosen as follows: $Gr\tau = 10,\ Grc = 5,\ \alpha = 5,\ B = -2,\ M = 0.5,\ K = 1.5,\ \omega t = 2\pi,\ t = 0.25,$ unless otherwise stated. We displayed the effect of each parameter on the flow in 2 – and 3 – dimensional graphs.

Figures 4.1a and 4.1b reveals effect of *B* on the velocity profiles due to the variations in ω*t*. It is observed from figure that the velocity near the plate exceeds at the plate *i.e.* the velocity overshoot occurs. Figure 2 the velocity variations with permeability factor *K*.



(a)



(b)

Figure 4,1: Velocity profile for various values of B

It is observed that greater increase in *K* results in an increase in the velocity. This is due to the fact increase in the value of *K* has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow.

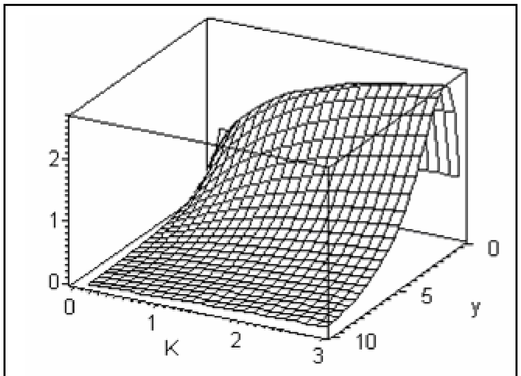


Figure 4,2: Velocity profile for various values of K

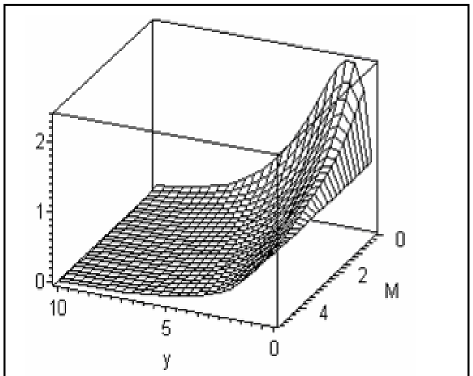


Figure 4.3: Velocity profile for various values of M

Figure 4.3 shows the effect of Hartmann's number *M* on the velocity field. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. The presence of a porous medium increases the resistance to flow resulting in decrease in the flow velocity. This behaviour is depicted by the decrease in the velocity as *K* decreases and when *K* = ∞ (*i.e.* the porous medium effect is vanished) the velocity is greater in the flow field. In Figure 4.4, we show the influence of mass Grashof number on the velocity profile. It is observed that velocity increases with increase in mass Grashof number. While figure 4.5 reveal the velocity variations with thermal Grashof number in cases of cooling and heating of the surface respectively.

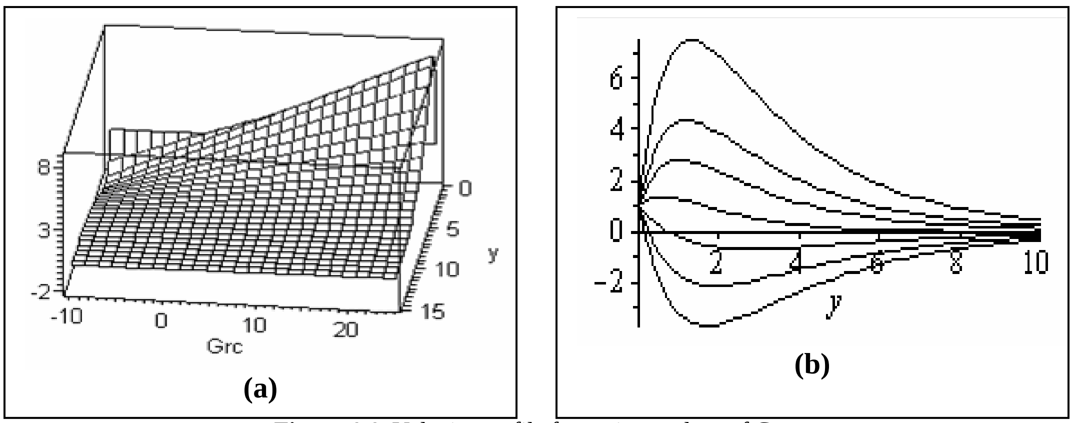


Figure 4.4: Velocity profile for various values of Grc

It is observed that greater cooling of surface (an increase in Grt) results in an increase in the velocity. It is due to the fact increase in the values of Grashof number and modified Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow. The reverse effect is observed in case of heating of the plate (Grt < 0).

