Journal of the Nigerian Association of Mathematical Physics
Volume **15 (November, 2009), pp 461 - 468**

Second law analysis of slip velocity on oscillatory MHD flow of stretched surface with radiative heat transfer and variable suction

[1]A. M. Okedoye and A. O. Adesanya
Department of Pure and Applied Mathematics,
Ladoke Akintola University of Technology,
P. M. B.* 4000. *Ogbomoso, Nigeria

Abstract

This paper reports the analytical calculation of entropy generation due to unsteady heat and mass transfer flow of an incompressible, electrically conducting, and viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability with radiative heat transfer and variable suction. The fluid and the plates are in a state of solid body rotation with constant angular velocity about the z-axis normal to the plates. Solution of an oscillatory boundary layer flow bounded by two horizontal flat plates, one of which is oscillating in its own plane and the other at rest, is developed by asymptotic expansion in order of epsilon for velocity, temperature and magnetic fields. The influences of the chemical reaction parameter, the thermal and mass Grashof numbers, heat generation/absorption and Hartmann number on total entropy generation were investigated, reported and discussed. A parametric study of all parameters involved was considered, and a representative set of results showing the effects are illustrated.

Keywords

Parallel plates, Radiation, Convective flow, irreversibility, Thermodynamics, Entropy, Viscous fluid, Magnetohydrodynamic fluid, Oscillatory flow

AMS Subject Classification: 76W05

Nomenclature

c_p: specific heat at constant pressure
v : fluid transverse velocity
k_0 = K0: permeability parameter
K : non-dimensional reaction parameter
T : fluid temperature
u: fluid axial velocity
C_j : skin-friction coefficient
Grc : mass Grashof number
$Gr\tau$: thermal Grashof number
M :Hartmann number
Nu : Nusselt number
Sh : Sherwood number
Pr : Prandtl number
Sc : Schmidt number
y : transverse or horizontal coordinate
c : non-dimensional concentration
$Gr\tau$: thermal Grashof number
t : time
$i = \sqrt{-1}$: complex identity

Greek Symbols

θ: non - dimensional fluid temperature
ϕ : heat generation/absorption coefficient
ω : angular velocity

Dimensionless Group

Grt : dimensionless thermal Grashof number
Grc : dimensionless mass Grashof number
ε: epsilon, $0 \le \varepsilon << 1$

Subscripts

ω : condition on the wall
∞: ambient condition

[1]**Corresponding author**

[1]e–mail address: micindex@yahoo.com,
[1]Telephone: +234-0803–568-8 453

1.0 Introduction

Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or boundary temperatures. Besides unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The phenomenon of heat and mass transfer is observed on buoyancy induced motions in the atmosphere, in bodies of water, quasi – solid bodies, such as earth and so on. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as polymer production and food processing.

Cookey et al [1] proposed a model for the study of MHD free convection flow past an infinite heated vertical plate in a porous medium, they observed that increased cooling of the plate was accompanied with an increase in the velocity. Taneja and Jain [2] looked at the Unsteady MHD flow in a porous medium in the presence of radiative heat where they obtained expressions for the velocity, temperature and rate of heat transfer. The study of fluid flow in porous media in the presence of radiative heat is of paramount importance in geothermal engineering and in astrophysics hence a lot of works have been reported in the literature. Cookey et al [1] has a good review of some of these works.

The contemporary trend in the field of heat transfer and thermal designs is the second Law (of Thermodynamics) analysis and its design-related concept of entropy generation minimization [3]. Entropy generation is associated with thermodynamic irreversibility, which is common in all types of heat transfer processes. Different sources of irreversibility are responsible for heat transfer's generation of entropy like heat transfer across finite temperature gradient, viscous effects, characteristics of convective heat transfer, etc. Thus entropy generation depends functionally on the local values of velocity and temperature in the domain of interest. Energy conversion processes are accompanied by an irreversible increase in entropy, which leads to a decrease in available energy.

Nag and Kumar [4] studied second Law optimization for convective heat transfer through a duct with constant heat flux. In their study, they plotted the variation of entropy generation versus the temperature difference of the bulk flow and the surface using a duty parameter. Shuja and Yilbas [5] analyzed the entropy generation in an impinging jet and [6], [7], [8] consider swirling jet impingement on an adiabatic wall for various flow conditions. The dissipation of energy takes the form of a sum of products of conjugate forces and fluxes associated to the problem under consideration; this was presented by the text of [9]. The fluxes are expressed as linear functions of all forces, as constitutive equations, subjected to the reciprocal relations of Onsager. Entropy generation in MagnetoHydroDynamic (MHD) flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction was studied and reported by [3]

The present paper reports an analytical determination of the entropy generation of unsteady oscillatory flow of an incompressible viscous, radiating, electrically conducting fluid past an infinite vertical wall with constant heat embedded in a porous medium of variable suction. Expressions are given for the velocity, temperature and induced magnetic field.

