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On the Arrhenius reacting flow over a stretching sheet in the presence of magnetic field

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Abstract

We present an analysis of the boundary layer flow of a reacting fluid. We show that the problem has a solution. We present an analytical solution for the limiting case of the Frank-Kamenetskii parameter ε. Numerical Results feature preliminary when $\varepsilon = O(1)$.

1.0 Introduction

In this paper, we investigate the characteristics of the solution of a reacting boundary layer equation of flow over a stretching sheet in the presence of a magnetic field.

Recently [4] investigated the boundary layer equation of flow over a stretching sheet in the presence of a magnetic field. The obtained analytical solution for the velocity field and temperature in the boundary layer was obtained numerically.

Earlier, Gupta and Gupta investigated heat and mass transfer in hydro magnetic fluid flow, over an isothermal stretching sheet. Chen and Char [1] extended the works of Gupta and Gupta to a non isothermal stretching sheet. On the other hand, [2] investigated extensible surface and presented an analytical solution for the boundary layer flow. Vajravelu and Nayfeh [6] investigated convective heat transfer.

Makinde [3] investigated a flow past a moving vertical porous plate. In this paper, we extended the works of [4] to an Arrhenius reacting flow. Thus when *n* in the power law equation is 1 and activation energy zero in our model we obtain the special case of [4].

2.0 Mathematical formulation

The appropriate continuity, momentum and energy steady equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma H_0{}^2}{\rho}u \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{QA}{\rho cp}\left(T - T_\infty\right)^n e^{-\frac{E}{RT}} \tag{2.3}$$

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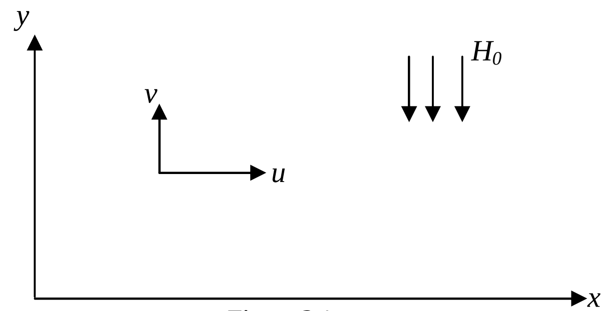

Figure 2.1:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0{}^2}{\rho}u \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{QA}{\rho cp}\left(T-T_\infty\right)^n e^{-\frac{E}{RT}} \tag{2.3}$$

respectively. The boundary conditions are

$$u=ax,\ v=0,\ T=T_w \ \text{ at } \ y=0 \tag{2.4}$$

$$u\to 0,\ T\to T_\infty \ \text{ as } \ y\to\infty \tag{2.5}$$

Here u and v are velocity components in x and y directions respectively, ρ is the density of the liquid, ν is the kinemetic viscosity, H_0 is the strength of the applied magnetiuc field, σ is the electrical conductivity of the fluid, T is the fluid temperature, α is the fluid thermal diffusivity and cp is the specific heat at constant. A is the pre-exponential factor, Q is the heat release per unit mass, E is the activation, R is universal gas constant and n is a natural number. In [4] $n = 1$, $E = 0$, and $A = 1$

3.0 Analysis

3.1 Existence of solution

Equations (2.1) – (2.5) admit self similar solutions of the form

$$u=ax\,f'(\eta),\ \ v=-\sqrt{a\nu}\ \ f(\eta),\ \ \eta=\sqrt{\frac{a}{\nu}}\ \ y$$

$$\theta=\frac{E}{RT_w}\left(\frac{T-T_\infty}{T_w-T_\infty}\right),\ \in\ =\frac{RT_w}{E}$$

$$R=\frac{\sigma H_0^2}{a\rho},\ \ P_r=\frac{\nu}{\alpha},\ \ N=\frac{QAe^{-\frac{E}{RT_w}}}{\rho cp\,a}$$

