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Effect of slip velocity on oscillatory MHD flow of stretched surface with radiative heat transfer and variable suction

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Abstract

The study of unsteady magnetohydrodynamic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability with radiative heat transfer and variable suction has been made. Analytical solution of an oscillatory boundary layer flow bounded by two horizontal flat plates, one of which is oscillating in its own plane and the other at rest, is developed by asymptotic expansion in order of epsilon for velocity, temperature and magnetic fields. The fluid and the plates are in a state of solid body rotation with constant angular velocity about the z-axis normal to the plates. The structure of the boundary layers is also discussed. Several known results of interest are found to follow as particular cases of the solution of the problem considered. A parametric study of all parameters involved was considered, and a representative set of results showing the effect of controlling parameters are illustrated.

Keywords
Second grade fluid; Parallel plates; Resonance; Magnetohydrodynamic fluid; Oscillation flow

AMS Subject Classification: 76W05

Nomenclature
v : fluid transverse velocity
T : fluid temperature
u: fluid axial velocity
k_0 = K0: permeability parameter
y : transverse or horizontal coordinate
$Gr\tau$: thermal Grashof number
M :Hartmann number
Nu : Nuselt number
t : time
$i = \sqrt{-1}$: complex identity
c_p: specific heat at constant pressure

Greek Symbols
θ: non - dimensional fluid temperature
ϕ : heat generation/absorption coefficient
ω : angular velocity

Dimensionless Group
Grt : dimensionless thermal Grashof number
Pr : Prandtl number
ε: epsilon, $0 \leq \varepsilon << 1$

Subscripts
ω : condition on the wall
∞: ambient condition

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1.0 Introduction

In industries and nature, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or boundary temperature. Besides unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The phenomenon of heat and mass transfer is observed on buoyancy induced motions in the atmosphere, in bodies of water, quasi – solid bodies, such as earth and so on. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as polymer production and food processing. Cookey et al [1] proposed a model for the study of MHD free convection flow past an infinite heated vertical plate in a porous medium in which they observed that increased cooling of the plate was accompanied with an increase in the velocity. Taneja and Jain [4] looked at the unsteady MHD flow in a porous medium in the presence of radiative heat where they obtained expressions for the velocity, temperature and rate of heat transfer. Ogulu and Prakash [2] studied oscillatory flow of an incompressible viscous, radiating, electrically conducting fluid past an infinite vertical wall. They show that near the plate, the velocity increases rapidly, attains a maximum value, then slowly fades away far from the limiting surface. The study of fluid flow in porous media in the presence of radiative heat is of paramount importance in geothermal engineering and in astrophysics hence a lot of work has been reported in the literature. Cookey et al [1] has a good review of some of these works.

We propose a two-dimensional oscillatory flow of an incompressible viscous, radiating, electrically conducting fluid past an infinite vertical wall with constant heat embedded in a porous medium of variable suction. Expressions are given for the velocity, temperature and induced magnetic field. All the above magnitudes are shown graphically, followed by a quantitative discussion for different values of the controlling parameters.

2.0 Governing equation

In this study we consider the two-dimensional oscillatory flow of an incompressible viscous, radiating, electrically conducting fluid past an infinite vertical wall with constant heat embedded in a porous medium of variable suction with velocity components (u', v') in the (x', y') direction. The limiting surface is moved in impulsively, with a constant velocity, either in the direction of the flow or in the opposite direction in the presence of transverse magnetic field. In Cartesian co – ordinate system, x-axis is assumed to be along the plate in the direction of the flow and y – axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. The induced magnetic field is not negligible, so that in the region considered, the magnetic field ($H = H_{x'}, H_{y'}, 0$). Within the framework of these assumptions, under the usual Boussinesq approximations, the magnetohydrodynamic flow relevant to the problem is governed by the set of equations