2.0 Governing equation

In this study we consider the two-dimensional oscillatory flow of an incompressible viscous, radiating, electrically conducting fluid past an infinite vertical wall with constant heat embedded in a porous medium of variable suction with velocity components (u', v') in the (x', y') direction. The limiting surface is moved in impulsively, with a constant velocity, either

in the direction of the flow or in the opposite direction in the presence of transverse magnetic field. In Cartesian co–ordinate system, *x*-axis is assumed to be along the plate in the direction of the flow and *y*–axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. The induced magnetic field is not negligible, so that in the region considered, $(H_{x'}, H_{y'}, 0)$. Within the framework of these assumptions, under the usual Boussinesq approximations, the magnetohydrodynamic flow relevant to the problem is governed by the set of equations proposed by [10].

$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+B\varepsilon e^{i\omega t})\frac{\partial u}{\partial y}=Gr\tau\theta+\frac{1}{4}\frac{\partial u_\infty}{\partial y}+\frac{\partial^2 u}{\partial y^2}-M^2u-\frac{u}{k_0(1+A\varepsilon e^{i\omega t})}+M\frac{\partial H}{\partial y} \tag{2.1}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-(1+B\varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}-R_a\theta \tag{2.2}$$

$$\frac{1}{4}\frac{\partial \mathrm{H}}{\partial t}-(1+B\varepsilon e^{i\omega t})\frac{\partial \mathrm{H}}{\partial y}=\frac{1}{Pm}\frac{\partial^2 \mathrm{H}}{\partial y^2}+M\frac{\partial u}{\partial y} \tag{2.3}$$

Subject to the boundary conditions, Shidlovskiy [11]

$$\left.\begin{aligned} &u=0,\ \theta=1+\varepsilon e^{i\omega t},\ \mathrm{H}=1+\varepsilon e^{i\omega t}\\ &u(y)\to 0,\ \theta(y)\to 0,\ \ \mathrm{H}(y)\to 0\ as\ y\to\infty \end{aligned}\right\} \tag{2.4}$$

3.0 Method of solution

To solve the problem as posed in equations (2.1) – (2.3), we seek a perturbative series expansion about ε for our dependent variables. This is justified since ε is small; thus we write

$$\left.\begin{aligned} u(y,t)&=u_0(y)+\varepsilon e^{i\omega t}\,u_1(y)+o(\varepsilon^2)+....\\ \theta(y,t)&=\theta_0(y)+\varepsilon e^{i\omega t}\,\theta_1(y)+o(\varepsilon^2)+....\\ \mathrm{H}(y,t)&=\mathrm{H}_0(y)+\varepsilon e^{i\omega t}\,\mathrm{H}_1(y)+o(\varepsilon^2)+.... \end{aligned}\right\} \tag{3.1}$$

For the stream we have $U=1+\varepsilon e^{i\omega t}$. Substituting equations (2.4) and (3.1) and the expression for the stream into equations (2.1), (2.2) and (2.3),equating the harmonic and non–harmonic terms and neglecting the coefficient of ε^2, we obtain the equations governing the steady state motion and the equations governing the transient.

These two sets of equations are now solved analytically for the velocity, magnetic and the temperature fields. The solutions are

$$\left.\begin{aligned} &\theta_0(y)=e^{-my}\\ &u_0(y)=a_9e^{-n_1y}+a_{12}\,e^{-my}\\ &H_0(y)=a_{13}e^{-n_1y}+a_{14}e^{-my}+(1-a_{13}-a_{14}) \end{aligned}\right\} \tag{3.2}$$

$$\left.\begin{aligned}\theta_1(y) &= a_1 e^{-my} + a_2 e^{-m_1 y}\\ u_1(y) &= a_{23}e^{-n_1 y} + a_{24}e^{-my} + a_{25}e^{-m_1 y} + a_{26} + a_{29}e^{-r_1 y}\\ H_1 &= a_{30}e^{-n_1 y} + a_{31}e^{-my} + a_{32}e^{-m_1 y} + a_{33}e^{-r_1 y} + a_{34}\end{aligned}\right\} \tag{3.3}$$

where $m = \frac{1}{2}\left(\Pr + \sqrt{\mathbf{Pr}^2 + 4\mathbf{P}rR_a}\right)$, $m_1 = \frac{1}{2}\left(\Pr + \sqrt{\Pr^2 - -4\Pr\left(\phi + \frac{i\omega}{4}\right)}\right)$,

