Journal of the Nigerian Association of Mathematical Physics
Volume **15 (November, 2009), pp 189 - 196**

Effects of both wellbore and reservoir properties on pressure and pressure derivative distribution of a horizontal well subject to complete external fluid drive

E. S. Adewole
Department of Petroleum Engineering;
University of Benin, Benin City, Nigeria.

Abstract

The task of clean oil production from a horizontal well under complete external fluid drive is a huge challenge to the operator who intends to exclude unwanted fluid production. In order to determine guidelines for drilling or modifying a horizontal well to achieve economic production therefore, dimensionless pressures and their derivative for such a well were studied theoretically, to investigate the effects of all wellbore and reservoir properties on overall well performance, assuming unsteady flow. For all sets of parameters considered, the dimensionless time of attainment of steady state, t_{Dss}, was calculated. This is the minimum dimensionless time beyond which clean oil production can no longer be guaranteed for a particular well completion. Results obtained show that the dimensionless time of arrival of external fluid at the wellbore increases with shorter dimensionless well lengths, unaffected by dimensionless well radius, and decreases with shorter reservoir size. Furthermore, well completion and re-completion practices can delay external fluids breakthrough. However, well eccentricity and perforation locations do not affect arrival times as well as well productivities. Finally, results also show that well productivities are affected inversely by dimensionless wellbore radius and directly by reservoir thickness and reservoir geometry.

Nomenclature

$Ei(-x) = \int_x^{\infty} \frac{e^{-u}}{u} du$,

$p_D = \frac{kh\Delta p}{141.2q\mu B}$;

$t_D = \frac{0.001056kt}{\mu\phi c_t L^2}$;

$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}}$ i = positions along x or y or z axes, *ft*;

$h_D = 1/L_D$;

Δ = drop;

p = pressure, psi;
k = permeability, *md;*
h = pay thickness, *ft*;
t = time, hours;
q = flow rate, STB/Day;
μ = oil viscosity, cp;
B = oil formation volume factor, bbl/STB;
c_t = total fluid compressibility, 1/psi; L = well length, *ft*;
erf = error function; τ = dimensionless dummy time.

Subscripts

x, y, z = x, y, or z, directions; D = dimensionless; w = wellbore; e = external

e-mail address: steveadewole@yahoo.com

Telephone: +234-08039237561

Journal of the Nigerian Association of Mathematical Physics Volume **15 (November, 2009), 189 - 196**

1.0 Introduction

For a horizontal well subject to complete external fluid drive, the period of clean oil production is of major concern to the operator considering economic gains. In other to achieve clean oil production, external (unwanted) fluid influx into the wellbore must be delayed or prevented permanently. To be able to understand the modalities for achieving delay or complete prevention of production of external fluid, a comprehensive fluid dynamics of such a reservoir system must be understood.

In this paper, the effects of wellbore and reservoir properties on overall wellbore pressure and pressure derivative distribution are studied theoretically. A mathematical pressure distribution expression will be derived for a horizontal well completely subject to external fluid drive. The distribution will contain all the parameters that are necessary for evaluating well flow performance. Pressure distribution shows all the possible flow regimes that are encountered during flow for a given well completion while pressure derivatives complement pressure distribution and are used to confirm the flow regimes exhibited on pressure distributions. Wellbore and reservoir properties will be varied to determine the arrival time of external fluid (steady state). As a result, options will be explored for delaying the arrival time and thus to prolong clean oil production.