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1 + B\varepsilon e^{i\omega t})\frac{\partial u}{\partial y} = Gr\,\tau\theta + \frac{1}{4}\frac{\partial u_\infty}{\partial y} + \frac{\partial^2 u}{\partial y^2} - M^2 u - \frac{u}{k_0(1 + A\varepsilon e^{i\omega t})} + M\frac{\partial H}{\partial y} \tag{2.1}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - (1 + B\varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - R_a\theta \tag{2.2}$$

$$\frac{1}{4}\frac{\partial \mathrm{H}}{\partial t}-(1+B\,\varepsilon\, e^{i\omega t})\frac{\partial \mathrm{H}}{\partial y}=\frac{1}{Pm}\frac{\partial^2 \mathrm{H}}{\partial y^2}+M\,\frac{\partial u}{\partial y} \tag{2.3}$$

Subject to the boundary conditions, [5].

$$\left.\begin{array}{l} u=0,\ \theta=1+\varepsilon e^{i\omega t},\ \mathrm{H}=1+\varepsilon e^{i\omega t} \\ u(y)\to 0,\ \theta(y)\to 0,\ \ \mathrm{H}(y)\to 0\ as\ y\to\infty \end{array}\right\} \tag{2.4}$$

3.0 Method of solution

To solve the problem posed in equations (2.1) – (2.3), we seek a perturbative series expansion about ε for our dependent variables. This is justified since ε is small; thus we write

$$\left.\begin{array}{l} u(y,t)=u_0(y)+\varepsilon e^{i\omega t}\,u_1(y)+o(\varepsilon^2)+.... \\ \theta(y,t)=\theta_0(y)+\varepsilon e^{i\omega t}\,\theta_1(y)+o(\varepsilon^2)+.... \\ \mathrm{H}(y,t)=\mathrm{H}_0(y)+\varepsilon e^{i\omega t}\,\mathrm{H}_1(y)+o(\varepsilon^2)+.... \end{array}\right\} \tag{3.1}$$

For the stream we have $U=1+\varepsilon e^{i\omega t}$. Substituting equations (2.4) and (3.1) and the expression for the stream into equations (2.1), (2.2) and (2.3), equating the harmonic and non – harmonic terms and neglecting the coefficient of ε^2, we obtain the equations governing the steady state motion and the equations governing the transient.

$$\frac{d^2\theta_0}{dy^2}+\Pr\frac{d\theta_0}{dy}-\Pr Ra\,\theta=0 \tag{3.2}$$
$$\theta_0(0)=1,\ \theta_0(y)\to 0\ \ as\ y\to\infty$$

$$\frac{d^2\mathrm{H}_0}{dy^2}+Pm\frac{d\mathrm{H}_0}{dy}=-\Pr M\,\frac{du_0}{dy} \tag{3.3}$$
$$\mathrm{H}_0(y)=1 at\ y=0_1\ \ \ \mathrm{H}_0(y)\to 0\ \ \ as\ y\to\infty$$

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-(M^2+\frac{1}{k})u_0=-Gr\,\tau\theta_0-M\,\frac{d\mathrm{H}_0}{dy} \tag{3.4}$$
$$u_0(y)=0\ \ at\ y=0,\ \ u_0(y)\to 0\ \ \ as\ y\to\infty$$

and

$$\frac{d^2\theta_1}{dy^2}+\Pr\frac{d\theta_1}{dy}+\Pr(\frac{i\omega}{4}-Ra)\,\theta_1=-\mathrm{B}\,\frac{d\theta_0}{dy} \tag{3.5}$$
$$\theta_1(y)=e^{i\omega t}\ at\ y=0,\ \ \theta_1(y)\to 0\ \ as\ y\to\infty$$

$$\frac{d^2\mathrm{H}_1}{dy^2}+Pm\,\frac{d\mathrm{H}_1}{dy}=-Pm\,\mathrm{B}\,\frac{d\mathrm{H}_0}{dy}-PmM\ \frac{du_1}{dy} \tag{3.6}$$
$$\mathrm{H}_1(y)=1\ \ at\ \ y=0,\ \ \mathrm{H}_1(y)\to 0\ \ \ as\ \ y\to\infty$$

$$\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-(M^2+\frac{1}{k}+\frac{i\omega}{4})u_1=-B\,\frac{du_0}{dy}-M\,\frac{d\mathrm{H}_1}{dy}-\frac{\mathrm{B}}{k_0}u_0-Gr\,\tau\theta_1-\frac{i\omega}{4} \tag{3.7}$$
$$u_1(y)=0\ \ at\ \ y=0,\ \ u_1(y)\to 0\ \ \ as\ \ y\to\infty$$

