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Unsteady MHD free convection flow and heat transfer along an infinite vertical porous plate under Arrhenius kinetics

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Abstract

Steady free convection flow of an electrically conducting fluid along an infinite vertical porous plate under Arrhenius kinetics are investigated in the presence of strong transverse magnetic field imposed perpendicularly to the plate .A similarity parameter length scale (h) as a function of time and the suction velocity are considered to be inversely proportional to this parameter. The coupled non-linear ordinary differential equations obtained are solved numerically using symbolic algebra package (MAPLE).The effects of various parameters on the velocity and temperature distributions are presented in graphs. The results show that (i)the velocity and the temperature of the fluid decrease with the increase in prandtl number (ii) fluid velocity decreases due to increase in the Hartmann number (iii) fluid velocity increases due to increase in Grashof number which agrees with natural phenomena because of the buoyancy force which assist the flow.

Keywords
Unsteady MHD, free convection, porous plate, Arrhenius kinetics, similarity solution.

1.0 Introduction

Heat transfer and fluid flow due to free convection in presence of magnetic field and internal heat generation along porous plate find useful applications in different branches of Science and Technology such as nuclear science, fire engineering ,combustion modeling, geophysical, heat exchanger, petroleum reservoir etc.

Sujit and Ioan [12] discussed unsteady boundary layer free convection flow of an incompressible electrically conducting viscoelastic second –order fluid over a vertically permeable flat plate where temperature and concentration differences are responsible for the convective buoyancy current. The flow was affected by a constant suction of fluid through the permeable wall in the presence of a temperature dependent heat source/sink and applied magnetic field. The problem was solved analytically and the results were presented in tables and graphs. The effect of various nondimensional parameters were investigated and discussed. Some of the several important findings of the results were (*i*) an increase of the velocity in the

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boundary layer with the increase of viscoelastic parameter was significant for higher values of the Grashof number. (*ii*) the combined effect of increasing the values of viscoelastic parameter, modified Grashof number and permeability parameter was to enhance the horizontal velocity profile largely in the boundary layer (*iii*) the effect of increasing the value of the Prandtl number was to decrease the skin-friction coefficient for small values of the viscoelastic parameter and its effect was to increase the skin- friction parameter for higher values of viscoelastic parameter in the presence of convective current generated by cooling of the boundary wall and concentration gradient (both *Gr* and *Gm* positive). Sivasankankaran et al [11] investigated the natural convection heat and mass transfer fluid past an inclined semi-infinite porous surface with heat generation using Lie group analysis. Their result revealed that the velocity and temperature of the fluid increases with the heat generation parameter. And also, the velocity of the fluid increases with the porosity parameter and temperature and concentration decreases with increase in the porosity

Parameter, Mansour et al [5] examined the effect of periodic heat and mass transfer on unsteady MHD free convection flow of a microplolar fluid through a porous medium past a vertical flat plate in slip- flow regime when suction velocity oscillates in time about a non-zero constant mean value, the effects of flow parameters and thermo physical properties on the flow, temperature and concentration fields across the boundary layer were investigated. The forms of wall shear stress, wall couple stress, Nusselt number and Sherwood number were derived. The results were shown in figures and tables followed by a quantitative discussion. Muthucumaraswamy and Janakiraman [6] discussed unsteady free convective flow of a viscous incompressible flow pass an infinite isothermal vertical oscillating plate in the presence of homogeneous chemical reaction of first order. The dimensionless governing equations were solved using Laplace transform technique. The plate temperature was raised to $T_w^{'}$ and the concentration level near the plate was also raised to $C_w^{'}$.The effect of velocity ,temperature and concentration were studied for different parameters like phase angle, chemical reaction parameter, Schmidt number and time were studied. It was observed that the velocity increases with decreasing phase angle ωt and chemical reaction parameter *k*. Sharma and Singh [11] investigated the effects of variable thermal conductivity and heat source/sink on flow of a viscous incompressible electrically conduction fluid in the presence of uniform transverses magnetic field and variable free stream near a stagnation point on a non-conduction stretching sheet. The equations of continuity, momentum and energy were transformed into ordinary differential equations and solved numerically using shooting method .The velocity and temperature distributions were discussed numerically and presented through graphs .Skin-friction coefficient and the Nusselt number at the sheet were derived, discussed numerically and their numerical values for various values of physical parameter were presented through tables.They showed that (*i*) fluid velocity decreases with the increase in the magnetic parameter M and Prandtl number. (*ii*) heat generation assists the flow considerably, as velocity profiles in the presence of heat generation are higher in comparison to absence of heat generation while the fluid temperature increases in the presence of heat generation hence the magnitude of temperature profiles are higher in presence of heat generation. Basant [2] discussed the effect of a transverse magnetic field on transient natural convection in a vertical channel due to asymmetric heating of channel walls. The solutions of the linear system of equations were derived by Laplace transform technique .Results were presented for the velocity and temperature profiles and skin friction at the hot and cold wall of the channel.