$n_1 = \frac{1}{6}\left(2a_3 - 36a_8 + a_7\right)$, $r_1 = \frac{1}{6}a_{27} - 6a_{28} + \frac{1}{3}a_{18}$

with $a_1 = \frac{-4i\mathrm{B}m}{\omega \Pr}$, $a_2 = e^{i\omega t} - a_1$, $a_{13} = \left(n_1^{\ 2} - n_1 - \left(M^2(1+Pm) + \frac{1}{k_0}\right)\right)\frac{a_9}{MPm}$,

$a_{14} = \left(m^2 - m - \left(M^2(1+Pm) + \frac{1}{k_0}\right)\right)\frac{a_{12}}{MPm}$

$a_{23} = \frac{-a_{19}}{n_1^{\ 3} - n_1^{\ 2}a_{18} + n_1 a_{17} - Pma_{16}}$, $a_{24} = \frac{-a_{20}}{m^3 - m^2 a_{18} + ma_{17} - Pma_{16}}$

$a_{25} = \frac{-a_{21}}{m_1^{\ 3} - m_1^{\ 2}a_{18} + m_1 a_{17} - Pma_{16}}$, $a_{26} = \frac{a_{27}}{Pma_{16}}$, $a_{29} = -\left(a_{23} + a_{24} + a_{25}\right)$

$$a_{30} = \frac{a_{23}\left(\left(M^2 + \frac{1}{k_0} + \frac{i\omega}{4} - PmM^2\right) + n_1 - n_1^{\ 2}\right) + a_9 B\left(n_1 - \frac{1}{k_0}\right) - MPmBa_{13}}{MPm}$$

$$a_{31} = \frac{a_{24}\left(\left(M^2(1-Pm) + \frac{1}{k_0} + \frac{i\omega}{4}\right) + m - m^2\right) + a_{12}B\left(m - \frac{1}{k_0}\right) - MPmBa_{14} - Gr\tau a_1}{MPm}$$

$$a_{32} = \frac{a_{25}\left(\left(M^2 + \frac{1}{k_0} + \frac{i\omega}{4} - PmM^2\right) + m_1 - m_1^{\ 2}\right) - Gr\tau a_2}{MPm} \quad a_{33} = \frac{a_{29}\left(\left(M^2 + \frac{1}{k_0} + \frac{i\omega}{4} - PmM^2\right) + r_1 - r_1^{\ 2}\right)}{MPm}$$

$a_3 = 1 + Pm$, $a_4 = -\left(M^2(1-Pm) + \frac{1}{k_0} + Pm\right)$, $a_5 = \left(M^2 + \frac{1}{k_0}\right)$, $a_6 = Gr\tau(m - Pm)$

$$a_7 = \left(-36a_3 a_4 + 108a_5 + 8a_3^3 + 12\sqrt{12a_4^3 - 3a_4^2 a_3^2 - 54a_4 a_3 a_5 + 81a_5^2 + 12a5a_3^3}\right)^{1/3}$$

$$a_8=\frac{\left(\frac{1}{3}a_4-\frac{1}{9}a_3{}^2\right)}{a_7},\quad a_{12}=\frac{-a_6}{m^3-a_3m^2+a_4m-a_5}\quad a_{19}=-B\left(a_9\left(n_1{}^2-\left(Pm+\frac{1}{k_0}\right)n_1+\frac{Pm}{k_0}\right)-a_3MPmn_1\right)$$

$$a_{20}=B\left(a_{912}\left(m^2-\left(Pm+\frac{1}{k_0}\right)m+\frac{Pm}{k_0}\right)-Gr\varpi a_1(m-Pm)+MmPma_{14}\right)$$

$$a_{21}=Gr\varpi a_2(m_1-Pm),\quad a_{22}=-\frac{Pm\omega}{4},\quad a_{27}=\left(-36a_{17}a_{18}+108a_{16}+8a_{18}{}^2\right.$$

$$\left.12\sqrt{12a_{17}{}^3-3a_{17}{}^2a_{18}{}^2-54a_{17}a_{18}a_{16}+a_{16}{}^2+12a_{16}a_{18}{}^2}\right)^{\frac{1}{3}},\quad a_{28}=\frac{\frac{1}{3}a_{17}-\frac{1}{9}a_{18}{}^2}{a_{27}}$$

The functions $u_0(y)$, $\theta_0(y)$ and $H_0(y)$ (given by equations (3.2)), are the mean velocity, the mean temperature and the mean induced magnetic field, respectively; and $u_1(y)$, $\theta_1(y)$ and $H_1(y)$ (given by equations (3.3)), are respectively, the oscillatory part velocity, the oscillatory part temperature and the oscillatory part induced magnetic field.

