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Application of modified power series method for the solution of system of differential equations

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Abstract

In this paper, we used modified power series method to solve nonlinear systems. Some examples were presented to show the ability of the method.

Keywords
Power Series, Nonlinear System, Ecosystem, Modelling.

1.0 Introduction.

The standard form of a system of ordinary differential equation of the first order with initial condition is considered as

$$\begin{aligned} \frac{dy_1}{dx} &= f_1\left(x, y_1, y_2, ..., y_n\right), y_1\left(x_0\right) = y_1 \\ \frac{dy_2}{dx_n} &= f_2\left(x, y_1, y_2, .., y_n\right), y_2\left(x_0\right) = y_2 \\ \frac{dy_n}{dx_n} &= f_n\left(x, y_1, y_2, ..., y_n\right),\ y_n\left(x_0\right) = y_n \end{aligned} \tag{1.1}$$

If the theoretical solution of the system (1.1) is $y(x)$, let y_n be an approximation to $y(x_n)$. We have: $y = \left[y_1, y_2, .., y_n\right]^T$, $f = \left[f_1, f_2, ..., f_n\right]^T$ and $y_n = \left[y_{11}, y_{21}, ..., y_{n1}\right]^T$. Where each equation represents the first derivatives of one of the unknown functions as a mapping depending on x with n unknown functions $f_1, f_2, ..., f_n$.We assumed f and y be vector function with sufficient differential [2, 4]. The solution of [1] can take the form.

$$y = y_0 + kx \tag{1.2}$$

where k is a vector function. Substituting (1,2) in (1.1) and neglecting higher orders terms, we have

$$Ak = B \tag{1.3}$$

where A and B are constant matrixes. The coefficient of x in (1.2) can therefore be determined from equation (1.2) and (1.3).

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2.0 Modified power series method

Let

$$f\left(x\right)= f_0 + f_1x+ f_2x^2 +...\left(f_n + c_1k_1 +...+ c_nk_n \right) x^n . \qquad (2.1)$$

where $c_1, c_2,\ldots,c_2$ are constant, $k_1, k_2,\ldots,k_n$ are basis vector k, n is size of vector k. Every element in y can be represented by (2.1), that is, $y_i = y_{i,0} + y_{i,1}x + y_{i,2}x^2 + ... + k_i x^n$ (2.2)

Substituting (2.2) in (1.1), we obtain $f_i \left(f_{i,n} + c_{i,1}, k + ... + c_{i,n}k_n \right) x^{n-j} + 0\left(x^{n-j+1} \right)$ (2.3)

Where f_i is the i^{th} element of $f\left(y, y', x\right)$ in 1.1 and j = 0 if $f\left(y, y', x\right) = \phi(y')$, otherwise 0.

Therefore, equation (1.3) becomes

$$A_{i,j} = c_{i,j} \qquad (2.4)$$

$$B_{i,j} = -f_{i,n} \qquad (2.5)$$

Solving equation (2.4) and (2.5) and substituting k_i into (2.2) we have y_i which are polynomial of degree n. Repeating the procedure, we get the power series solution of (1.1).

3.0 Numerical examples

***Example* 3.1.**

Consider the predator prey system:

$$x'(t) = x(t) - x(t)y(t) \qquad (3.1)$$

$$y'(t) = -y(t) - x(t)y(t) \qquad (3.2)$$

$$x(0) = 1, \quad y(0) = 0.5$$

Applying (2.2) in equations (3.1-3.2), we have

$$x_1(t) = 1 + k_1t \qquad (3.3)$$

$$y_1(t) = \frac{1}{2} + k_2t \qquad (3.4)$$

Substituting for (3.3) and (3.4) in (3.1) and (3.2), we have

$$x_1(t) = 1 + \frac{1}{2}t \qquad (3.5)$$

$$y_1(t) = \frac{1}{2} \qquad (3.6)$$

Similarly

$$x_2(t) = 1 + \frac{1}{2}t + \frac{1}{8}t^2 \qquad (3.7)$$

$$y_2(t) = \frac{1}{2} + \frac{1}{8}t^2 \qquad (3.8)$$

We therefore have;

$$x_3(t) = 1 + \frac{t}{2} + \frac{t^2}{8} - \frac{t^3}{48} \qquad (3.9)$$

$$y_3(t) = \frac{1}{2} + \frac{t^2}{8} + \frac{t^3}{48} \qquad (3.10)$$

as the fourth approximate solution to the problem

***Example* 3.2**

Consider a linear system

$$x' = y(t) \qquad (3.11)$$

$$y'(t) = 2x(t) - y(t) \qquad (3.12)$$

$$x(0)=1,\ y(0)=-1$$

Use (2.2) in (3.11) and (3.12),we have $x_1(t)=1+k_1t$ (3.13)