[1], [2] and [3] have studied pressure distribution in a horizontal well in a reservoir with both bottom water and top gas cap drives. [1] and [2] solved this problem using semi-analytic method considering aquifer boundaries as constant-pressure boundaries. This is similar to [4], [5] and [6], who also represented bottom water as constant-pressure boundaries. In this paper, all the influx boundaries are represented as constant-pressure boundaries and the chemical composition of the external fluids is not considered. This affords the possibility of determining the influence of all operational parameters, either collectively or separately, with time. All the source functions describing the relevant constant-pressure boundaries will be selected from [7] and [8]

2.0 Reservoir and horizontal well model description

The reservoir and wellbore description used in this study is shown below in Figure 2.1. The origin of the well is at the well center and all dimensionless parameters are measured from this center. Hence, the coordinate of the origin is $(x_D , y_D , z_D) = (L_D/2, y_{wD}, z_{wD})$, while the coordinate of the extremities is (x_{eD} , y_{eD} , h_D). Directional permeabilities are assumed constant and the effects of wellbore storage and skin are not considered.

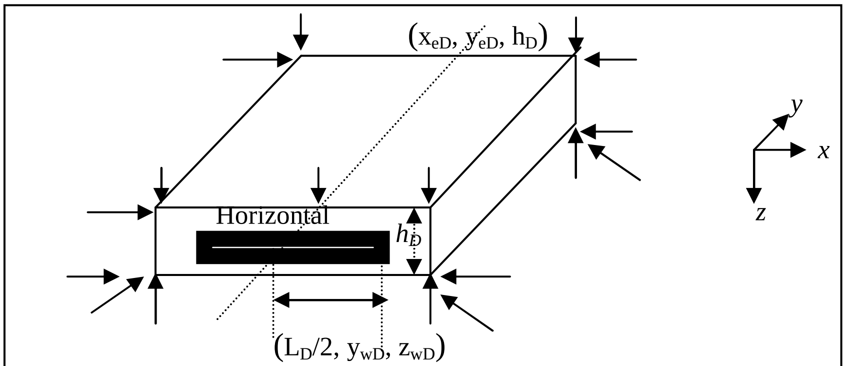


Figure 2.1: Reservoir and horizontal well model showing complete external fluid drive.

3.0 Mathematical description of axes of fluid flow

According to [7] and [8], mathematical relationships for the axes of flow are selected as follows:

***x-axis*:** Infinite slab source in an infinite slab reservoir with prescribed pressure at the extremities. Hence, the relevant source function is

$$s(x_D,t_D)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2t_D}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\sin\frac{n\pi x_{wD}}{x_{eD}}\sin\frac{n\pi x_D}{x_{eD}}1) \tag{3.1}$$

***y-axis*:** Because the well cannot be practically located in the middle of the reservoir along this axis, the source is an infinite source in an infinite plane reservoir with both ends subject to aquifer influx, that is

$$s(y_D,t_D)=\sum_{m=1}^{\infty}\exp(-\frac{m^2\pi^2t_D}{y_{eD}^2}\sin\frac{m\pi y_{wD}}{y_{eD}}\sin\frac{m\pi y_D}{y_{eD}} \tag{3.2}$$

***z-axis**:* Along the z-axis, the well experiences driving forces from both the top and bottom ends, both capable of fomenting steady-state. Therefore, the source is an infinite slab source from an infinite plane reservoir, given as

$$(z_D,t_D)=\frac{1}{h_D}\sum_{l=1}^{\infty}\exp(-\frac{l^2\pi^2t_D}{h_D^2}\sin\frac{l\pi z_D}{h_D}\sin\frac{l\pi z_{wD}}{h_D} \tag{3.3}$$

Therefore, the dimensionless pressure distribution is given as

$$p_D=8\int_0^{t_D}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2t_D}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\sin\frac{n\pi x_{wD}}{x_{eD}}\sin\frac{n\pi x_D}{x_{eD}}.\exp(-\frac{m^2\pi^2t_D}{y_{eD}^2})\sin\frac{m\pi y_{wD}}{y_{eD}}\sin\frac{m\pi y_D}{y_{eD}}.$$

$$\exp(-\frac{l^2\pi^2t_D}{h_D^2})\sin\frac{l\pi z_D}{h_D}\sin\frac{l\pi z_{wD}}{h_D}d\tau \tag{3.4}$$

