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MHD flow and heat transfer of a viscous reacting fluid over a stretching sheet.

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Abstract

This paper presents a boundary layer flow analysis for a viscous, incompressible, electrically conducting reacting fluid over a stretching sheet in the presence of a magnetic field. It is shown that the Hartmann, Prandtl and the Eckert numbers have effect on the velocity and temperature fields.

Keywords
Boundary layer, MHD, Heat transfer, Stretching sheet, reacting fluid

1.0 Introduction

Recently Ayeni [1] investigated the boundary layer flow characteristics of a reacting fluid in the presence of a magnetic field. The fluid is viscous and the sheet is stretching he obtained some interesting properties of the flow. However, he neglected viscous dissipation and quadratic term in velocity

The thesis of Ayeni [1] extended the paper of Makinde and Gbolagade [13] assumed only a heat source, no viscous dissipation and they also neglected the quadratic term. Their numerical results form only part of [1]. Also, recently Jat and Chaudhary [8] examined MHD flow and heat transfer over a stretching sheet .The fluid is not reacting but he included viscous dissipation and the quadratic term.

Flow and heat transfer of an incompressible viscous fluid over a stretching sheet has wide important applications in several manufacturing processes .So the study of heat transfer and flow field are necessary for determining the quality of the final products of such processes [10] .Crane [4] studied flow over a linearly stretching sheet in an ambient fluid and gave a similarity solution in closed form for the two dimensional problem.

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Gupta and Gupta [7], Carrgher and Crane [3], Dutta et al [6], Chiam [4],Magyari and Keller [9,10], Mahapatra and Gupta [11,12] studied the heat transfer in the steady two dimensional stagnation point flow of a viscous incompressible Newtonian viscoelastic fluids over a horizontal stretching sheet assuming a constant surface temperature

In this paper we carried an investigation into a fluid flow over a stretching sheet for which the Arrhenius reaction law applied. We further assume viscous dissipation and quadratic term in the velocity.We determine the criteria for existence of a solution and the problem was solved numerically.

2.0 Mathematical formulation

The appropriate continuity, momentum and energy equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0^2}{\rho}u \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma\mu e^2 H_0^2}{\rho c_p}u^2+\frac{QA}{\rho C_P}e^{-\frac{E}{RT}} \tag{2.3}$$

The boundary conditions are:

$$\begin{aligned} &y=0, \quad u=u_w=ax, v=0, T=T_w \\ &y=\infty, u=0. T=T_\infty \end{aligned} \tag{2.4}$$

where H_0 is magnetic field , ν is the kinematic viscosity,μ is the dynamic viscosity, μ_e is the magnetic permeability, σ is the electrical conductivity ,α is thermal diffusivity,ρ is density of fluid, c_p is specific heat at constant pressure, *T* is temperature, *u* and *v* are velocity components along *x* and *y* axes respectively. *Q* is heat release per unit mass, *A* is the pre-exponential factor and E is the activation energy, while a is a constant. Following [1], we seek self similar solutions of form

$$u = ax\, f^{'}(\eta),\ v = -\sqrt{av}\ f(\eta),\ \eta = \sqrt{\frac{a}{v}}\ y$$

$$\theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty},\ \in = \frac{RT_\infty}{E}$$

We obtain

$$f^{'''} - f^{'2} + ff^{''} = H_a^2 f^{'} \tag{2.5}$$

$$\frac{1}{\Pr}\theta^{''} + f\,\theta^{'} + Ec(f^{''})^2 + H_a^2 Ec(f^{'})^2 + \delta\, e^{\frac{\theta}{\alpha + \in\theta}} \tag{2.6}$$

$$f(0) = 0,\ f^{'}(0) = 1,\ f^{'}(\infty) = 0,\ \theta(0) = 1, \theta(\infty) = 0 \tag{2.7}$$

where

Ec = Eckert number, H_a = Hartman number, δ= Frank-kamentskii number
Pr = Prandtl number

3.0 Method of solution

As shown in [13]

$$f = \frac{1 - e^{-m\eta}}{m} \tag{3.1}$$

where $m^2 = 1 + H_a^2$, we are left to solve equation (2.6) subject to

$$\theta(0) = 1,\ \theta(\infty) = 0 \tag{3.2}$$

***Theorem* 3.1**

Problem (2.6) which satisfies (3.2) has a solution. The solution is unique if $\theta'(0) = \alpha$ is fixed.

