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The transmission properties of the motion of the oval and round windows

O. H. Adagba
Department of Industrial Mathematics and Applied Statistics
Ebonyi State University,
Abakaliki, Nigeria.

Abstract

A mathematical model describing the motion of the oval and round windows is studied. The exact solutions of the equations of motion are obtained. For certain model configurations, the displacement patterns of the motions consist of sinusoidal waveforms that are in cycles were obtained. The qualitative effect of a transmitted pressure along the tympanic canal is discussed.

1.0 Introduction

Sound waves are collected by the pinna and directed through the external auditory meatus to impinge on the tympanic membrane or eardrum. The vibrations then pass through the middle ear, which incidentally, is open to the throat through the Eustachian tube. The movement of air pressure through the tube equalizes the air pressure on both sides of the eardrum. In human beings, vibrations are normally carried through the middle ear by a series of three bones: the hammer, anvil and stirrup (or malleus, incus and stapes respectively) known as the ossiscles. The train of bones connect the tympanic membrane with the oval window of the cochlea, the inevitable organ where vibrations are converted into nerve impulses. But in conducting vibrations, these bones also increase their strength, that is, the pressure they exert. Pressure is force divided by the area on which the force acts, and the area of the tympanic membrane is about 30 times that of the oval window. At the tympanic membrane, sound pressure received are relatively lost, most is transmitted to the cochlea amplified to a 22-fold greater pressure on the inner ear (for more details see Cal et al [1]. Dalhoff et al [2], Ghaffari et al [3].

The mechanical forces that are transmitted by the bones of the middle ear are transformed into hydraulic pressure variation when the stapes strike the oval window. The cochlea is filled with fluid, and so any pressure applied to the oval window is transmitted through it, just as the pressure applied to the car's brake pedal is transmitted through its hydraulic system to the wheels. Some of this pressure continues as a wave to the far end of the cochlea and back through the tympanic canal to the round window,

e-mail address: henela2006@yahoo.com
Telephone: 08036258406

which moves out or in and allows the pressure inside the cochlea to remain relatively constant. Most of the pressure applied by the oval windows is transmitted to the basilar membrane, which responds with a vibration of its own, see Montgomery [7], Rajan [8], Temchin et al [9].

In this paper, a mathematical model which is a system of partial differential equations is studied, see [5]. However, there are relatively few papers dealing with the study of motion of oval and round windows. For instance, in solving linear partial differential equation, one comes across differential equations containing several parameters with auxiliary conditions that the solutions satisfy a boundary condition at several points.

2.0 The mathematical model

A mathematical Model describing the motion of the oval and round windows is formulated as follows:

The model is an enclosed two dimensional cavity and the basilar membrane appears in it as thin plate immersed in the fluid. Thus, we assume liberalized two dimensional potential flow in the configuration depicted in Figure 2.1 below.

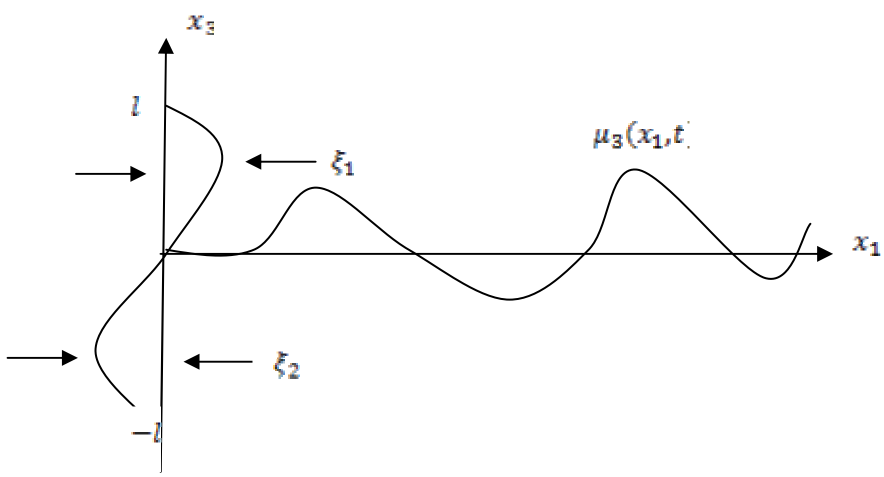

Figure2.1: Potential flow Model of Cochlea

The x_1 and x_3 are the fluid variables of enclosed two dimensionals cavity. The upper domain where $x_3 > o$ is denoted by l while for $x_3 < 0$ is denoted by $-l$ and $x_1 = L$ is the end of the cavity.