These two sets of equations are now solved analytically for the velocity, magnetic and the temperature fields. The solutions of equations (3.2) – (3.7) are

$$\theta_0(y)=e^{-my} \quad (3.8)$$

$$u_0(y)=a_9e^{-n_1y}+a_{12}e^{-my} \quad (3.9)$$

$$H_0(y)=a_{13}e^{-n_1y}+a_{14}e^{-my}+(1-a_{13}-a_{14}) \quad (3.10)$$

$$\theta_1(y)=a_1e^{-my}+a_2e^{-m_1y} \quad (3.11)$$

$$u_1(y)=a_{23}e^{-n_1y}+a_{24}e^{-my}+a_{25}e^{-m_1y}+a_{26}+a_{29}e^{-r_1y} \quad (3.12)$$

$$H_1=a_{30}e^{-n_1y}+a_{31}e^{-my}+a_{32}e^{-m_1y}+a_{33}e^{-r_1y}+a_{34} \quad (3.13)$$

where $m=\frac{1}{2}\left(\Pr+\sqrt{\mathbf{Pr}^2+4\mathbf{P}rR_a}\right),\ m_1=\frac{1}{2}\left(\Pr+\sqrt{\Pr^2--4\Pr\left(\phi+\frac{i\omega}{4}\right)}\right),$

$n_1=\frac{1}{6}(2a_3-36a_8+a_7),\ r_1=\frac{1}{6}a_{27}-6a_{28}+\frac{1}{3}a_{18}$

with $a_1=\frac{-4i\text{B}m}{\omega\Pr},\ a_2=e^{i\omega t}-a_1,\ a_{13}=\left(n_1^2-n_1-\left(M^2(1+Pm)+\frac{1}{k_0}\right)\right)\frac{a_9}{MPm},$

$a_{14}=\left(m^2-m-\left(M^2(1+Pm)+\frac{1}{k_0}\right)\right)\frac{a_{12}}{MPm}\quad a_{23}=\frac{-a_{19}}{n_1^3-n_1^2a_{18}+n_1a_{17}-Pma_{16}},$

$a_{24}=\frac{-a_{20}}{m^3-m^2a_{18}+ma_{17}-Pma_{16}},\ a_{25}=\frac{-a_{21}}{m_1^3-m_1^2a_{18}+m_1a_{17}-Pma_{16}},\ a_{26}=\frac{a_{27}}{Pma_{16}},$

$a_{29}=-(a_{23}+a_{24}+a_{25}),\ a_{30}=\frac{a_{23}\left(\left(M^2+\frac{1}{k_0}+\frac{i\omega}{4}-PmM^2\right)+n_1-n_1^2\right)+a_9B\left(n_1-\frac{1}{k_0}\right)-MPmBa_{13}}{MPm}$

$a_{31}=\frac{a_{24}\left(\left(M^2(1-Pm)+\frac{1}{k_0}+\frac{i\omega}{4}\right)+m-m^2\right)+a_{12}B\left(m-\frac{1}{k_0}\right)-MPmBa_{14}-Gr\,\tau a_1}{MPm}$

$a_{32}=\frac{a_{25}\left(\left(M^2+\frac{1}{k_0}+\frac{i\omega}{4}-PmM^2\right)+m_1-m_1^2\right)-Gr\,\tau a_2}{MPm},\ a_{33}=\frac{a_{29}\left(\left(M^2+\frac{1}{k_0}+\frac{i\omega}{4}-PmM^2\right)+r_1-r_1^2\right)}{MPm}$

$a_3=1+Pm,\ a_4=-\left(M^2(1-Pm)+\frac{1}{k_0}+Pm\right),\quad a_5=\left(M^2+\frac{1}{k_0}\right),\ a_6=Gr\,\tau(m-Pm)$