To prove theorem 3.1 we need the following theorem due to Derrick and Grossman. Consider the initial value system,

$$\begin{aligned} X_1' &= f_1(X_1,...X_{n,}t),\ X_1(t_o) = X_{10} \\ X_2' &= f_2(X_1,...X_{n,}t),\ X_2(t_o) = X_{20} \\ &.\quad . \\ .X_n' &= f_n(X_1,...X_n,t),\ X_n(t_o) = X_{n0} \end{aligned} \tag{3.3}$$

T*heorem* 3.2

(Derricks and Grossman) [5] If the partial derivatives are continuous and bounded in the region D of definition, then problem (3.3) has a unique solution. We now prove our theorem

Proof

Let $y_1 = \eta, y_2 = f(\eta), y_3 = \theta_1, y_4 = f^{'}, y_5 = \theta^{'}, y_6 = f^{''}$, then

$$y_1' = 1 = f_1(y_1,..,y_6),\; y_1(0) = 0$$
$$y_2' = y_4 = f_2(y_1,..,y_6),\; y_2(0) = 0$$
$$y_3' = y_5 = f_3(y_1,..,y_6),\; y_3(0) = 1$$
$$y_4' = y_6 = f_4(y_1,..,y_6),\; y_4(0) = 1$$
$$y_5' = -\Pr\left(\frac{y_5}{\sqrt{1+H_a^2}}\left(1-e^{-\sqrt{1+H_a^2}\eta}\right) + Ecy_6^2 + H_a^2 Ecy_4^2 + \delta e^{\frac{y_3}{1+\in y_3}}\right) \text{ when } \alpha = 1$$
$$= f_5(y_1,..,y_6),\; y_5(0) = G \text{ (to be determined but finite)}$$
$$y_6' = y_4^2 - \frac{y_6}{\sqrt{1+H_a^2}}\left(1-e^{-\sqrt{1+H_a^2}y_1}\right) + H_a^2 y_4, y_6(0) = J \text{(to be determined but finite)}$$

Clearly, $\frac{\partial f_i}{\partial y_j}, i, j = 1,...,6$ are continuous and bounded hence the problem has solution. When G and J are fixed, the solution is unique.

4.0 Numerical solution

We plot the graphs of θ(η), *f*(η) for various values of the parameters. Of particular interest is the fact that θ(η) when compare with results obtained by Jat and Chandhary [8], has

a maximum in the interval $(0,\infty)$ for some values of the parameter as shown in figure 4.1, figure 4.3, figure 4.4, figure 4.5 and figure 4.6 . A rigorous proof of the phenomenon is the subject of another paper.

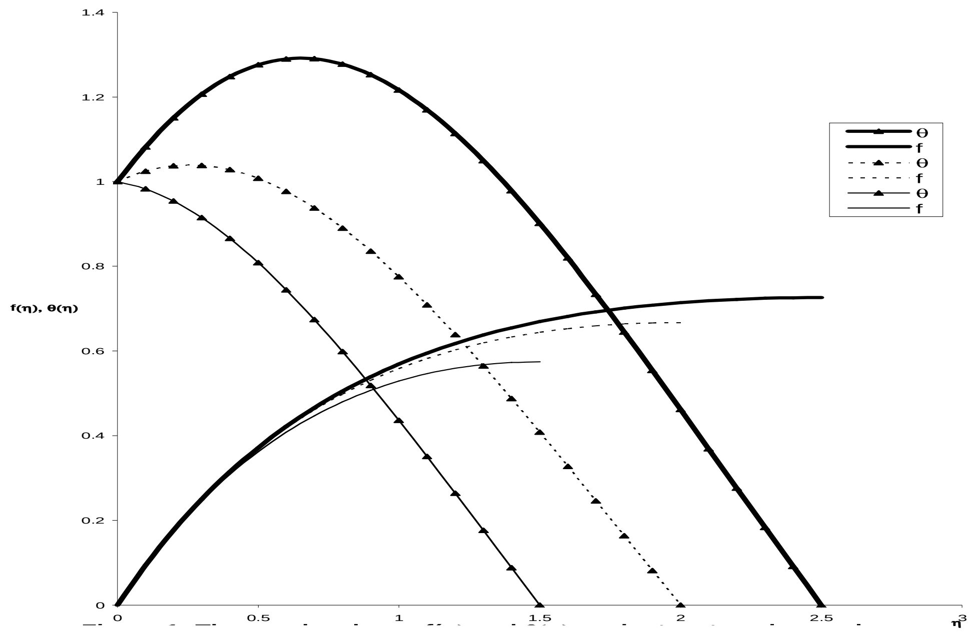

Figure 4.1: The graphs shows *f*(η) against η at various values of infinity and for η = 0.1; Pr = 0.71, Ec = 0.2; Ha = 0.5; δ = 0.5; α = 1; ε = 0.01