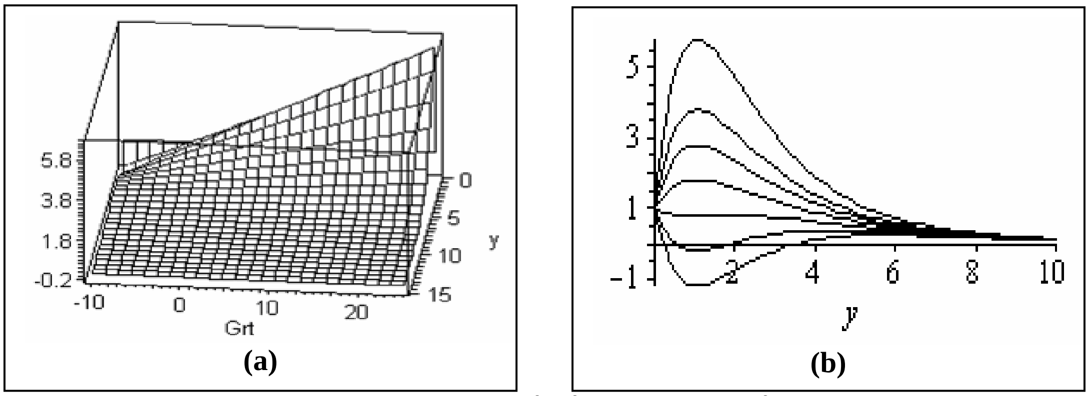


Figure 4.5: Velocity profile for various values of Grt

Figure 4.6 shows the effect of reaction parameter α on the velocity field. We noticed that for a destructive chemical reaction $(\alpha > 0)$, the velocity reduces as α increases, while in a generative chemical reaction $(\alpha < 0)$, the velocity oscillate with the phase angle ωt and reduces away from the plate.

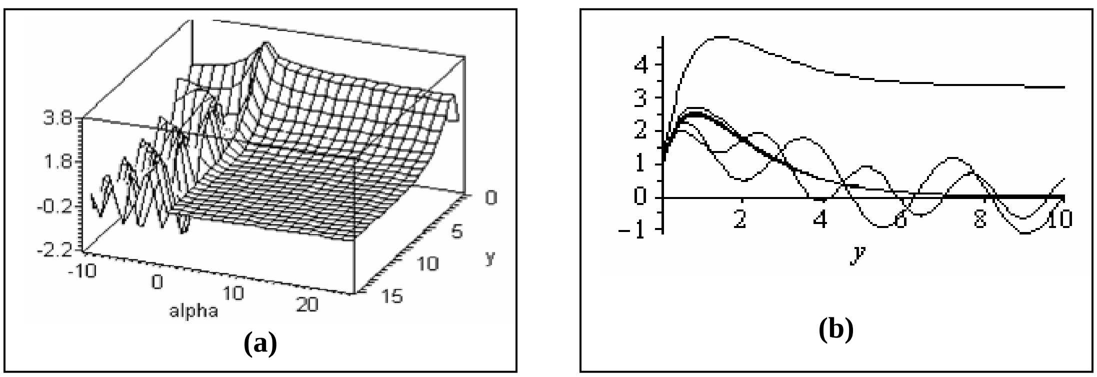


Figure 4.6: Velocity profile for various values of α

Figure 4.7 depicts the temperature profiles against y (distance from plate). The magnitude of temperature is maximum at the plate and then decays to zero asymptotically,

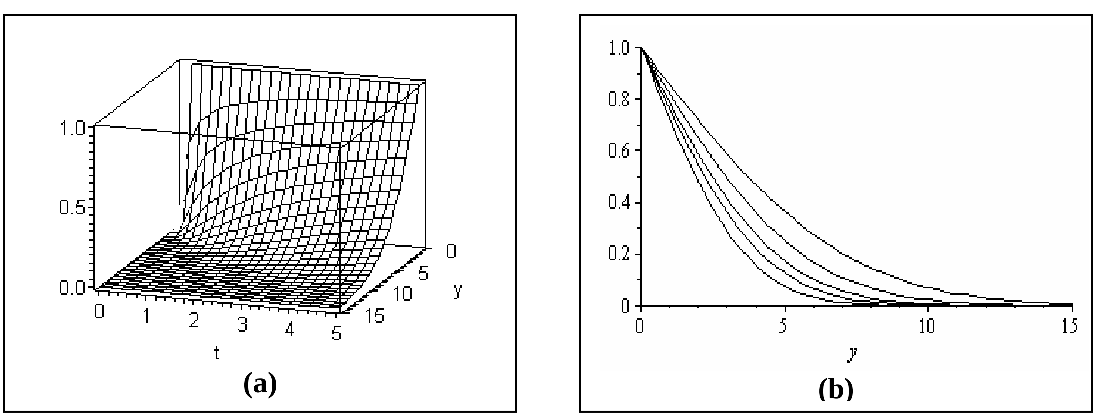


Figure 4.7: Temperature profile at different time t

Figure 4.7 show the temperature field at different time during the flow process. It is observed that temperature field increases as the flow progresses. It is show in the figure that temperature increases as time increases.