Like the cases discussed in [4] and [6], p_D would attain steady state upon oil production at t_D given as

$$t_D=t_{Dss}\geq\frac{1}{\pi^2\left[\frac{1}{x_{eD}^2}+\frac{1}{y_{eD}^2}+\frac{1}{h_D^2}\right]}. \tag{3.5}$$

Therefore, using the principle of superposition for flow beyond $t_D\geq t_{Dss}$ and, considering at least a two-rate change

$$p_D = -\frac{\alpha h_D}{8} Ei(-\frac{r_D^2}{4t_D}) +$$

$$\int_{t_{Dss}}^{t_D} 8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2 t_D}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\sin\frac{n\pi x_{wD}}{x_{eD}}\sin\frac{n\pi x_D}{x_{eD}}.\exp(-\frac{m^2\pi^2 t_D}{y_{eD}^2})\sin\frac{m\pi y_{wD}}{y_{eD}}\sin\frac{m\pi y_D}{y_{eD}}.$$

$$\exp(-\frac{l^2\pi^2 t_D}{h_D^2})\sin\frac{l\pi z_D}{h_D}\sin\frac{l\pi z_{wD}}{h_D} \qquad (3.6)$$

From (3.6), it follows that the pressure derivative can be derived as [9, 10].

$$t_D\frac{\partial p_D}{\partial t_D} = \frac{h_D}{4}e^{-r_D^2/4t_D} + 8\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{l=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2 t_D}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\sin\frac{n\pi x_{wD}}{x_{eD}}\sin\frac{n\pi x_D}{x_{eD}}.\exp(-\frac{m^2\pi^2 t_D}{y_{eD}^2})\sin\frac{m\pi y_{wD}}{y_{eD}}\sin\frac{m\pi y_D}{y_{eD}}.$$

$$\exp(-\frac{l^2\pi^2 t_D}{h_D^2})\sin\frac{l\pi z_D}{h_D}\sin\frac{l\pi z_{wD}}{h_D}. \qquad (3.7)$$

4.0 Dimensionless pressure and dimensionless pressure derivative computations

Equation (3.6) is solved numerically using Gauss-Legendre and Gauss-Laguerre quadratures [9]. Values of l, *m*, and *n* are set as unity for wellbore computation, where only one image is obtainable. Several values of external dimensionless parameters, i.e., x_{eD}, y_{eD}, of the reservoir are tested. Wellbore dimensionless radius, r_{wD}, is assumed to be the same as wellbore width, y_{wD}. Furthermore, $z_D = z_{wD} + r_{wD}$ (well location along the z-axis). An isotropic reservoir is assumed. Hence, from the definition of dimensionless parameters, $L_D = 1/h_D$. In addition, values of $x_{eD} > 2.0$)mean incomplete well penetration while lower values mean complete penetration. In all computations, central well location is assumed, i.e., $z_D = 0.5h_D$. Both infinite conductivity ($x_D = 0.732$) and uniform flux conditions ($x_D = 1.0$ and 0.0) were tested. For each set of parameters, a corresponding value of t_{Dss} is computed using (3.5). Dimensionless pressure derivatives were computed directly from (3.7).

5.0 Results and discussion

Several dimensionless wellbore radii r_{wD} = 0.002, 0.0002, 0.00005, 0.0005 and 0.0001 were considered. Dimensionless radii of 0.0001, 0.00005 and 0.0005 were tested for the purpose of validating results with those of [4]. Dimensionless reservoir thickness, h_D = 1.0, 2.0, 25 and 10 (corresponding to dimensionless wellbore length L_D of 1, 0.5, 0.04 and 0.1, respectively) were also selected. These values were also used in [4]. Points of perforations along the axis of the well length $x_D = x_{wD}$. Values of $x_D = 0.732$ for infinite conductivity and x_D =0.0 and1.0 for uniform flux were considered for every variation in both reservoir and wellbore parameter. External reservoir dimensionless lengths were selected to simulate rectangular and square geometry.