The effects of Hartman number (M) and time parameter (t) on the velocity profiles and skin friction was extensively discussed.. It was noted that the effect of Hartmann number (M) in the present problem was similar to that of the Darcy number Da in the porous medium .The effect of Hartmann number on velocity and skin-friction was more decisive in steady state.

Eldabe et al [3] examined MHD of a Non-Newtonian unsteady flow of an incompressible fluid under the effect of couple stresses and a uniform external magnetic field by using the Eyring powel model. In the first approximation the solution was obtained by using the mathematica computational program with assuming a pulsatile pressure gradient in the direction of the motion. In the second order approximation a numerical solution of the non-linear partial differential equation was obtained by using a finite difference method. The effect of different parameters were discussed with the help of graphs in the two cases. They showed that the velocity decreases with increasing in both Hartmann number (*Ha*), couple stress and Reynolds number. Phiri and Makinde [7] reported the effects of chemical reaction with heat and mass transfer on unsteady flow of an electrically conducting viscous fluid past an accelerating vertical porous plate in the presence of a transversely imposed external magnetic field and heat generation or absorption. The plate was embedded in a uniform Dacian porous medium in order to allow for possible fluid wall suction or blowing and has a variable wall temperature and concentration. The coupled partial differential equations describing the conservation of mass ,momentum and energy were obtained and solved analytically. Graphical results for the velocity, temperature and concentration profiles as well as for the local skin friction, Nusselt number and Sherwood number were obtained for various parametric conditions to show interesting aspects of solution.

Their results revealed among other things that for positive values of the buoyancy parameter, the fluid velocity, temperature and concentration profiles decrease with increasing values of chemical reaction parameter, the Schmidt number, heat absorption parameter, Hartmann number and porosity parameter. Seddeek and Aboeldahab [9] reported radiation effect on unsteady free convection flow of an electrically conducting gray gas near equilibrium in the optically thin limit along an infinite vertical porous plate in presence of strong transverse magnetic field imposed perpendicularly to the plate, taking Hall currents into account. A similarly parameter length scale (h) as a function of time and the suction velocity were considered to be inversely proportional to this parameter. Similarity equations were then derived and solved numerically using the shooting method. The numerical values of skin-friction and the rate of heat transfer were represented in a table. The effects of radiation parameter, Hall parameter and magnetic field parameters were discussed and shown graphically. They observed that increasing the thermal radiation parameter produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase. Also, the fluid velocity decreases as magnetic parameter increases.

In all the above papers non investigated Arrhenius reacting flow. Also, it is clear that some of heat generation source includes Arrhenius reaction .Thus, we shall study Unsteady free convection flow of an electrically conducting fluid along an infinite vertical porous plate under Arrhenius kinetics in the presence of strong transverse magnetic field imposed perpendicularly to the plate and prominently we shall discuss the effect of each embedded parameter on the flow structure graphically.

2.0 Mathematical formulation of the problem

Consider unsteady free convection flow of a viscous incompressible and electrically conducting fluid, along an infinite vertical plate subjected to time dependent suction velocity and the flowing fluid is reacting. The fluid is assumed to be in the *x*-direction which is taken

along the plate in the upward direction and the *y*-axis perpendicular to it. A uniform strong magnetic field β_0 is assumed to be applied in the *y*-direction and the induced magnetic field of the flow is negligible in comparison with the applied one which corresponds to very small magnetic Reynolds number [11] .Also, electrical field due to polarisation of charges and Hall effects are neglected. Incorporating the Bousessinesq approximation within the boundary layer, the governing equations of continuity, momentum and energy respectively are given by