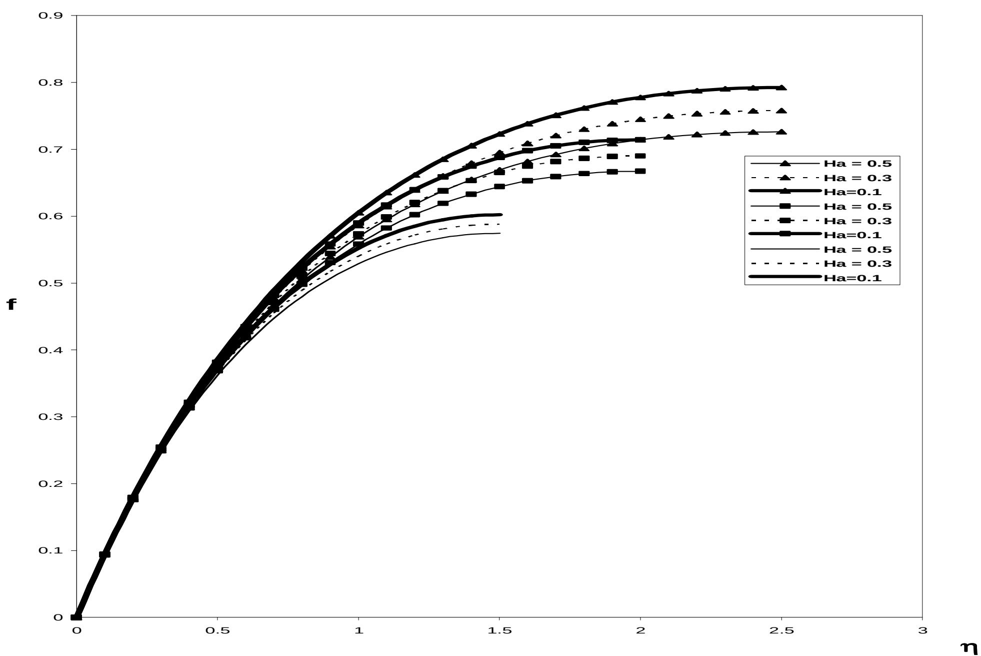

Figure 4.2: The graphs shows *f* against η at various values of infinity and for various Ha

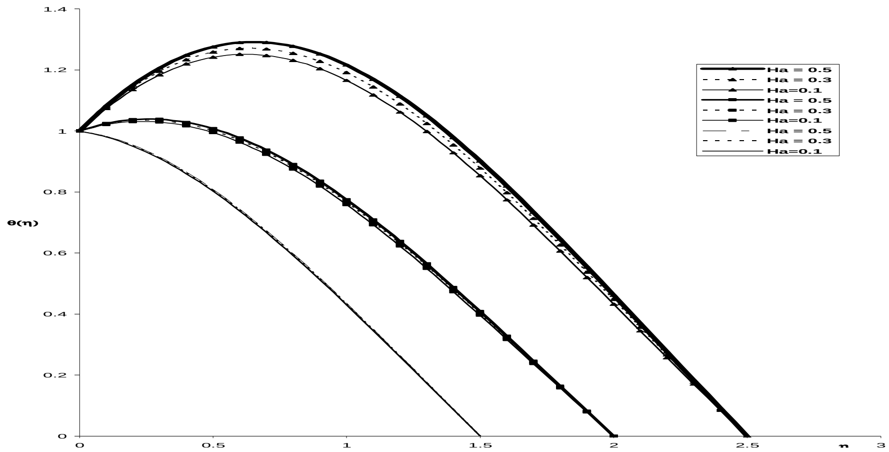

Figure 4.3: The graphs shows *θ(η)* against η at various values of infinity and for various Ha