such that equations (2.2) – (2.5) become

$$f'''-f'^2+ff''=Rf' \tag{3.1}$$

$$-P_r\ f\theta'=\theta''+N\theta^n e^{\frac{\theta}{1+\in\theta}} \tag{3.2}$$

$$f(0)=0,\ \ f'(0)=1\ \ f'(\infty)=0 \tag{3.3}$$

$$\theta(0) = 1/\in, \ \theta(\infty) = 0 \qquad (3.4)$$

Here *R*, P_r and *N* represent the Chandrasekhar number, Prandtl number and [4] have given the analytical solution for the momentum equation as

$$f(\eta) = \frac{1 - e^{-m\eta}}{m}, \quad u = axe^{-m\eta}, \ v = \sqrt{av}\left(\frac{1 - e^{-m\mu}}{m}\right),$$

where $m = \sqrt{1+R}$.

Unlike equation (3.1), equation (3.2) has no analytical solution. But we shall now show that equation (3.3) has a numerical solution which satisfies (3.4)

***Theorem* 3.1**

Equation (3.2) which satisfies (3.4) has a solution when f satisfies (3.2) and (3.3).

In the proof we shall need the following:

Consider the initial value system

$$\left.\begin{array}{l} x_1^1 = f_1(x_1, \Lambda, x_n, t), \ x_1(t_0) = x_{10} \\ x_2^1 = f_2(x_1, \Lambda, x_n, t), \ x_2(t_0) = x_{20} \\ x_n^1 = f_n(x_1, \Lambda, x_n, t), \ x_n(t_0) = x_{n0} \end{array}\right| \qquad (*)$$

***Theorem* 3.2** [7]

If the partial derivatives $\frac{\partial f_i}{\partial x_j}, i, j = 1, \Lambda, n$ are continuous in the region *D* of definition, then problem (*) has a unique solution.

We are now in a position to prove our theorem.

Proof

Let $x_1 = \eta$, $x_2 = \theta$, $x_3 = \theta^1$ then

$$x_1^1 = 1, \ x_1(0) = 0 \qquad (3.5)$$

$$x_2^1 = x_3, \ x_2(0) = \alpha \ (say), \ \alpha \text{ finite.} \qquad (3.6)$$

$$x_3^1 = -N \exp\left(\frac{x_2}{1 + \in x_2}\right) - P_r \ f \ x_3 \qquad (3.7)$$

That is

$$x_1^1 = f_1(x_1, x_2, x_3) = 1$$

$$x_2^1 = f_2(x_1, x_2, x_3) = x_3$$

$$x_3^1 = f_3(x_1, x_2, x_3) = -NX_2 \exp\left(\frac{x_2}{1 + \in x_2}\right) - p_r \ f(x_1) x_3$$

Clearly $\frac{\partial fi}{\partial xj}$ are continuous. Hence by the above theorem, the problem has a unique solution. ■

***Remark* 3.1**

This theorem only states that our problem has a solution. The solution is only unique when the initial gradient is fixed. Our problem may indeed have more than one solution.

3.2 Asymptotic analysis

Suppose the Frank-Kamenetskii parameter N is small, i.e $0<N<<1$. We may neglect the Arrhenious term $\theta^n \exp\left(\frac{\theta}{1+\in\theta}\right)$. We then solve

$$-p_r f\theta^1 = \theta^{11},\ \theta(0)=1/\in \quad \theta(\infty) = 0 \qquad (3.8)$$

Equation (3.8) admits analytical solution. In fact,

$$\in\theta(\eta)=\exp\left(\frac{p_r}{m^2}\right)\int_0^{\eta}\exp-\left(\frac{p_r}{m}\right)\left[S+\frac{1}{m}\exp(-ms)\right]ds-\exp\left(\frac{p_r}{m^2}\right)\int_0^{\infty}\exp-\left(\frac{p_r}{m}\right)\left[S+\frac{1}{m}\exp(-ms)\right]ds$$**T**

***heorem* 3.3** [8]

Let

$$u'' + g(x)u' \perp h(x)u = f(x) \qquad (**)$$

$$u(a) = \gamma_1,\ u'(a) = \gamma_2 \qquad (***)$$

where function *f, g* and *h* are defined in (*a, b*) with *g* and *h* bounded.