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma\beta_0^2}{\rho}u \tag{2.2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{AQ}{\rho c_p}e^{\frac{E}{RT}} \tag{2.3}$$

where

u : Velocity along *x*-direction
v : Velocity along *y*-direction
t : Time variable
υ : Kinematic viscosity, $\frac{\mu}{\rho}$
g : Acceleration due earth gravity
β : Coefficient of volume expansion
T : Temperature of the fluid in boundary layer.
T_∞ : Temperature of the fluid far away from plate.
σ : Electrical conductivity.
c_p : Specific heat at constant pressure
k : Thermal conductivity
Q : Heat release
A : Pre-exponent factor
R : Universal gas constant
ρ : Density
μ : Dynamic viscosity
β_0 : Magnetic field

The boundary conditions are

$$y = 0: u = 0,\ v = v(t),\ T = T_w \tag{2.4}$$

$$y \to \infty : u \to 0,\ T \to T_\infty \tag{2.5}$$

3.0 Method of solution

In order to obtain a local similarity solution in terms of the problem under consideration, we introduce a time dependent length scale h as

$$h = h(t) \tag{3.1}$$

In terms of this length scale, a convenient solution of equation (2.1) is considered to be in the following form

$$v = v(t) = -v_0\frac{\upsilon}{h} \tag{3.2}$$

where v_0 is a nondimensional transpiration parameter which positive for suction and negative for injection or blowing.

We now introduce the following dimensionless variables

$$\eta = \frac{y}{h},\ \theta(\eta) = \frac{E(T - T_\infty)}{RT_\infty^2},\ u = u_0 f(\eta) \tag{3.3}$$

Introducing the relations (3.1) - (3.3) into equations (2.2) and (2.3) ,we obtain (by using the analysis of Hasimoto [4], Sattar and Hossain [8] and Alam et al [1]) the following dimensionless non-linear ordinary differential equations which are locally similar in time but not explicitly time dependent

$$\frac{-h}{\upsilon}\frac{\partial h}{\partial t}\eta f' - v_0 f' = f'' + Gr\in\theta - H_a^2 f \tag{3.4}$$

$$\frac{-h}{\upsilon}\frac{\partial h}{\partial t}\eta\theta' - v_0\theta' = \frac{1}{\Pr}\theta'' + \delta e^{\frac{\theta}{1+\in\theta}} \tag{3.5}$$

where the Grashof number, $Gr = \frac{g\beta T_\infty h^2}{u_0\upsilon}$, the activation energy parameter $\in = \frac{RT_\infty}{E}$, Hartman number $H_a = \beta_0 h\sqrt{\frac{\sigma}{\rho\upsilon}}$, prandtl number $\frac{\rho\upsilon c_p}{k}$,the Frank-kameneskii parameter $\delta = \frac{AQE\,\upsilon\, e^{-\frac{E}{RT_\infty}}}{\rho c_p RT_\infty^2 h^2}$.η is similarity variable, u_0 is uniform characteristic velocity, f is dimensionless stream function, θ is dimensionless temperature .

The boundary conditions corresponding to equations (3.5) and (3.6) are

$$f = 0,\ \theta = 1 \text{ at } \eta = 0 \tag{3.7}$$

$$f \to 0,\ \theta \to 0 \text{ at } \eta \to \infty \tag{3.8}$$

Equations (3.5) and (3.6) are similar except for the term $\left(\frac{h}{\nu}\right)\left(\frac{\partial h}{\partial t}\right)$,where t appears explicitly. Thus the similarity condition requires that $\left(\frac{h}{\nu}\right)\left(\frac{\partial h}{\partial t}\right)$ must be constant. Hence it is assumed that

$$\left(\frac{h}{\nu}\right)\left(\frac{\partial h}{\partial t}\right) = c \tag{3.9}$$

where c is an arbitrary constant.

At c = 2 and by integrating equation (3.9) ,one obtains $h = 2\sqrt{\nu t}$ which defines the well-established scaling parameter for unsteady boundary layer problem [8].Hence, the similarity equations are obtained as

$$f'' + (2\eta + v_0) f' + Gr \in \theta - H_a^2 f' = 0 \tag{3.10}$$

$$\theta'' + \Pr(2\eta + v_0)\theta' + \Pr\delta e^{\frac{\theta}{1+\in\theta}} = 0 \tag{3.11}$$

The governing equations (3.10) and (3.11) are coupled non-linear ordinary differential equations and it is solved numerically using symbolic algebra package (MAPLE). The equations were not difficult to handle using the package. Attention was paid to the various parameters that occurs in the equations (3.10) and (3.11). The effect of these parameters on the flow are presented in figures (4.1 – 4.12).