$a_7=\left(-36a_3a_4+108a_5+8a_3^3+12\sqrt{12a_4^3-3a_4^2a_3^2-54a_4a_3a_5+81a_5^2+12a5a_3^3}\right)^{1/3}$

$a_8=\frac{\left(\frac{1}{3}a_4-\frac{1}{9}a_3^2\right)}{a_7},\ a_{12}=\frac{-a_6}{m^3-a_3m^2+a_4m-a_5}\quad a_{19}=-B\left(a_9\left(n_1^2-\left(Pm+\frac{1}{k_0}\right)n_1+\frac{Pm}{k_0}\right)-a_3MPmn_1\right)$

$a_{20}=B\left(a_{912}\left(m^2-\left(Pm+\frac{1}{k_0}\right)m+\frac{Pm}{k_0}\right)-Gr\,\tau a_1(m-Pm)+MmPma_{14}\right)$

$$a_{21} = Gr\tau a_2(m_1 - Pm),\; a_{22} = -\frac{Pm\omega}{4},\; a_{27} = \Big(-36a_{17}a_{18} + 108a_{16} + 8a_{18}{}^2$$

$$12\sqrt{12a_{17}{}^3 - 3a_{17}{}^2a_{18}{}^2 - 54a_{17}a_{18}a_{16} + a_{16}{}^2 + 12a_{16}a_{18}{}^2}\Big)^{\frac{1}{3}},\; a_{28} = \frac{\frac{1}{3}a_{17} - \frac{1}{9}a_{18}{}^2}{a_{27}}$$

The functions $u_0(y)$, $\theta_0(y)$ and $H_0(y)$ are the mean velocity, the mean temperature and the mean induced magnetic field, respectively; and $u_1(y)$, $\theta_1(y)$ and $H_1(y)$ are, respectively, the oscillatory part velocity, the oscillatory part temperature and the oscillatory part induced magnetic field.

Now substituting equations (3.8) – (3.13) into equation (3.1), we obtain the required expressions for velocity, temperature and magnetic induction;

$$u(y,t) = a_9e^{-n_1y} + a_{12}e^{-my} + \varepsilon e^{i\omega t}\left(a_{23}e^{-n_1y} + a_{24}e^{-my} + a_{25}e^{-m_1y} + a_{26} + a_{29}e^{-r_1y}\right) \quad (3.14)$$

$$\theta(y,t) = e^{-my} + \varepsilon e^{i\omega t}\left(a_1e^{-my} + a_2e^{-m_1y}\right) \quad (3.15)$$

$$H(y,t) = a_{13}e^{-n_1y} + a_{14}e^{-my} + (1 - a_{13} - a_{14}) + \varepsilon e^{i\omega t}\left(a_{30}e^{-n_1y} + a_{31}e^{-my} + a_{32}e^{-m_1y} + a_{33}e^{-r_1y} + a_{34}\right) \quad (3.16)$$

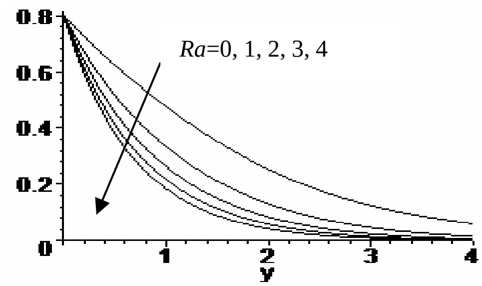

Figure 3.1: Temperature distribution for various values of radiation parameter

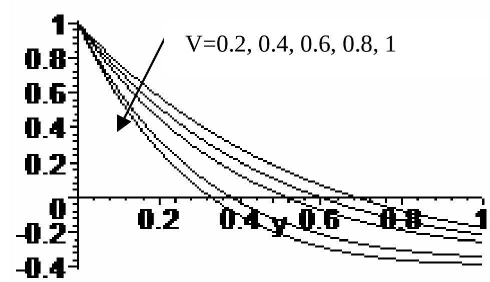

Figure 3.4: Magnetic field for various values *V*.