Now substituting equations (2.1) – (2.2) into equation (3.1), we obtain the required expressions for velocity and temperature;

$$u(y,t)=a_9e^{-n_1y}+a_{12}e^{-my}+\varepsilon e^{i\omega t}\left(a_{23}e^{-n_1y}+a_{24}e^{-my}+a_{25}e^{-m_1y}+a_{26}+a_{29}e^{-r_1y}\right)\quad(3.4)$$

$$\theta(y,t)=e^{-my}+\varepsilon e^{i\omega t}\left(a_1e^{-my}+a_2e^{-m_1y}\right)\quad(3.5)$$

Having obtained expressions for velocity, temperature and magnetic induction we now obtain a function for the entropy generation.

4.0 Entropy generation rate

For an incompressible Newtonian fluid, the local entropy generation rate is given by Okedoye et al (see [3]):

$$\Gamma=\frac{\mu}{T}\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right)-\frac{q}{T^2}\left(\frac{\partial T}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha S_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha K_\alpha\mu_\alpha$$

On the right hand side of the above equation, the first term is due to fluid friction, the second is due to mass diffusion and the third term is due to heat conduction. The fourth term is due to heat transfer induced by mass diffusion and the fifth is due to chemical reactions. In the case of non – reactive mixture, the heat due to diffusion is negligible, and thus the entropy generation rate is rewritten as

$$\Gamma=\frac{\mu}{T}\left(\frac{\partial u_i}{\partial x_j}\right)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{1}{T}\sum_\alpha J\alpha_i\left(\frac{\partial u_\alpha}{\partial x_i}\right)-\frac{q}{T^2}\left(\frac{\partial T}{\partial x_i}\right)$$

In convective heat and mass transfer and MHD flow, irreversibility arises due to the heat transfer, the viscous effects and the mass transfer. The entropy generation rate is expressed as the sum of contributions due to viscous, thermal and diffusive effects, and thus it depends functionally on the local values of temperature, velocity and concentration in the domain of interest.

According to Bejan [12], the characteristic entropy transfer rate is given by:

$$\Gamma_0 = k\left(\frac{\Delta T}{LT_0}\right)$$

where k, L, To and ΔT are respectively, the thermal conductivity, the characteristic length of the enclosure, a reference temperature and a reference temperature difference.

Following Okedoye et al [3], dimensionless entropy Generation rate is given as

$$\Gamma_n = \left(\frac{\partial \theta}{\partial y}\right)^2 + \lambda_1\left(\frac{\partial u}{\partial y}\right)^2 \qquad (4.1)$$

We can now define the followings:

$$\Gamma_{n,h} = \left(\frac{\partial \theta}{\partial y}\right)^2, \ \Gamma_{n,f} = \lambda_1\left(\frac{\partial u}{\partial y}\right)^2,$$

where $\Gamma_{n,h}$ and $\Gamma_{n,f}$ are thermal and viscous irreversibility respectively. Dimensionless terms denoted λ_1, and called irreversibilities distribution ratio, are given by:

$$\lambda_1 = \frac{\mu T_0}{k}\left(\frac{a}{L(\Delta T)}\right)^2$$

where T_0 is the reference temperature, which in our case, the bulk is the bulk temperature.

The local entropy generation rate is a function of temperature and velocity gradients in the y directions in the entire calculation domain.