5.1 Effects of dimensionless well length, L_D

As shown in Table 5.1, dimensionless presure drops generally increase with increasing dimensionless well length (increasing dimensionless pay thickness). Conversely, the dimensionless pressure drops increase with reservoirs with thinner pay for an isotropic reservoir. Also, dimensionless pressure gradients increase with increasing dimensionless well length. The dimensionless time for attainment of steady state (arrival time of external fluid) increases with shorter well lengths or the thicker the reservoir. This means, however, that longer wells give more oil production per day per psi pressure drop.

Table 5.1: Effects of dimensionless reservoir thickness, h_d, on dimensionless pressure and their derivatives

Dimensionless Time, t_D	**$h_D = 3$**		**$h_D = 4$**		**$h_D = 5$**		**$h_D = 10$**	
	p_D	**$t_D p_D'$**	**p_D**	**$t_D p_D'$**	**p_D**	**$t_D p_D'$**	**p_D**	**$t_D p_D'$**
0.00001	5.7877	0.7498	7.7169	0.9998	9.6461	1.2497	19.2922	2.4994
0.0001	7.5146	0.75	10.0195	1.0	12.5243	1.25	25.0486	2.4999
0.001	9.2415	0.75	12.3220	1.0	15.4026	1.25	30.8051	1.25
0.01	10.9685	0.75	14.6246	1.0	18.2808	1.25	36.5616	1.25
0.1	12.6954	0.75	16.9272	1.0	21.1590	1.25	42.3180	1.25
1	14.4223	0.75	19.2298	1.0	24.0372	1.25	48.0745	1.25
10	163.149	0.75	21.5324	1.0	26.9155	1.25	53.8310	1.25
100	178.806	0.6837	23.8350	1.0	29.7937	1.25	59.5874	1.25

5.2 Effects of dimensionless Wellbore radius, $r_{w\,D}$

For a constant dimensionless pay thickness h_D =0.04 (L_D =25), reservior geometry (x_{eD}= 10 and y_{eD}=1000), dimensionless pressure and derivatives were computed for $r_{WD} = 5 \times 10^{-5}$, 5×10^{-4} and 10^{-4} and the results are tabulated in Table 5.2**.** It is observed that smaller wellbore dimensionless radii give larger dimensionless wellbore pressure. It should be noted that r_{WD} values do not affect t_{Dss} . All the dimensionless wellbore pressures obtained are similar to those of [4] at $t_D < t_{Dss}$. As usual, all the derivatives obtained show various degrees of decline from maximum points at p_D values beyond $t_D \geq t_{Dss}$. However, all the derivatives obtained are unaffected by dimensionless radius.

For all h_D, $(\Delta p_D / \Delta 1n\, t_D) \approx 0.575\, h_D$ and $t_D\, \partial\, p_D / \partial t_D \approx 0.25 h_D$ before $t_D = t_{DSS}$ for all r_{wD}, irrespective of point of perforation and reservoir geometry tested, especially at early flow times; i.e., at $t_D < t_{Dss}$. These relationships can thus be used to establish the actual productive dimensionless length, L_D, exposed to flow.

Table 5.2: Effects of dimensionless wellbore radius, r_{wd} *on* dimensionless pressure and their derivatives

Dimensionless Time, t_D	**$r_{wD} = 5 \times 10^{-5}$**		**$r_{wD} = 5 \times 10^{-4}$**		**$r_{wD} = 1 \times 10^{-4}$**	
	p_D	**$t_D p_D'$**	**p_D**	**$t_D p_D'$**	**p_D**	**$t_D p_D'$**
0.00001	2.27580	0.25	1.1245	0.2484	1.9292	0.2499
0.0001	2.8514	0.25	1.7001	0.2498	2.5049	0.25
0.001	3.4271	0.25	2.2758	0.25	3.0805	0.25
0.01	4.0027	0.25	2.8514	0.25	3.6562	0.25
0.1	4.5784	0.25	3.4271	0.25	4.2318	0.25