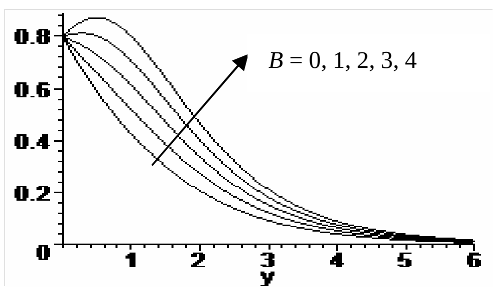

Figure 3.2: Temperature distribution for various values of suction parameter

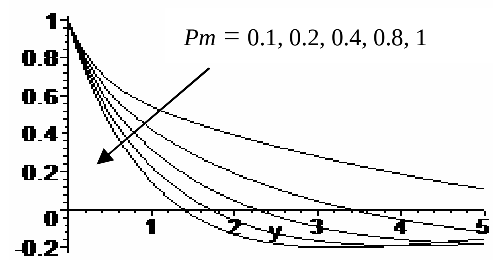

Figure 3.5: Magnetic field for various various values of *Pm*

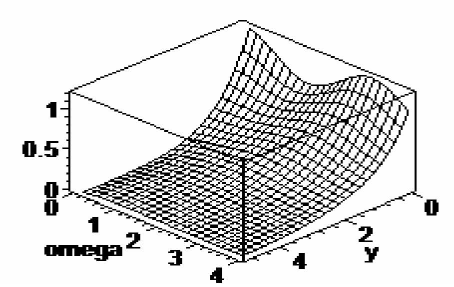

Figure 3.3: Temperature distribution for various values of ω

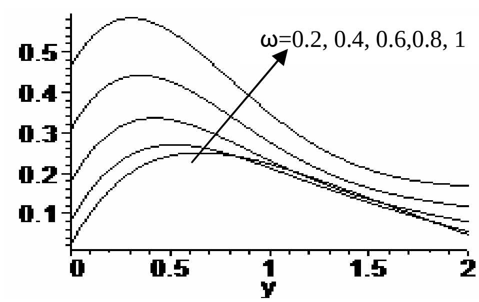

Figure 3.6: Magnetic field for various various values of ω

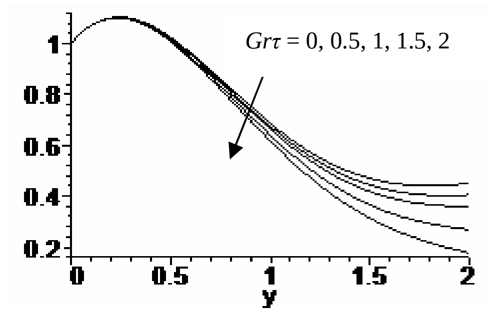

Figure 3.7: Magnetic field for various thermal Grashof number

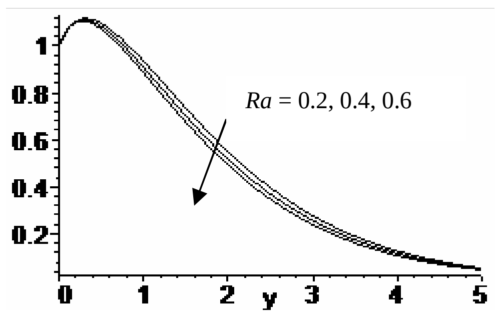

Figure 3.8: Magnetic field for various various values of radiation parameter

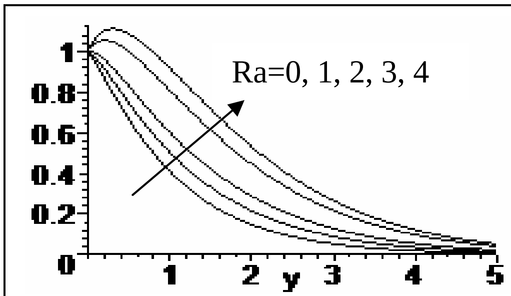

Figure 3.9: Magnetic field for various values B

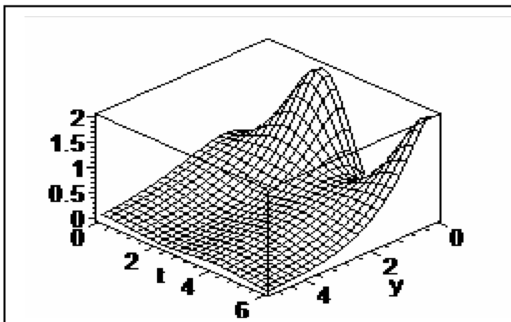

Figure 3.10: Magnetic field distribution as a function of (y,t)