4.0 Results and discussion

Figures 4.1 and 4.2 show the decrease in fluid velocity and fluid temperature when the suction parameter v_0 is increased i.e. more fluid withdrawn through the plate .Both boundary layer and thermal boundary layer thickness decrease with increase in suction parameter v_0.This observation is in agreement with those reported in [7,10] . Figures 4.3 and 4.4 show that there is no noticeable change on the fluid velocity and fluid temperature when the activation energy

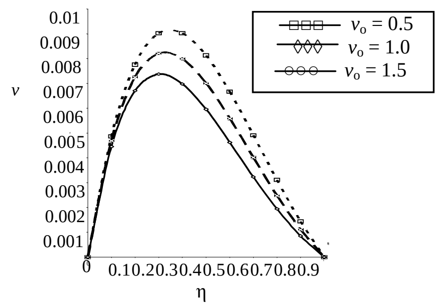

Figure 4.1: Velocity distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, ε = 0.1, δ = 0.1

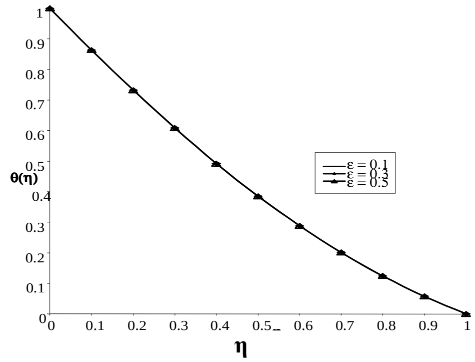

Figure 4.4: Temperature distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, ε = 0.5, δ = 0.1

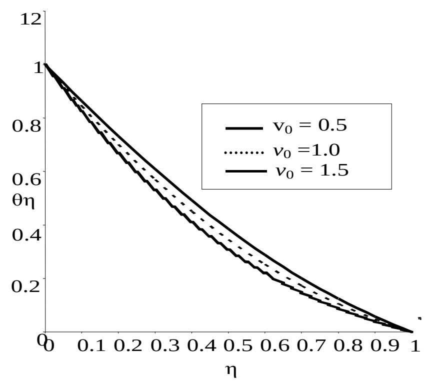

Figure 4.2: Temperature distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, ε = .01, δ =

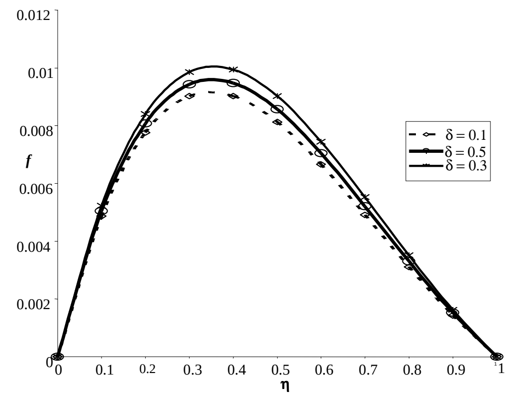

Figure 4.5: Velocity distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, $v_o = 0.5$, ε = 0.1

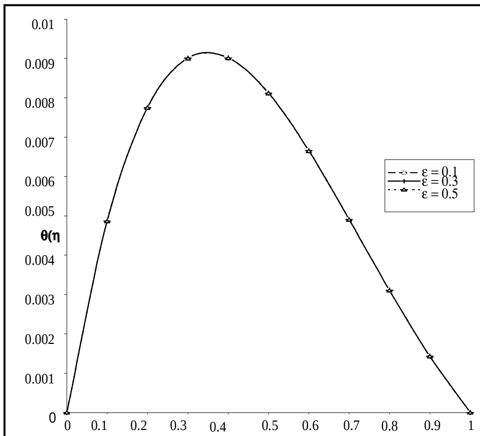

Figure 4.3: Velocity distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, $v_o = 0.5$, δ = 0.1