Using the above equation, on substituting equations (3.4) and (3.5) into (4.1) for irreversibilities, we have

$$\Gamma_n = \left(-me^{-my} + \varepsilon e^{i\omega t}\left(-a_1 me^{-my} - a_2 m_1 e^{-m_1 y}\right)\right)^2 + \lambda\left(-a_9 n_1 e^{-n_1 y} - a_{12} me^{-my} + \right.$$
$$\left. \varepsilon e^{i\omega t}\left(-a_{23} n_1 e^{-n_1 y} - a_9 m_1 e^{-m_1 y} - a_{24} me^{-my} - a_{25} m_1 e^{-m_1 y} - a_{29} r_1 e^{-r_1 y}\right)\right)^2$$

5.0 Discussion

In this study, dimensionless numbers (ω, *Grτ*, *Pr*, *M*, *Ra*, *B* and *Pm*) are used in the governing combined heat and mass transfer equations. The exploitation of the entropy generation equation limits our choices of the Prandtl and the Schmidt numbers to the case of plasma only. Furthermore, the Prandtl and Schmidt numbers are fixed at 0.71 and 0.6 respectively. The analytical simulations presented in this work has been conducted in order to study the effects of the thermal Grashof number, radiation parameter, suction parameter, magnetic Prandtl number and the Hartmann number on entropy generation.

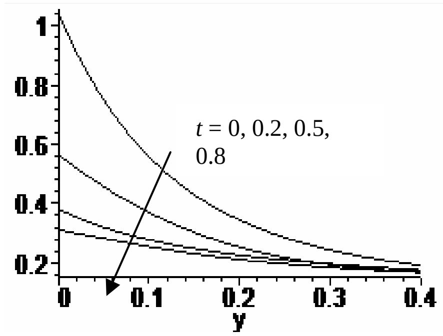

Figure 5.1: Variation of Entropy Generation with time

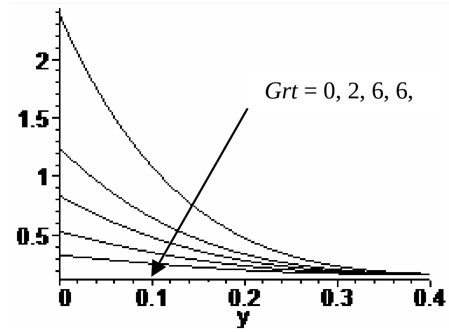

Figure 5.2: Entropy Generation various thermal Grashof number for

For small thermal Grashof number, there is practically little or no convection and the entropy generation due to fluid friction is zero, consequently the total entropy generation is reduced to the entropy generation due to heat transfer. At higher Grashof number heat transfer due to convection begins to play a significant role increasing the flow velocity and in turn the entropy generation due to the viscous effects. Also the isotherms are deformed increasing the temperature gradient and consequently the entropy generation due to heat transfer.

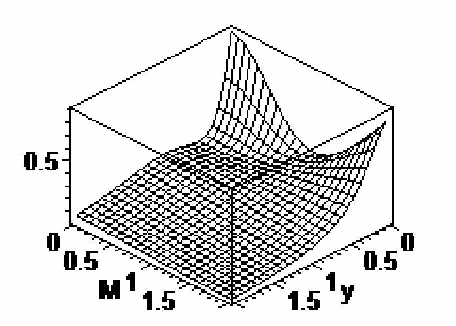

Figure 5.3: Variation of Entropy Generation with Hartmann number

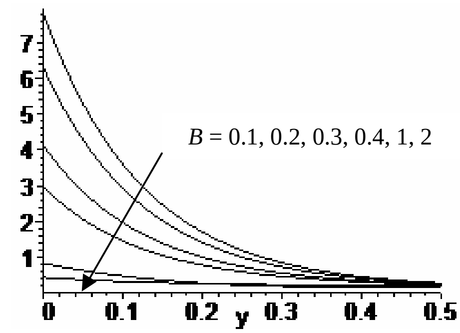

Figure 5.6: Entropy Generation for various various suction parameter

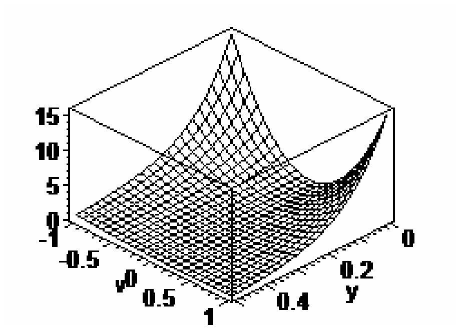

Figure 5.4: Entropy Generation for various limiting surface velocity

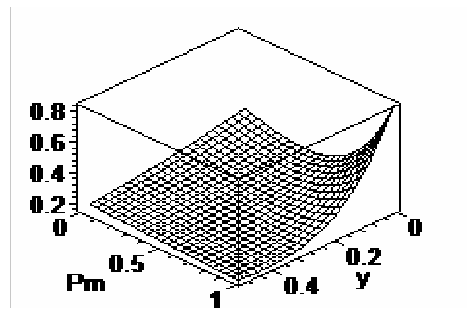

Figure 5.7: Distribution of Entropy Generation for various magnetic Prandtl number

