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Viscous dissipation effect on the flow through a horizontal porous channel with temperature dependent viscosity

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Abstract

This work investigates the flow of fluid through a horizontal channel filled with a porous media with a temperature dependent viscosity. The influence of Darcy number on the velocity and temperature was thoroughly investigated. It is observed that high Darcy number leads to a higher velocity and that velocity is parabolic while reversal flow takes place at low Darcy number, while at very low Darcy number, oscillation and instabilities of flow is observed. It is also observed that as the brinkman number increases the temperature profile increases.

1.0 Introduction

The growing interest in the field of flow through porous media stems from the fact that heat and mass transfer occurs in many engineering applications, geophysical and biological applications in which porous media are present. By using a generalized form of Darcy's Law where the convective acceleration and viscous stress are important. Raptis [3] studied the viscous boundary through a very porous medium bounded by a horizontal semi-infinite plate. Kafoussias [2] studied the flow through a porous medium in the presence of heat transfer but neglected the viscous dissipation effect. Gideon and Eletta [1] further studied the viscous dissipation effect on the flow through a very porous media. While flow through parallel ducts has engaged the attention of several researchers, horizontal parallel ducts filled with porous materials in the presence of viscous dissipation have been much neglected. But the present paper is focused on studying the effect of viscous dissipation on the flow through a parallel duct through a porous media with a temperature dependent viscosity.

2.0 Problem formulation

Consider a laminar flow between two horizontal parallel walls. The lower wall is fixed while the upper wall moves at a speed u_0 in its own plane. The temperature of the two walls are kept constant at T_0. If the speed u_0 is sufficiently high, dissipation caused by shear can affect the flow and temperature distribution.

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In the region $0 < y < a$. The flow is one-dimensional in the $x -$ direction. The viscous stress $\mu\frac{du}{dy}$ cannot vary along y if there is no pressure gradient. Conservation of momentum in the $x -$ direction demands that for a fully developed flow

$$\frac{d}{dy}\left[\mu\frac{du}{dy}\right] + \frac{\mu}{k}u = 0 \qquad (2.1)$$

On the other hand, energy conservation requires that the rate of heat diffusion be balanced by the rate of heat viscous dissipation such that

$$\alpha\frac{d^2T}{dy^2} + \mu\left[\frac{du}{dy}\right]^2 = 0 \qquad (2.2)$$

We shall consider a fluid whose viscosity varies with the local temperature according to

$$\mu = \mu_0 e^{-\alpha[T - T_0]} \qquad (2.3)$$

where T_0 is the wall temperature. As boundary conditions along the

$$\begin{aligned} & y = 0, y = a \\ & U = 0. T = T_0 \text{ on } y = 0 \\ & U = U_0, T = T_0 \text{ on } y = a \end{aligned} \qquad (2.4)$$

Using the following dimensionless variables

$$\theta = \frac{T - T_0}{T_0}, Y = \frac{y}{a}, U = \frac{u}{u_0}$$

$$\frac{d}{dy}\left[\mu_0 e^{-\beta\theta}\frac{du}{dy}\right] + \frac{\mu_0}{k}e^{-\beta\theta}U = 0 \qquad (2.5)$$

$$\Rightarrow \qquad \frac{d}{dy}\left[e^{-\beta\theta}\frac{du}{dy}\right] + \frac{e^{-\beta\theta}}{Da}u = 0 \qquad (2.6)$$

where Da = Darcy number. Similarly,

$$\frac{d^2\theta}{dy^2} + \varepsilon e^{-\beta\theta}\left[\frac{du}{dy}\right]^2 = 0 \qquad (2.7)$$

where $\varepsilon = \frac{\mu_0 U_0^2}{\alpha T_0}$ = Brinkman Number

3.0 Method of Solution

We shall use the transformations, $t = e^{-\beta\theta}$ in (2.6) such that $\theta = -\frac{1}{\beta}\ln t$ and we have

$$\frac{d}{dy}\left[t\frac{du}{dy}\right] + \frac{t}{Da}u = 0 \qquad (3.1)$$

This simplifies in to;

$$t\frac{d^2u}{dy^2} + \frac{dt}{dy}.\frac{du}{dy} + \frac{tu}{Da} = 0 \qquad (3.2$$

and (3.1) becomes $$\frac{K}{\beta} t \frac{d^2 t}{dy^2} - \frac{K}{\beta}\left(\frac{dt}{dy}\right)^2 + \varepsilon t^3 \left(\frac{du}{dy}\right)^2 = 0 \qquad (3.3)$$

which gives a pair of non-linear differential equations. We shall proceed to solve (3.2) and (3.3) numerically using the Runge-Kuta Method for a system of equation. We shall use $t = 1$, $u = 0$, $\theta = 0$ for $y = 0$ and $t = 1$, $u = 1$, $\theta = 0$ for $y = 1$ then