On $x_1 = 0, \quad 0 < x_3 < l$, the equation of motion of the oval window is given by:

$$m_0 \frac{\partial^2 \overline{\xi}_1}{\partial t^2} + r_0 \frac{\partial \overline{\xi}_1}{\partial t} + k_0 \overline{\xi}_1 = \overline{\rho}_0 - \overline{\rho}_1(0, x_3, t) \tag{2.1}$$

The velocity at the oval window and that of the fluid at the point of contact is given by:

$$\frac{\partial \overline{\xi}_1}{\partial t} = \frac{\partial \overline{\phi}_1}{\partial x_1}, \; on \;\; x_1 = 0 \tag{2.2}$$

On $x_1 = 0, \quad -l < x_3 < 0$, the equation of motion of the round window is given by:

$$m_0 \frac{\partial^2 \bar{\xi}_2}{\partial t^2} + r_0 \frac{\partial \bar{\xi}_2}{\partial t} + k_0 \bar{\xi}_2 = -\bar{p}_2(0, x_3, t) \qquad (2.3)$$

The velocity at the round window and that of the fluid at the point of contact is given by:

$$\frac{\partial \bar{\xi}_2}{\partial t} = \frac{\partial \bar{\phi}_2}{\partial x_1}, \; on \;\; x_1 = 0 \qquad (2.4)$$

$\bar{\xi}_1$ and $\bar{\xi}_2$ are the displacements of the oval and round windows respectively. Equations (1 – 4) valid on $x_1 = 0$ are equations of motion of the oval and round windows with their boundary conditions respectively, which were adapted from the work of Lesser and Berkley [4, 5].

The constant values of the parameter for the oval and round windows are denoted by m_0, mass per unit area, r_0, damping in dyne sec/cm^3 and k_0, stiffness in dyne /cm^3. We seek a solution such that the field variables will be proportional to $e^{i\omega t} = e^{st}$.

We write

$\phi = \mathrm{Re}(\bar{\phi} e^{st})$, $u_3 = \mathrm{Re}(\bar{u}_3 e^{st})$, $p_i = \mathrm{Re}(\bar{p}_i e^{st})$, $\xi = \mathrm{Re}(\bar{\xi} e^{st})$.

On $x_1 = 0, \quad 0 < x_3 < l$,

$$m_0 s^2 \xi_1 + r_0 s \xi_1 + k_0 \xi_1 = p_0 - p_1(0, x_3) \quad - \qquad (2.5)$$

with the boundary condition

$$s\xi_1 = \frac{\partial \phi_1}{\partial x_1} \qquad (2.6)$$

On $x_1 = 0, \quad -l < x_3 < 0$,

$$m_0 s^2 \xi_2 + r_0 s \xi_2 + k_0 \xi_2 = -p_2(0, x_3) \; - \quad - \qquad (2.7)$$

with boundary condition

$$s\xi_2 = \frac{\partial \phi_2}{\partial x_1} \quad - \quad - \qquad (2.8)$$

To solve equations (2.5) and (2.7)

$$m_0 s^2 \xi_1 + r_0 s \xi_1 + k_0 \xi_1 = p_0 - p_1(0, x_3)$$

$$m_0 s^2 \xi_2 + r_0 s \xi_2 + k_0 \xi_2 = -p_2(0, x_3)$$

$$s\xi_1 = \frac{p_0 - p_1(0, x_3)}{z(x_3 \, \omega)} \quad - \quad - \quad - \quad - \qquad (2.9)$$

where

$$z(x_3 \omega) = m_0 s + r_0 + \frac{k_0}{s} \quad - \quad -- \qquad (2.10)$$

Applying equation (2.6) at $x_1 = 0$, gives

$$s\xi_1 = \frac{\partial \phi_1}{\partial x_1} = -\frac{\beta\lambda \sin\lambda L \cosh(l - x_3)}{\cos\lambda L \cosh\lambda l} = \frac{p_0 - p_1(0, x_3)}{z(x_3 \, \omega)} \qquad (2.11)$$

(See Mbah and Adagba [6]).

Generally, β is given by

$$\beta = \frac{p_0 - p_1(0,x_3)\cos\lambda L\cosh\lambda l}{\lambda\sin\lambda L\cosh(l-x_3)z(x_3\,\omega)} \qquad - \qquad -(2.12)$$

Similarly, the equation for the round window is

$$m_0 s^2\xi_2 + r_0 s\xi_2 + k_0\xi_2 = -p_2(0,x_3)$$

Using equation (2.10) it can be written compactly as

$$s\xi_2 = -\frac{p_2(0,x_3)}{z(x_3,\omega)} \qquad - \qquad (2.13)$$

Applying (2.8) to (2.13) gives

$$s\xi_2 = \frac{\partial\phi_1}{\partial x_1} = -\frac{\gamma\lambda\sin\lambda L\cosh(l+x_3)}{\cos\lambda L\cosh\lambda l} = -\frac{p_2(0,x_3)}{z(x_3,\omega)} \qquad (2.14)$$

Generally, $\gamma = \frac{p_2(0,x_3)\cos\lambda L\cosh\lambda l}{\lambda\sin\lambda L\cosh(l+x_3)z(x_3,\omega)}$

Using the fact that $p_2 = p_0 - p_1$, the problem as presented can be simplified by redefining the arbitrary time functions in the introduction of the velocity potentials. Thus, for region 2, where

$$-l < x_3 < 0, \;\; \phi_1 = \phi_2 - \frac{p_0}{\rho s}.$$

Allowing $p_0 = 0$, we obtain $-\rho s\phi_1 = p_1$. Hence p_1 valued at $x_1 = 0$, gives

$$p_1(0,x_3) = \frac{-\beta s\rho\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}\,l} \quad - \qquad - \qquad (2.15)$$