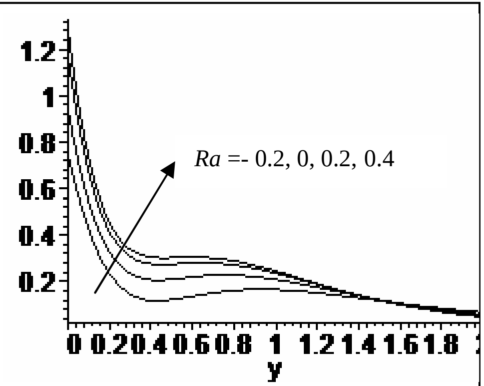

Figure 5.5: Variation of Entropy Generation with radiation parameter

We carried out the analysis using the values $\varepsilon = 0.2$, $Pm = 1$, $Pr = 0.71$, $M = 0.5$, $\omega t = \pi/2$, $Gr\tau = 2$, $V = 0.2$ and $Ra = 0.2$ except where stated otherwise. The positive values indicate that the impulsive velocity of the limiting surface is in a direction opposite to that of the flow.

In figure 5.1 and 5.2, we show the distribution of entropy generation at various time. It could be seen that entropy generation decreases as either time or the thermal grashof number increases. From figure 5.3, we see that entropy generation decreases away from the surface as Hartmann number increases and oscillate within the fluid body. The effect of limiting surface velocity (V) on the entropy generation is shown in figure 5.4, for the limiting surface moving in a direction of the flow or opposite direction to that of the flow. It was discovered that as $|V|$ increases the entropy generation increases. We also observe that increase in radiation parameter brings about increase in entropy generation as shown in figure 5.5. We show figure 5.6, the effect of suction parameter B on the entropy generation. It was discovered that entropy generation decreases as suction parameter increases, while in figure 5.7, we noticed that increase in magnetic Prandtl number Pm increases the entropy generation.

References

[1]. C. I. Cookey, A. Ogulu and V. B. Omubo-Pepple 2003: "Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time dependent suction". International Journal of Heat and Mass Transfer. In Press.

[2]. R. Taneja and N. C. Jain 2002: "Unsteady MHD flow with radiation through porous medium in slip flow regime". Modelling Sim. & Control B AMSE vol. 71, no. 8, p23.

[3]. A. M. Okedoye, O. T. Lamidi and R. O. Ayeni 2007: Entropy generation in MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction. The Journal of Mathematical Association of Nigeria, vol. 34, Number 2A Mathematical series, pp. 52–57.

[4]. Nag, P. K. ; Kumar, N. 1989, Second Law optimization of convective heat transfer through a duct with constant heat flux. *Int. J. Energy Res*., 13, 537-543.

[5]. Shuja, S. Z. ; Yilbas, B. S. 2001, Budair, M. O. Local entropy generation in an impinging jet minimum entropy concept evaluating various turbulence models. *Computer Methods in Applied Mechanics and Engineering*, 190, 3623-3644.

[6]. Shuja, S. Z. ; Yilbas, B. S. 2001, A laminar swirling jet impingement on to an adiabatic wall effect of inlet velocity profiles. *Int. J. Numerical Methods for Heat and Fluid Flow*, 11, 237-254.

[7]. Shuja, S. Z. ; Yilbas, B. S. ; Budair, M. O. 2002, Investigation into a confined laminar swirling jet and entropy production. *Int. J. Numerical Methods for Heat and Fluid Flow*, 12, 870-887.

[8]. Shuja, S. Z. 2003, Yilbas, B. S. ; Rashid, M. Confined swirling jet impingement onto an adiabatic wall. *Int. J. Heat Mass Transfer*, 46, 2947-2955.

[9]. De Groot, S. R. 1966, *Thermodynamics of irreversible processes*, North-Holland, Amesterdam,. 28.

[10]. Okedoye A. M. and Bello O. A. 2008: MHD flow is of a uniformly stretched vertical permeable surface under oscillatory suction velocity. To appear.

[11]. V. P. Shidlovskiy 1961: Introduction to the Dynamics of a Rarefied Gas. Elsevier, New York.

[12]. Bejan, A., 1996. Entropy generation minimization, CRC Press, New york,