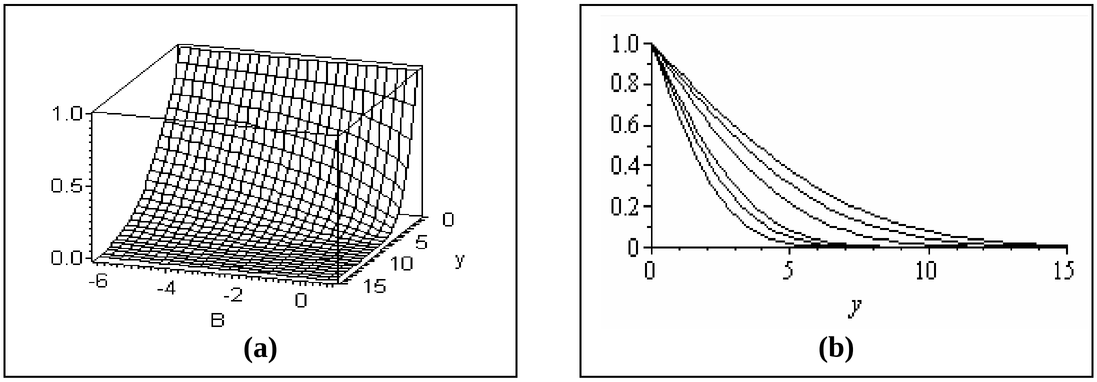


Figure 4.8: Temperature profile for various values of B

In figure 4.8, we show the temperature profiles for various values radiation parameter. It could be seen that heat generation ($B < 0$) increases the temperature field.

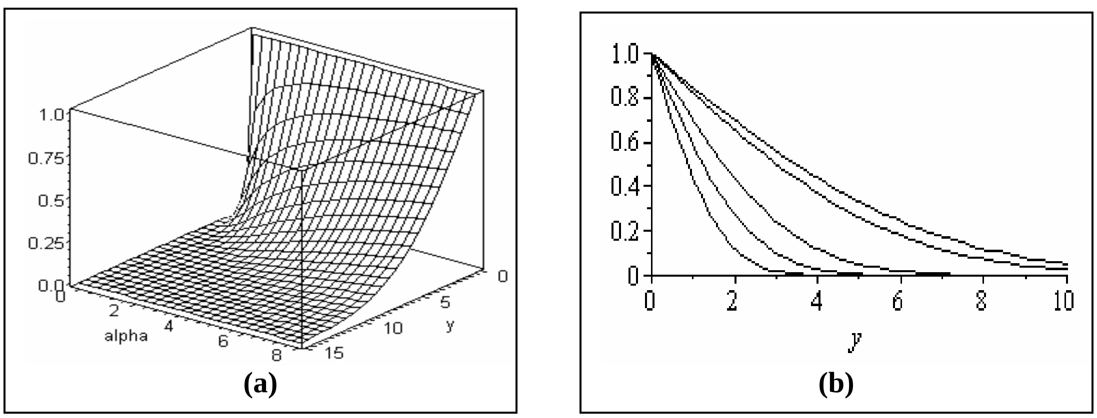


Figure 9: Concentration profile for various values of α

Figures 4.9 display the effect of reaction parameter α on the concentration field. Like temperature, the concentration is maximum at the surface and falls exponentially.The Concentration decreases with an increase in reaction parameter. Further, it is noted that concentration falls slowly and steadily. It should also be mentioned that for generative chemical reaction, the concentration increases with time as long as there continues to be injection of reactive species in to the flow field. Figure 10 depicts rate of heat transfer at the wall against time *t* for different values of radiation parameters. The Nusselt increases with an increase in *B*, and in figure 11 we display the effect of reaction parameter on Sherwood number. It is observed that an increase in reaction parameter reduces the Sherwood number.