Suppose that $u_1(x)$ *and* $u_2(x)$ are solutions of (**) in (*a, b*) which satisfy (***). Then $u_1 \equiv u_2$ in (*a,b*).

***Theorem* 3.4**

Let $n = 1$ and $E = 0$ Then problem (2.1) – (2.5) has a unique solution.

Proof

Clearly $f(\eta) = \frac{1-e^{-m\eta}}{m},\ u = axe^{-m\eta}$ and $v = \sqrt{av}\left(\frac{1-e^{-m\eta}}{m}\right)$ and they are unique. Also, $\theta'' + p_r f\theta' + N\theta = 0,\ \theta(0) = 1$ and $\theta'(0) = \alpha(say)$. Hence by [8] Theorem θ is also unique. This completes the proof for a $\rightarrow \infty$. ■

***Theorem* 3.5** [8]

Let

$$u'' + g(x)u' \perp h(x)u = f(x) \qquad (****)$$

$$u(a) = \gamma_1,\ u'(a) = \gamma_2 \qquad (*****)$$

where function *f, g* and *h* are defined in (*a, b*) with *g* and *h* bounded.

Suppose that $u_1(x)$ *and* $u_2(x)$ are solutions of (****) in (*a, b*) which satisfy. Then $u_1 \equiv u_2$ in (*a,b*).

***Theorem* 3.6**

Let $n = 1$ and $E = 0$ Then problem (2.1) – (2.5) has a unique solution.

Proof

Clearly $f(\eta) = \frac{1-e^{-m\eta}}{m},\ u = axe^{-m\eta}$ and $v = \sqrt{av}\left(\frac{1-e^{-m\eta}}{m}\right)$ and they are unique. Also, $\theta'' + p_r f\theta' + N\theta = 0,\ \theta(0) = 1$ *and* $\theta'(0) = \alpha(say)$. Hence by [6] Theorem θ is also unique. This completes the proof ■

***Theorem* 3.7** [6]

Let $u'' + g(x)u'' + h(x)u = f(x), x>a,\ U(a) = \gamma_1,\ U''(a) = \gamma_2$

The function $Z_1(X)$ and $Z_2(X)$ are said to be upper and lower solutions if $Z_1 + g(X)Z_1 + h(X)Z_1 \geq f(X)$ and $Z_2'' + g(X)\, Z_2' + h(X)Z_2 \leq f(X)$, $Z_2(a) \leq \gamma$ and $Z_2'(a) \leq \gamma_2$ respectively. That is $Z_2(X) \leq U(X) \leq Z_1(X)$.

***Theorem* 3.8**

Let $\theta'' + Prf(\eta)\theta' + N\theta = \theta$, $N > 0$, $\theta(0) = 1$, $\theta'(0) = \alpha$. Then $Z_1(X) = 1$ and $Z_2(X) = 0$, are upper and lower solutions i.e $0 \leq \theta \leq 1$

Proof

Let $Z_1(X) = 1$. Then $Z_1' = 0$, $Z_1'' = 0$ and $N > 0$. Let $Z_2 = 0$. Then $Z_2' = 0$, $Z_2'' = 0$ and $N(0) = 0$. Hence $0 \leq \theta \leq 1$. This completes the proof ■

3.3 Numerical results

We present the numerical result for the various parameters as shown in Figure 3.1..

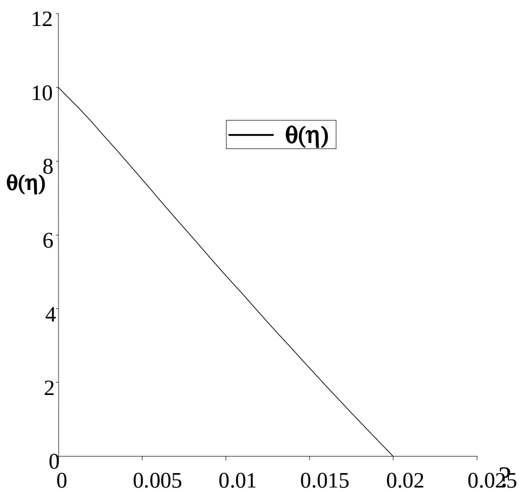

Figure 3.1: The graph shows the dimensionless temperature θ(η) against the position η when ε - 0.1