$$y(t)=-1+k_2t \quad (3.14)$$

Substitute equations (3.13) and (3.14), in (3.14) and (3.12), we obtained

$$x_1(t)=1-t \quad (3.15)$$

$$y_1(t)=-1+3t \quad (3.16)$$

Similarly $x_2(t)=1-t+\frac{3}{2}t^2$ (3.17)

$$y_2(t)=-1+3t-\frac{5t^3}{2} \quad (3.18)$$

and $x_3(t)=1-t+\frac{3t^2}{2}-\frac{5t^3}{6}$ (3.19)

$$y_3(t)=-1+3t+\frac{5t^2}{2}-\frac{11t^3}{6} \quad (3.20)$$

$$x_4(t)=-1+t+\frac{3}{2}t^2-\frac{5t^3}{6}+\frac{11t^4}{24} \quad (3.21)$$

$$y_4(t)=-1+3t-\frac{5t^2}{2}+\frac{11t^3}{6}-\frac{21t^4}{24} \quad (3.22)$$

also $x_5(t)=1-t+\frac{3t^2}{2}-\frac{5t^3}{6}+\frac{11t^4}{24}-\frac{21t^5}{120}$ (3.23)

$$y_5(t)=-1+3t-\frac{5t^2}{2}+\frac{11t^3}{6}-\frac{21t^4}{24}+\frac{43t^5}{120} \quad (3.24)$$

Rearranging equations (3.23) and (3,24), we have

$$x(t)=\frac{2}{3}\left(1-2t+\left(\frac{-2t}{2!}\right)^2+\frac{(-2t)^3}{3!}+\frac{1}{3}\left(1+t+\frac{t^2}{2!}+\frac{t^3}{3!}+........\right)\right) \quad (3.25)$$

$$\approx \frac{2}{3}e^{-2t}+\frac{1}{3}e^{t} \quad (3.26)$$

$$y(t)=-\frac{4}{3}\left(1-2t+\frac{(-2t)^2}{2!}+\frac{(-2t)^3}{3!}+\frac{(-2t)^4}{4!}+\frac{1}{3}\left(1+t+\frac{t^2}{2!}++\frac{t^4}{4!}+..........\right)\right) \quad (3.27)$$

$$\approx -\frac{4}{3}e^{-2t}+\frac{1}{3}e^{t}$$

This is the exact solution.

Example **3.3**

Consider the following system of non-homogeneous differential equations [2].

$$\frac{dy_1}{dx}=y_3-\cos x,\ y_1(0)=1 \quad (3.28)$$

$$\frac{dy_2}{dx}=y_3-e^{x} \quad ,\ y_2(0)=0 \quad (3.29)$$

$$\frac{dy_3}{dx} = y_1 - y_2, y_3(0) = 2 \quad (3.30)$$

Applying the method $y_{1,1} = 1 + k_1 x$ (3.31)

$$y_{2,1} = k_2 x \quad (3.32)$$

$$y_{3,1} = 2 + k_3 x \quad (3.33)$$

Substituting for (3.31-3.33) in (3.28-3.30), we have $y_{1,1} = 1 + x$ (3.34)

$$y_{2,1} = x \quad (3.35)$$

$$y_{3,1} = 2 + x \quad (3.36)$$

Second iteration becomes $y_{1,2} = 1 + x + \frac{1}{2}x^2$ (3.37)

$$y_{2,2} = x \quad (3.38)$$

$$y_{3,2} = 2 + x \quad (3.39)$$

Then third iteration $y_{1,3} = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots$ (3.40)

$$y_{2,3} = x - \frac{1}{6}x^3 + \ldots. \quad (3.41)$$

$$y_{3,3} = 2 + x + \frac{1}{6}x^3 + \ldots \quad (3.42)$$

Similarly, $y_{1,4} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \ldots$ (3.43)

$$y_{2,4} = x - \frac{x^3}{6} + \frac{x^4}{24} + \ldots \quad (3.44)$$

$$y_{3,4} = 2 + x + \frac{x^3}{6} + \frac{x^4}{12} + \ldots \quad (3.45)$$

The iteration converges to $y_1 \approx e^x$ (3.46)

$$y_2 \approx \sin x \quad (3.47)$$

$$y_3 \approx e^x + \cos x \quad (3.48)$$

Equations (3.46-3.48) are the exact solution to equations (3.1-3.2).

4.0 Conclusion

In this paper, we applied the power series method for solving linear and nonlinear systems of boundary value problems. The method gives more realistic series solution that converge rapidly [7].

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