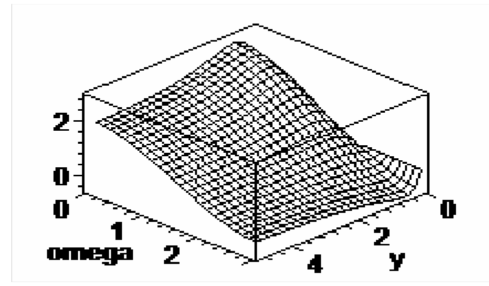

Figure 3.11: Velocity distribution to a function (y. ω)

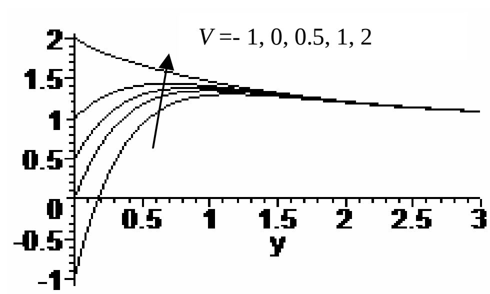

Figure 3.12: Velocity distribution for various values of V

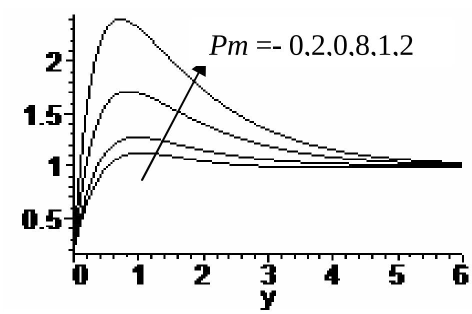

Figure 3.13: Velocity distribution for various values of magnetic Prandtl number

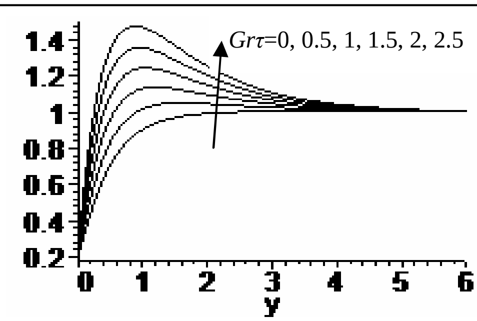

Figure 3.14: Velocity distribution for various values of thermal Grashof number

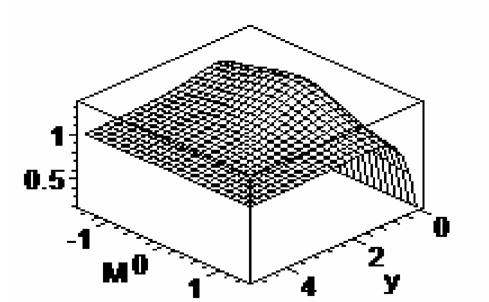

Figure 3.15: Velocity distribution for various values of Hartmann number

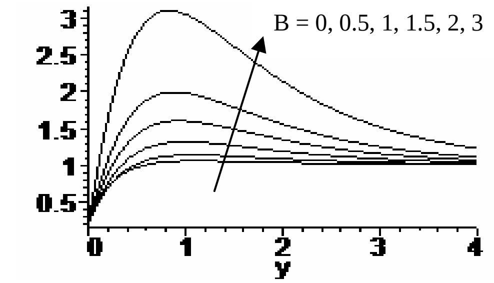

Figure 3.17: Velocity distribution for variousvalues of suction parameter

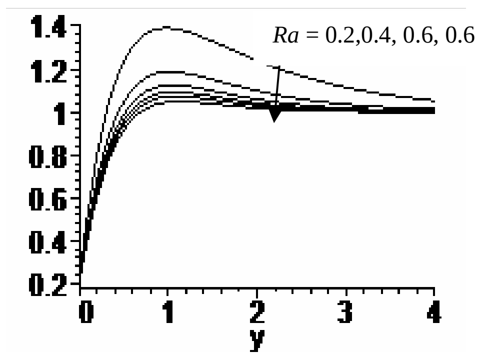

Figure 3.16: Velocity distribution for various values of *Ra*

Having obtained expressions for velocity, temperature and magnetic induction we then use a computer software package (Mapple 8.1 release) to build up the real and imaginary parts and their graphical representation is presented for analysis.