1	5.1540	0.25	4.0027	0.25	4.8074	0.25
10	5.7297	0.25	4.5784	0.25	5.3831	0.25
100	5.7329	0.0253	4.5816	0.0253	5.3863	0.0253

5.3 Effects of reservoir geometry

For a fixed dimensionless wellbore radius, dimensionless wellbore pressures and dimensionless pressure derivatives were computed for various ratios of x_{eD}/y_{eD}. The results are shown in Figures 5.1 to 5.8. The figures show that generally both rectangular and square reservoir geometry yield the same results at early flow dimensionless times. But, at late dimensionless times, rectangular geometry with larger values dimensionless length x_{eD} offer larger dimensionless pressures (i.e., larger drawdown), than larger dimensionless width y_{eD} is. This therefore means that the length of the well should always be aligned with the length axis of the reservoir pattern for optimum oil recovery. However, the time to attainment of steady state is shorter for smaller reservoir area. All the derivatives obtained conform to $t_D\partial p_D/\partial t_D \approx h_D/4$ for all $t_D \le t_{Dss}$ as discussed earlier.

5.4 Effects of infinite conductivity and uniform flux completion

Infinite conductivity (x_D = 0.732) and uniform flux (x_D = 0.0 and 1.0) completions were tested for their effects on wellbore productivity, just as other effects were investigated. In all the results obtained, there is no difference in dimensionless pressure drops and dimensionless pressure derivatives for all the cases. Results obtained are, however, slightly different from those in [4] (for a model with a least one sealing external boundary), but only at early dimensionless times. Therefore, for a reservoir experiencing complete external fluid influx, a horizontal well drilled in such reservoir (1) could be perforated anywhere along the length of the well in terms of productivity consideration, and (2) attainment of steady-state does not depend on perforation point. Dimensionless derivatives are also unaffected by perforation points.

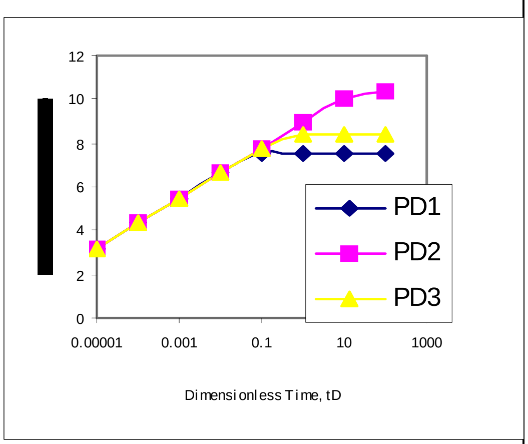


Figure 5.1: Dimensionless pressure for h_D = 2.0, r_{wD} =0.0002;
p_{D1}: $x_{eD} = y_{eD}$ = 1.0
p_{D2}: $x_{eD} = y_{eD}$ = 10.0
p_{D3}: x_{eD} = 10.0, y_{eD} = 1.0

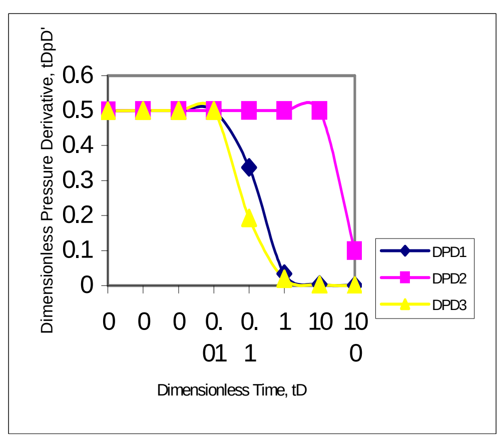
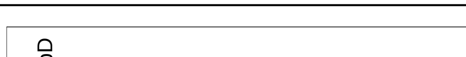
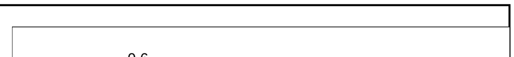