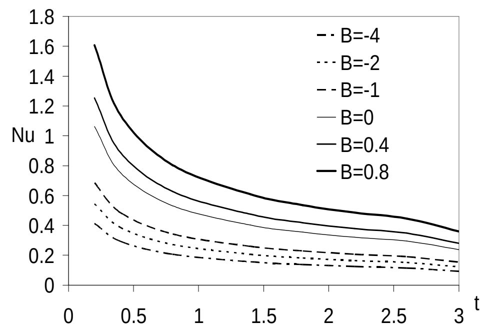


Figure 4.10: Variation of parameter ***B*** on the wall heat transfer *Nu*

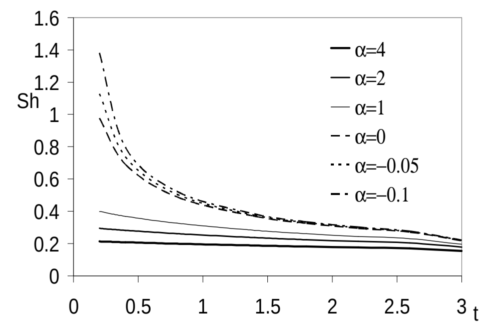


Figure 11: Variation of parameter α on the Sherwood number *Sh*

In table 4.1 we display the effect of heat radiation, reaction parameter, Grashof numbers, Hartmann number and permeability factor on the Skin-fiction. The skin-friction increases with *M* due to enhanced Lorentz force which imports additional momentum in the boundary layer. On the other hand, the skin-fiction decreases with increasing *K*, Grc and $Gr\tau$. The magnitude of skin-friction reduces as either heat generation/absorption coefficient or reaction parameter increases.

Table 4.1: Variation of parameters M, $Gr\tau$, Grc, α, K and B on the wall shear stress c_f

M	Grc	$Gr\tau$	α	K	B	c_f	M	Grc	$Gr\tau$	α	K	B	c_f
0	5	10	2	1	-4	4.3954	1	5	10	0	1	-4	4.1929
1	5	10	2	1	-4	3.4558	1	5	10	1	1	-4	3.5620
2	5	10	2	1	-4	2.9575	1	5	10	2	1	-4	3.4264
3	5	10	2	1	-4	2.6284	1	5	10	3	1	-4	3.3460
4	5	10	2	1	-4	2.3872	1	5	10	4	1	-4	3.2891
1	-4	10	2	1	-4	0.3206	1	5	10	5	1	-4	3.2453
1	-2	10	2	1	-4	1.4434	1	5	10	2	4	-4	6.6952
1	0	10	2	1	-4	2.5661	1	5	10	2	3	-4	6.4880
1	2	10	2	1	-4	3.6889	1	5	10	2	2	-4	6.1342
1	4	10	2	1	-4	4.8117	1	5	10	2	1	-4	5.3730
1	5	20	2	1	-4	13.0714	1	5	10	2	1	-0.5	4.3652
1	5	10	2	1	-4	7.9392	1	5	10	2	1	-1	4.7073
1	5	5	2	1	-4	5.3730	1	5	10	2	1	0	6.8514
1	5	0	2	1	-4	2.8069	1	5	10	2	1	2	4.5706
1	5	-5	2	1	-4	0.2408	1	5	10	2	1	4	3.9354
1	5	-10	2	1	-4	0.0143	1	5	10	2	1	6	3.6374

5.0 Conclusion

In this paper, MHD free convection flow past an oscillating plate in the presence of heat generation/absorption and chemical reaction is presented. Results are presented graphically to illustrate the variation of velocity, temperature, concentration, skin-friction, Sherwood and Nusselt numbers with various parameters. In this study, the following conclusions are set out:

(1) In case of cooling of the plate (Grt > 0), the velocity decreases with an increase in magnetic and reaction parameters. On the other hand, it increases with an increase in the value of thermal Grashof number and mass Grashof number and permeability parameter.

(2) In case of cooling of the plate (Grt > 0), the velocity increases with an increase in magnetic and reaction parameters. On the other hand, it decreases with an increase in the value of thermal Grashof number and mass Grashof number and permeability parameter.

(3) The concentration increases with an increase in reaction parameter and time.

(4) The temperature increases with an increase in heat generation and time.

(5) Nusselt number increases with an increase in heat absorption while temperature decreases with an increase in heat generation.

(6) Sherwood number reduces with an increase in destructive chemical reaction, while it increases with an increase in generative chemical reaction

(7) The skin-friction increases with *M*, and decreases with increasing *K*, Grc and $Gr\tau$. While magnitude of skin-friction reduces as either heat generation/absorption coefficient or reaction parameter increases.

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