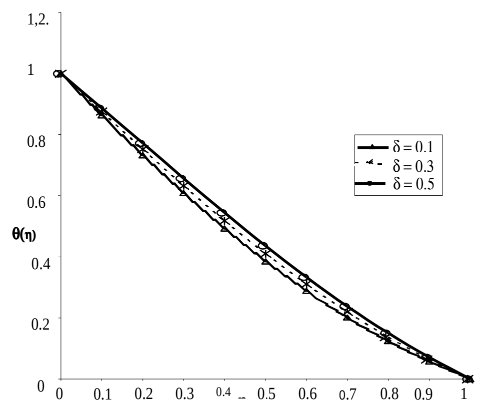

Figure 4.6: Temperature distribution versus η when $Ha = 0.5$, $Gr = 2$, $Pr = 0.71$, $v_o = 0.5$, ε = 0.1

parameter ε is increased. Figure 4.5 shows that with increase in the value of δ the velocity profiles increase considerably when $0.1 \leq \delta \leq 0.5$ .It is interesting to note that the maximum of velocity profiles is achieved at almost same value of η .Also, the temperature profiles (figure 4.6) increases as δ increases. Figure 4.7 shows that with the increase in the value *Pr* the

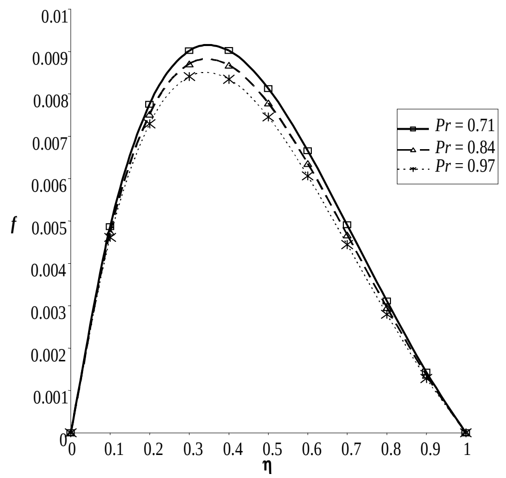

Figure 4.7: Velocity distribution versus η when Ha = 0.5, Gr = 2, v_o = 0.5, δ = 0,1, ε = 0.1

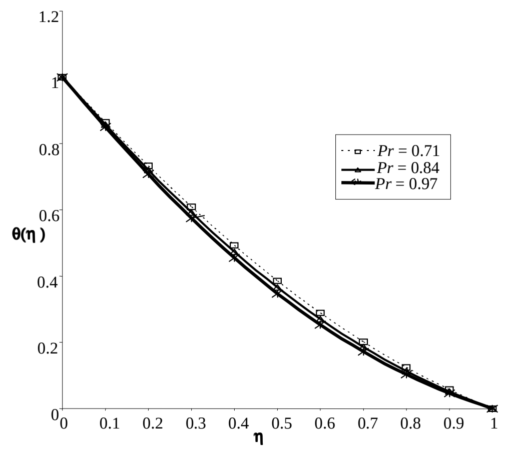

Figure 4.8: Temperature distribution versus η when Ha = 0.5, Gr = 2, v_o = 0.5, δ = 0,1, ε = 0.1

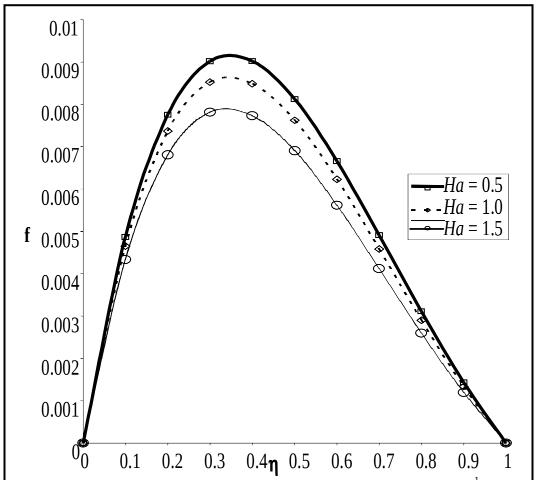

Figure 4.9: Velocity distribution versus η when , Gr = 2, Pr = 0.71, v_o = 0.5, δ = 0,1, ε = 0.1

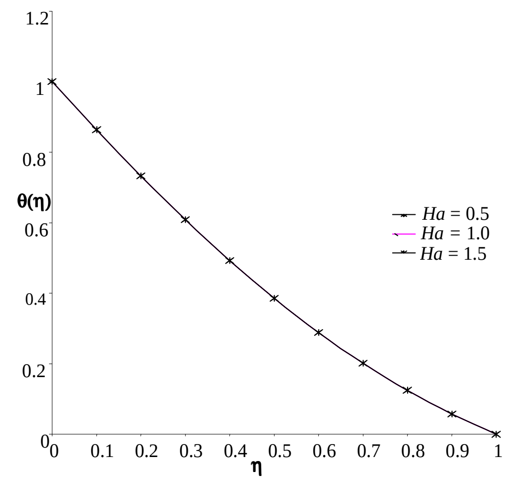

Figure 4.10: Temperature distribution versus η when Gr = 2, Pr = 0.71, v_o = 0.5, δ = 0,1, ε = 0.1