$$\frac{du}{dy} = 1 \text{ for } y = 0$$

$$\frac{du}{dy} = 0 \text{ for } y = 0 \qquad (3.4)$$

4.0 Numerical Result

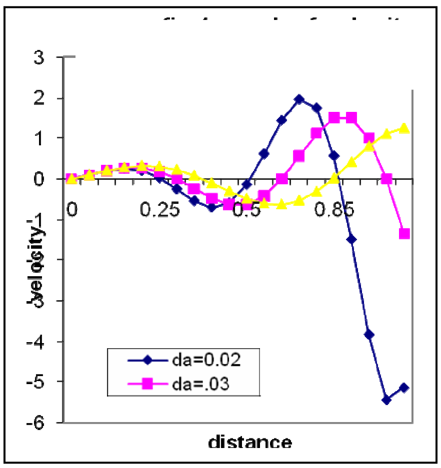


Figure 4.1: Graph of velocity against distance

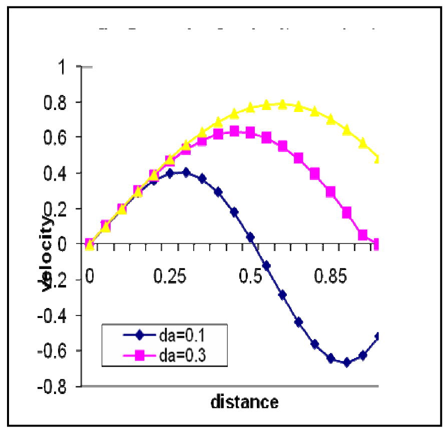


Figure 4.2: Graph of velocity against distance

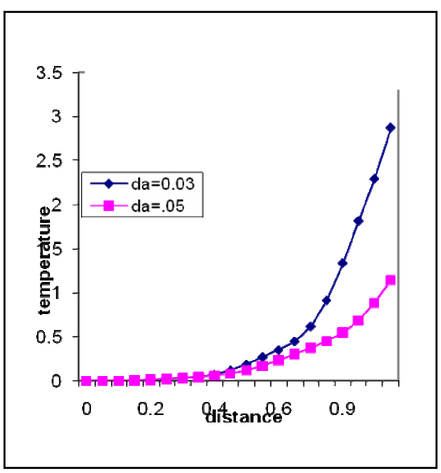


Figure 4.3: Graph of temperature against distance

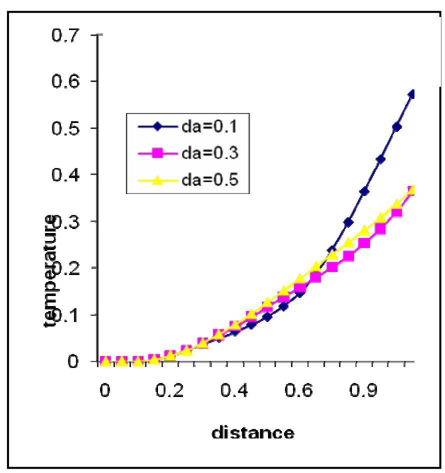
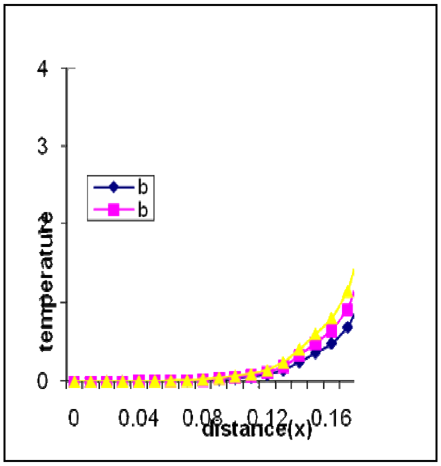


Figure 4.4: Graph of temperature against distance



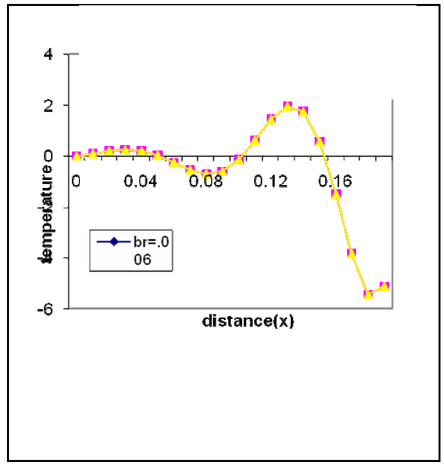


Figure 4.5: Graph of temperature against distance for difference brinkman number

Figure 4.6: Graph of velocity against distance for different brinkman number

A computer code was designed to solve (3.2) and (3.3) numerically using the Runge-Kuta Method for $\beta = 0.001, \varepsilon = 0.004$, $K = 0.1$ for various values of the Darcy number. Figure 4.1 shows the Variation of Velocity for various values of the Darcy Numbers between 0.02 and 0.05. It can be seen that reversal flow occurs compared to the fluid flow in Figure 4.2 where the flow is purely parabolic for Da = 0.3 and 0.5.

Figures 4,3 and 4.4 show the Variation of Temperature with Distance for Da = 0.05 and Da = 0.03. It can be inferred that the temperature increases with distance and as Darcy number increases, the temperature decreases.

In Figure 4.5, we show the relationship between the Temperature and Distance for various values of Brinkman number. It is observed that higher Brinkman number leads to higher temperature profile while similar flow reversal are observed for the velocity profile as seen in Figure 4.6.

5.0 Conclusion

The problem of flow through a horizontal Duct filled with porous material with viscous dissipation effect of a temperature dependent viscosity has been effectively studied. It is observed that the velocity of flow experiences a reversal flow for very low Darcy number which is as a result of the dissipation effect while the flow is purely parabolic for high Darcy number. It is also observed that the temperature increases for low Darcy number which is as a result of viscous dissipation and internal energy of the fluid while higher brinkman numbers leads to higher temperature profile.

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