Figure 5.2: Dimensionless Pressure Derivative for h_D = 2.0, r_{wD} =0.0002;
p_{D1}: $x_{eD} = y_{eD}$ = 1.0
p_{D2}: $x_{eD} = y_{eD}$ = 10.0
p_{D3}: x_{eD} = 10.0, y_{eD} = 1.0





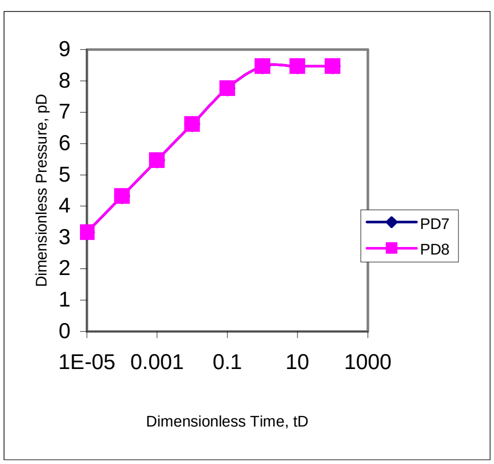


Figure 5.5: Dimensionless Pressure for $h_D = 2.0$, $r_{wD} = 0.0002$;
p_{D7}: $x_{eD} = 1$, $y_{eD} = 100$
p_{D8}: $x_{eD} = 1$, $y_{eD} = 1000$

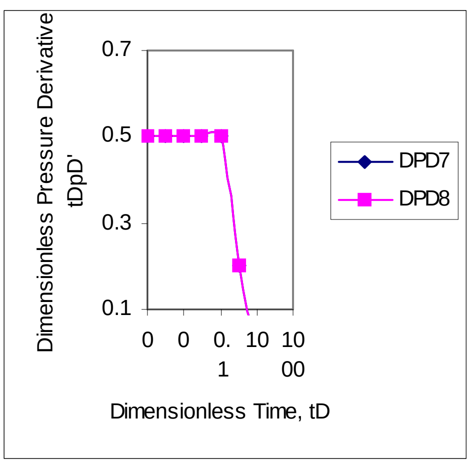


Figure 5.6: Dimensionless pressure derivative for $h_D = 2.0$, $r_{wD} = 0.0002$;
p_{D7}: $x_{eD} = 1$, $y_{eD} = 100$
p_{D8}: $x_{eD} = 1$, $y_{eD} = 1000$

5.5 Effect of well stand-off

The effects of well stand-off, z_{wD}, were investigated by computing dimensionless pressures and derivatives for z_D = 0.5, 0.25, 0.125 and 0.0625. The results are shown in Table 5.3. For all stand-off considered, the results remain the same showing that well stand-off does not affect well productivities and pressure derivatives. However, depending on the properties of the encroaching fluid, well stand off is usually decided to delay breakthrough into the well in spite of results obtained here analytically. Similar results are obtained in [4] and [9].

Table 5.3: Effects of dimensionless well stand-off on dimensionless pressures and dimensionless pressure derivatives

Dimensionless Time, t_D	Dimensionless Pressure and Dimensionless Pressure Derivative, p_D ($t_D\, p_D$')

0.00005	0.0722(0.0100)	0.0722(0.0100)	0.0722(0.0100)	0.0722(0.0100)
0.0001	0.1002(0.0100)	0.1002(0.0100)	0.1002(0.0100)	0.1002(0.0100)
0.001	0.1232(0.0100)	0.1232(0.0100)	0.1232(0.0100)	0.1232(0.0100)
0.01	0.1462(0.0100)	0.1462(0.0100)	0.1462(0.0100)	0.1462(0.0100)
0.1	0.1511(0.0016)	0.1511(0.0016)	0.1511(0.0016)	0.1511(0.0016)
1	0.1511(0.0002)	0.1511(0.0002)	0.1511(0.0002)	0.1511(0.0002)
10	0.1511(0)	0.1511(0)	0.1511(0)	0.1511(0)
100	0.1511(0)	0.1511(0)	0.1511(0)	0.1511(0)
1000	0.1511(0)	0.1511(0)	0.1511(0)	0.1511(0)

6.0 Conclusion

When the drive energy for oil production in a horizontal well is derived from external fluids in all directions, this study reveals important operational factors that may affect well productivity and clean oil production. Dimensionless wellbore pressures and their derivatives were studied to understand how they are affected by both wellbore and reservoir properties. The following conclusions can be drawn from the results obtained.