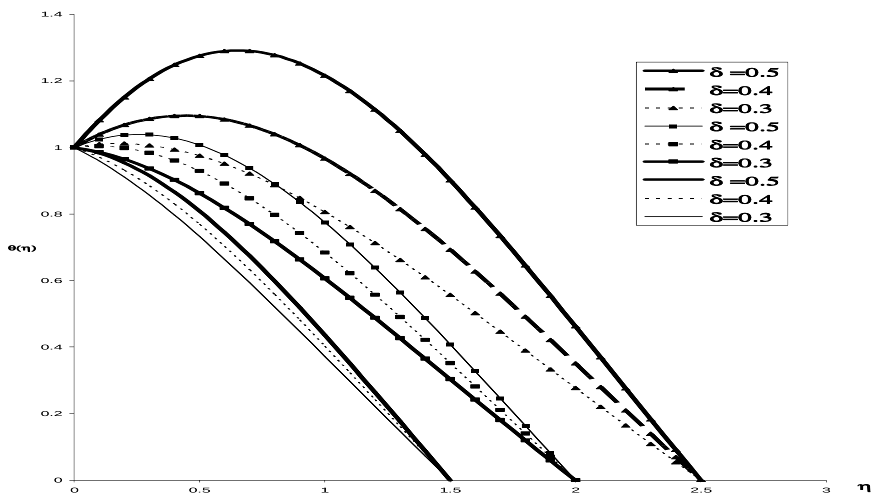

Figure 4.4: The graphs shows *θ* against η at various values of infinity and for various δ

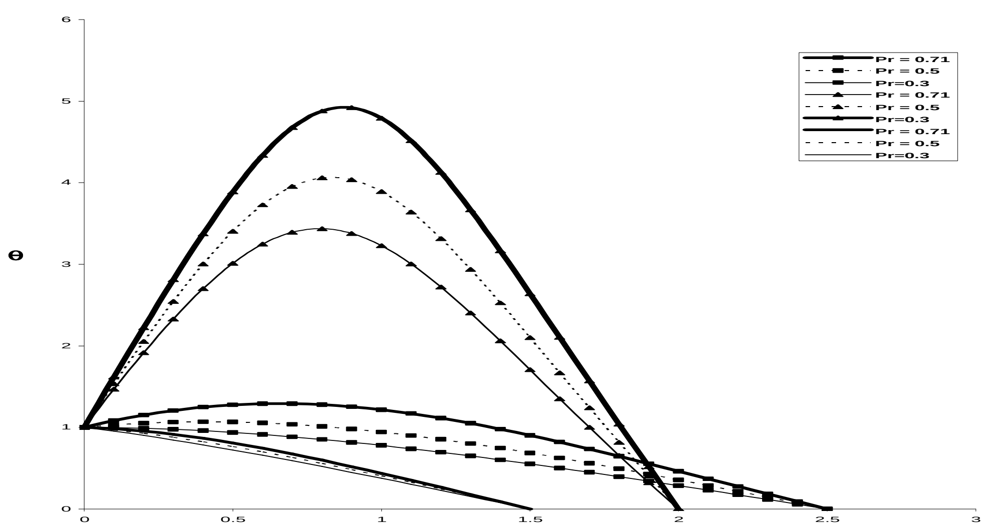

Figure 4.5: The graphs show θ against η of various values of infinity and for various of Pr

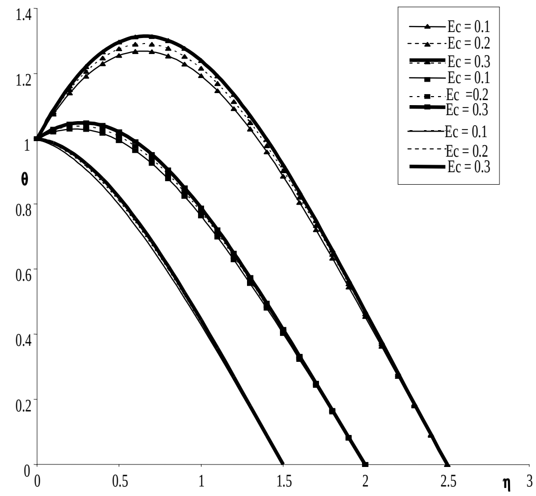

Figure 4.6: The graphs show θ against η of various values of infinity and for various of Ec

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