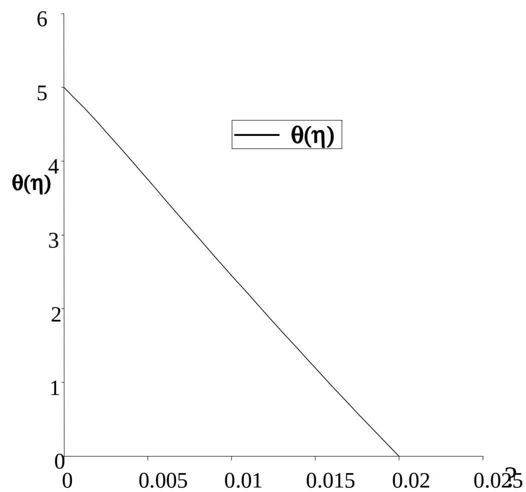

Figure 3.2: The graph shows the dimensionless temperature θ(η) against the position η when ε = 0.2.

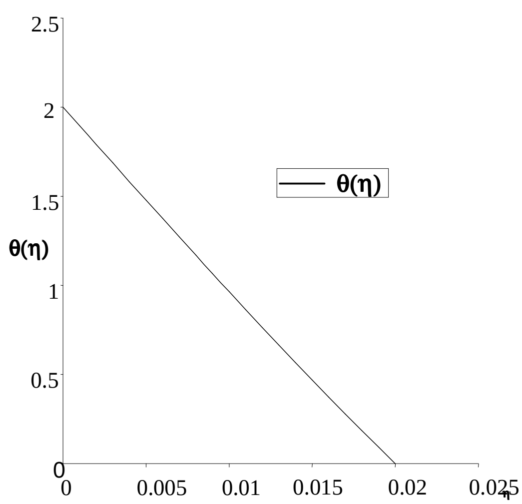

Figure 3.3: The graph shows the dimensionless temperature θ{η) against the position η when ε = 0.5

4.0 Discussion of results

We have presented a boundary layer analysis of a reacting flow. We show the flow properties and numerical results show that the activation energy parameter has influence on the temperature .It is easily seen in figures 2-4 that the wall temperature is $\frac{1}{\varepsilon}(10,5,2)$ respectively. The higher the wall temperature the longer the point where its influence is not felt.

Appendix

```
Program bode (input, output);
const h=0.001;
m=1.2247;
N=0.1;
e=0.1;
Pr=0.71;
var x1, x2, x3 : Array[0..20] of real;
Gv, f1a, f1b, f1c, f2a, f2b, f2c:real;
i:integer;
Begin
write ('supply the Guess value:');
readln(Gv);
writeln('i':3,' x1':12,' x2':12,' x3':12);
x1[0]:=0;
x2[0]:=10;
x3[0]:=Gv;
for i:=0 to 20 do
Begin
f1a:=h;
 f1b:=h*x3[i];
 f1c:=h*d*(-N*x2[i]*exp(x2[i]/1+e*x2[i]))-(Pr*((1-exp(-m*x1[i]))/m)*x3[i]);
 f2a:=h;
 f2b:=h*(x3[i]+f1c);
 f2c:=h*d*(-N*(x2[i]+f1b)*exp((x2[i]+f1b)/1+e*(x2[i]+f1b)))-
 (Pr*((1-exp(-m*(x1[i]+f1a)))/m)*(x3[i]+f1c));
 x1[i+1]:=x1[i]+0.5*(f1a+f2a);
 x2[i+1]:=x2[i]+0.5*(f1b+f2b);
 x3[i+1]:=x3[i]+0.5*(f1c+f2c);
 writeln(i:3, x1[i]:12:4, x2[i]:12:4, x3[i]:12:4);
 End;
 readln;
 End
```

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