(1) Both drilling and well completion practices can influence the period of clean oil production.

(2) For all h_D, $(\Delta p_D/\Delta \ln t_D) \approx 0.575h_D$ and $t_D\partial p_D/\partial t_D \approx h_D/4$ at $t_D < t_{Ds}$, irrespective of x_{eD}, y_{eD} x_D and y_{wD}.

(3) Well eccentricity plays no significant role in influencing horizontal well productivity for all parameters considered, but has to be suitably determined for optimum delay of external fluid breakthrough.

(4) Reservoir geometry affect well productivity only at late dimensionless time and offers larger productivity if $x_{eD} > y_{eD}$.

(5) The point of the perforations does not affect well productivity for the model under study.

(6) Smaller dimensionless well radii yield larger well productivity than larger wells.

References

[1] Papatzacos, P., Herring, T.R., Martinsen, R. and Skjaeveland, S.M.: "Cone Breakthrough Time for Horizontal Wells," SPERE, Trans., AIME, 291, p.311 – 318.

[2] Papatzacos, P.: "Gas Coning by a Horizontal Well as a Moving Boundary Problem," Proc., European Conference on Mathemaatics of Oil Recovery, Cambridge, July 25 – 27, 1989.

[3] Kuchuck, F.J., Goode, P.A., Wilkinson, D.J. and Thambynayagam, R.K.M.: "Pressure-Transient Behaviour for Horizontal Wells with and without a Gas Cap or Aquifer," SPERE, Trans., 1991, AIME, 291.

[4] Ozkan, E. and Raghavan, R.: "Performance of Horizontal Wells Subject to Bottom Water Drive," SPERE (Aug. 1990) 375 – 383, *Trans*., 289.

[5] Adewale, E. S. Pressure Gradient and Pressure Derivative Characteristics of a Horizontal Well subject to Bottom Water Drive Mechamism. Journal of the Nigerian Association of Mathematics Physics, Vol. 12, 2008.

[6] Adewole, E.S. Rai, B.M. and Audu, T.O.K.: "Breakthrough Time in a Horizontal Well Subject to Simultaneous Bottom Water and Gas Cap Drives," *Nigerian Journal for Industrial Systems and Studies*, Vol. 2, No.3, 2003, p. 39-46.

[7] Gringarten, A.C. and Ramey, H.J.Jr.: "The Use of Source and Green's Functions in Solving Unsteady Flow Problems in Reservoirs," SPE *Trans*., AIME, 1973, 225-285.

[8] Carslaw, H.S. and Jaeger, J.C.: *Conduction of Heat through Solids*, 2nd Ed. Oxford University Press, London, England, 1959.

[9] Bourdet, D., Ayoub, J. A. and Pirard, Y. M. Use of the pressure derivative in well test interpretation, SPE Formation Evaluation June 1989, 293 – 302ar, A. Application of the PD function to interference Analysis, IPT 1990, 1465 - 70

[10] Tiab, D. and Ku

[11] Adewole, E.S.: "The Use of Gauss-Legendre and Gauss-Laguerre Quadratures in Solving Problems involving Oil Flow in Horizontal Wells, *Abacus: J. of the Mathematical Ass. of Nigeria*, vol. 34, Number 2B, Mathematical Series, Sept. 2007, p. 380-386.

[12] Ozkan, E., Raghavan, R. and Joshi, S.: "Horizontal Well Pressure Analysis," SPERE, Dec. 1989, Trans., AIME, 567 – 575.