4.0 Discussion

In order to point out the effect of the temperature and magnetic field on the velocity and on all the other quantities, the following discussion is set out. We carried out the analysis using the values $\varepsilon = 0.2$, $Pm = 1$, $Pr = 0.71$, $M = 0.5$, $\omega t = \pi/2$, $V = 0.2$ and Ra = -0.2 for the parameters except where stated otherwise.

Figures 3.1, 3.2 and 3.3 show the temperature distribution for radiation parameter, suction parameter and the free stream oscillation, from which we observe the temperature decreases away from the plate. The temperature decreases with increase in the radiation parameter as shown in figure 3.1, which is in agreement with the work reported by Cookey et al [1] and Ogulu and Prakash [2]. In figure 3.2, the temperature, however, increases with increase in suction parameter. Furthermore, we observe that the magnitude of the temperature maximum for a relatively lager suction parameter. The free stream oscillation give rise to temperature fluctuation with significant effect near the limiting surface than far way from the plate, as displayed in figure 3.3.

We displayed in figures 3.4 – 3.10, the magnetic field for limiting surface velocity, the free stream oscillation, thermal grashof number, radiation parameter, suction parameter and the magnetic distribution at different time, it could also be seen that the magnetic field decreases away from the plate. Displayed in figure 3.4 is the effect of limiting surface velocity on the induced magnetic field distribution. We see from this figure that increase in the plate velocity brings about decrease in the magnitude of the induced magnetic field, and the magnetic boundary layer reduces with increase in the plate velocity.

In figure 3.5, it could be seen that the induced magnetic field reduces as magnetic Prandtl number increases. The induced magnetic field increases as the free – stream oscillation frequency increases, with maximum close to the surface as shown in figure 3.6. The effect of thermal grashof number corresponding to heating of the plate is shown in figure 3.7. We observe that the induced magnetic field increases as thermal grashof number reduces. In figure 3.8, we show that the magnetic field reduces as the radiation parameter increase. At a relatively higher value(s) of suction parameter B, the induced magnetic field increase to a maximum close to the surface and uniformly decreases away from the limiting surface, shown in figure 3.9. This is in agreement with the work reported by Kim. Figure 3.10 shows the variation of induced magnetic field with time. We observe that oscillation is more pronounced near the limiting surface but dies out away from the surface. However the induced magnetic field reduces as the position element increases.

The variation of velocity distribution for different values of the control parameters are shown in figures 3.11 – 3.17. In this case, the velocity increases and reach a maximum, decreases to the free – stream velocity U. In figure 3.11, we show that as free – stream oscillation frequency increases, the velocity decreases. We also noticed from this figure that the maximum in the velocity field is removed for higher value of oscillation frequency, which makes the free – stream velocity to be the maximum velocity during the flow. The variation in the velocity distribution with respect to the limiting surface velocity is shown in figure 3.12. It could be seen that velocity increases the V increases. In figure 3.13 and 3.14, we observed that velocity increases as either magnetic Prandtl number or the thermal grashof number increases. While in figure 3.15, we see that as Hartmann number increase the velocity reduces with maximum value only occurs at $M=0$. We observed from figure 3.16 that velocity decrease as radiation parameter increases. Suction velocity has increasing effect as it increase on the velocity with emergence of maximum as suction parameter increases as shown in figure 3.17.

Our preliminary results for the velocity show that near the plate, the velocity increases rapidly, attains a maximum value, then slowly tends to stream velocity far from the plate, whereas the induced magnetic field and the temperature decreases away from the plate.

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