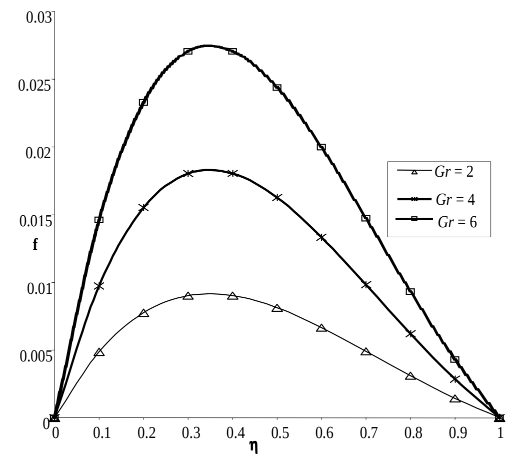

Figure 4.11: Velocity distribution versus η when Ha = 0.5,, Pr = 0.71, v_o = 0.1, ε = 0.1, δ – 0.1

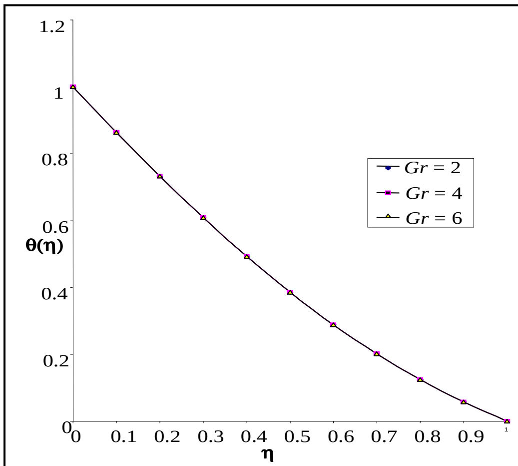

Figure 4.12: Temperature distribution versus η when Ha = 0.5, Pr = 0.71, v_o = 0.5, δ = 0,1, ε = 0.1

velocity profiles decrease considerably when $0.71 \le Pr \le 0.97$. Also, it can be observed that boundary layer thickness decreases with the increase in prandtl number .This is in agreement with the findings in [9,10,12] .The temperature profiles decrease with the increase in the prandtl number which is seen in figure 4.8. The Hartmann number (*Ha*) represents the importance of magnetic field on the flow. The presence of transverse magnetic field sets in Lorentz force, which results in retarding force on the velocity field and therefore as Hartmann number (*Ha*) increases ,so does the retarding force and hence the velocity profiles decrease (see figure 4.9), while the Hartmann number (*Ha*) has negligible effects on the temperature profile see figure 4.10. From figure 4.11, it is observed that fluid velocity increases when Grashof number (Gr) is increase which agrees with natural phenomena because of the buoyancy force which assist the flow while Grashof number does not has effect on the temperature of the flow see figure 4.12.

5.0 Conclusion

Unsteady MHD free convection flow and heat transfer along an infinite vertical porous plate under Arrhenius kinetics has been studied. The governing equations are solved numerically under the prescribed boundary conditions using symbolic algebra package (MAPLE). It can be concluded that (*i*) the velocity and the temperature of the fluid decrease with the increase in prandtl number (*ii*) fluid velocity decreases due to increase in the Hartmann number (*iii*) fluid velocity increases due to increase in Grashof number which agrees with natural phenomena because of the buoyancy force which assist the flow. From the present study we can make the following conclusions

(*i*) Using magnetic field we can control the flow characteristics and heat transfer (see figures 4.9, 4.10).

(*ii*) the velocity profiles increase where as temperature profiles decreases with increase of free convection current (see figures 4.11, 4.12).

(*iii*) the suction stabilizes the boundary layer growth (figures 4.1, 4.2).

(*iv*) it is observed that the damping forces exerted by the Hartman number play a similar role to the resistance offered by the presence of the solid matrix in the porous medium.

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