Substituting (2.15) into (2.11), we obtain

$$\xi_1 = -\beta\omega\frac{\sqrt{\frac{\rho}{\alpha_3}}\sin\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{s\cos\omega\sqrt{\frac{\rho}{\alpha_3}}L\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}\,l} = \frac{\rho\beta\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{z(x_3,\omega)\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}\,L}$$

Recalling that $\xi_1 = Re\{\xi_1 e^{st}\}$

$$\xi_1 = \frac{-\beta\omega\sqrt{\frac{\rho}{\alpha_3}}\tan\omega\sqrt{\frac{\rho}{\alpha_3}}\sin\omega t\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)}{\omega\cos\omega\sqrt{\frac{\rho}{\alpha_3}}\,l}$$

$$= \frac{\beta\rho\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l-x_3)\left(r_0\cos\omega t\left(m_0\omega - \frac{k_0}{\omega}\right)\sin\omega t\right)}{r_0{}^2 + \left(m_0\omega - \frac{k_0}{\omega}\right)^2\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}\,l} \qquad (2.16)$$

Similarly,

$$\xi_2 = \frac{\beta\omega\sqrt{\frac{\rho}{\alpha_3}}\tan\omega\sqrt{\frac{\rho}{\alpha_3}}L\sin\omega t\cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)}{\omega\cos\omega\sqrt{\frac{\rho}{\alpha_3}}\,l}$$

$$= \frac{-\beta\rho \cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(l+x_3)\left(r_0 \cos\omega t\left(m_0\omega-\frac{k_0}{\omega}\right)\sin\omega t\right)}{{r_0}^2+\left(m_0\omega-\frac{k_0}{\omega}\right)^2 \cosh\omega\sqrt{\frac{\rho}{\alpha_3}}l} \quad - \qquad (2.17)$$

3.0 Discussion

We seek for a solution such that the field variables will be proportional to $e^{i\omega t} = e^{st}$. Hence, the first step in the method of analysis of the model is the choice of values for the constants. The values for the constants are:

$\alpha_3 = 2$ (calculated), $s = i\omega, |\omega| = 10^4,\ m_0 = 0.05,\ k_0 = 10^7 e^{-1.5x_3}$,

$r_0 = 30000\, e^{-1.5x_3}$, adapted from the works of Lesser and Berkley [4,5].

$\rho = 1\ and\ L = 35mm$ with the parameters having the same meaning as earlier stated.

ζ_1 and ζ_2 are the displacements of the oval and round, windows respectively.

Equations (2.1 – 2.4) valid on $x_1 = 0$ are equations of the oval and round windows with their boundary conditions.

Equations (2.16, 2.17) were used by Math card 7 Professional to plot figures 3.1 and 3.2 which are the displacements at the oval and round windows respectively. These depict sinusoidal waveforms that are in cycles. The direction of propagation is in opposite directions, which agrees with the theory, that is, as the oval window bulges outwards, the converse will be the case of the round window.

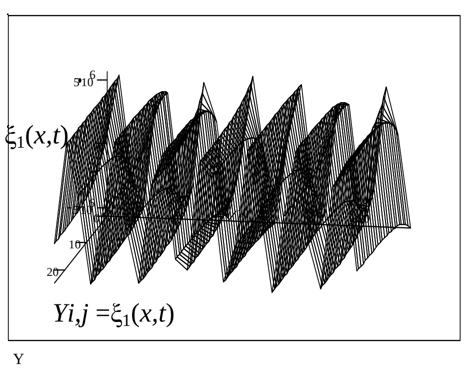

Figure 3.1: Displacement at the oval window

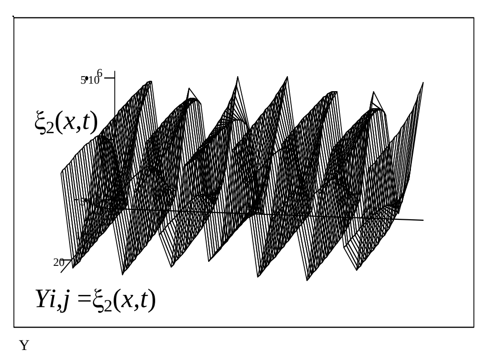

Figure 3.2: Displacement at the round window

4.0 Conclusion

The problem of cochlea model [4,5] is not considered here and a paper treating this problem has been published. The conclusion of this exploratory effort, it seems pertinent to point out a desirable course for future research. Input signals of finite duration and various displacements both sequentially and simultaneously should be considered to obtain a better understanding of the transient behaviour. The motion of oval and round windows in relation to their transmission and physical properties should be investigated, and the qualitative results presented in this article can be seen on sinusoidal waveforms in cycles and auditory thresholds (see references) should be re-examined in the light of the finding from more sophisticated models. The result of such research would permit a meaningful quantitative comparison of the model's behaviour to the physiological evidence.

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