Journal of the Nigerian Association of Mathematical Physics
Volume **15 (November, 2009), pp 37 - 46**
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Effects of static pre-loading on the dynamic stability of a column on nonlinear foundations stressed by a step load

[1]A. M. Ette and [2]W. I. Osuji
[1,2]Department of Mathematics and Computer Science
Federal University of Technology Owerri
Owerri, Imo State, Nigeria

Abstract

This paper presents, from strictly analytical consideration, the dynamic analysis of a finite column stressed by a step load but in the presence of a previously imposed static load. The results show that (a) the dynamic buckling load for this type of loading is relatively higher than that of a similar column stressed by a step load without pre-static loading. Thus, the step load provides a lower bound for such loads (b) it is possible to relate the dynamic buckling load to its static equivalent and thereby by-passing the labour of repeating the entire process for different imperfection parameters.

1 0 Introduction

Analyses of the dynamic stability of structural materials under various dynamic loading conditions have received serious attention in the past forty years or so. Relevant researches have led to the conclusion that the dynamic buckling load is strongly dependent on the initial geometric imperfection and the duration of load application, among other parameters. However, in almost all reported investigations concerning columns and for whatever loading history ever prescribed, it has , to our knowledge, always been assumed that the structure is free from any form of static pre-loading. Thus the assumption often made is that the structure is suddenly trapped by the stipulated load applied just after the initial time. In this investigation, we however take a detour from what appears to be the usual assumption and consider the dynamic stability of an imperfect finite column resting on nonlinear elastic foundations, trapped by a step load but in the presence of a previously applied static pressure. We note that the static pre-loading phenomenon is of utmost practical importance and examples of this type of loading include (a) a submarine resting on the bottom of an ocean (static pre-loading condition), but subjected to a sudden blast loading (depth charge), (b) airplanes or fuel tanks under gust loading, (c) submerged pipelines under impact loading, and (d) jet engines casing under rotor imbalance, among others. Since the main thrust of this investigation is the dynamic

[1]Corresponding author:
[1]e-mail address: tonimonsette@yahoo.com
[1]Telephone: +234-08037760191

analysis, it is hereby suggested that in an attempt to model the static (or quasi-static) pre-loading condition, the static pressure must be gradually applied at such a rate as to keep the inertial effect as negligible as possible so that, in the final analysis, the static pre-loading emanates as static analysis. In other words, the column is first loaded slowly and statically to a level below the static buckling load, and, subsequently, is subjected to a sudden step load of constant magnitude for all time.

The effect of static pre-loading on the dynamic stability of structures was first investigated by Simitses [1], while Tanov and Tabiei [2] used numerical investigations to study the beheviour of cylindrical laminated shells subjected to a sudden applied lateral pressure. Other relevant studies include Gu et al [3], Huyan and Simitses [4], Shaw et al [5] and Giliat and Aboudi [6], among others.

The analysis presented here is organized in the following order: (a) we first analyze the static behaviour of the structure under a static pre-loading condition and obtain both the maximum lateral displacement w_a and the static buckling load λ_s in this case. (b) next, we analyze the dynamic response of the system and obtain both the maximum displacement U_a and the dynamic buckling load $\lambda_D^{(1)}$ of the structure under a step load, assuming homogeneous initial conditions. (c) lastly, the effect of static pre-loading is obtained as a superposition of (a) and (b) to determine the dynamic buckling load $\lambda_D^{(2)}$ which follows [7] from the maximization

$$\frac{d\lambda}{dw_c}=0,\ \ w_c=w_a(\lambda)+U_a(\lambda) \tag{1.1}$$

where both $w_a(\lambda)$ *and* $U_a(\lambda)$ depend on the load parameter λ. We define $\lambda_D^{(2)}$ as the maximum load parameter for which the solution of the pre-statically loaded column under a step load remains bounded for all time.

2.0 General formulation

The usual differential equation [7] satisfied by the lateral displacement $W(X,T)$, of a finite column on a nonlinear (cubic) elastic foundation under a dynamic load $P(T)$, is

$$m_0W_{,TT}+EIW_{,XXXX}+2P(T)W_{,XX}+k_1W-\alpha k_3W^3=-2P(T)\frac{d^2\bar{W}}{dX^2}, T > 0 \tag{2.1}$$

$$0 < X < \pi \tag{2.2}$$

where m_0 is the mass per unit length, EI is the bending stiffness, E *and* I are the young's modulus and moment of inertia respectively and X *and* T are the axial coordinate and the time variable respectively, while a subscript following a comma indicates partial differentiation. The column rests on a nonlinear (cubic) elastic foundation that produces a restoring force per unit length of $k_1W-\alpha k_3W^3$, where $k_1>0, k_3>0$ are constants and α is the imperfection sensitivity parameter which is such that if $\alpha=1$, the nonlinear elastic foundation is said to be a "softening" type where as if $\alpha=-1$, the foundation is said to be a "hardening" type. $\bar{W}$ is a stress-free twice differentiable imperfection, and all nonlinearities greater that the cubic are neglected. We now introduce the following nondimensional quantities:

$$X=\left(\frac{k_1}{E}\right)^{\frac{1}{4}}x\ ,w=\left(\frac{k_3}{k_1}\right)^{\frac{1}{2}}W\ ,\in\overline{w}=\left(\frac{k_3}{k_1}\right)^{\frac{1}{2}}\overline{W}\ ,\ t=\left(\frac{k_1}{m_0}\right)^{\frac{1}{2}}T,\lambda f(t)=\frac{P(T)}{2(EIk_1)^{\frac{1}{2}}}\quad(2.3)$$ O

n substituting (2.3) into (2.1) and (2.2), we have

$$w_{,tt}+w_{,xxxx}+2\lambda f(t)w_{,xx}+w-\alpha w^3=-2\lambda f(t)\in\frac{d^2\overline{w}}{dx^2},t>0,\ 0<x<\pi\quad(2.4a)$$

$$w=w_{,xx}\ at\ x=0,\pi\quad(2.4b)$$

where $\in$ is a small parameter satisfying the inequality $0<\in<1$ while λ similarly satisfies the inequality $0<\lambda<1$.

3.0 Static Pre-loading analysis

In this case, we ignore the inertia term as well as set $f(t)\equiv 1$ in (2.4a), and get

$$w_{,xxxx}+2\lambda w_{,xx}+w-\alpha w^3=-\ 2\lambda\in\frac{d^2\overline{w}}{dx^2},0<x<\pi;w=w_{,xx}\ at\ x=0,\pi\quad(3.1)$$

The classical buckling load λ_C is obtained from an eigen-value problem obtained by letting $\in=0$, and ignoring the nonlinear term in (3.1), to get $\lambda_C=1$. Based on the boundary conditions in (3.1), we let

$$\overline{w}=\overline{a}_m\sin mx,m=1,2,3\Lambda\ ,\left|\overline{a}_m\right|<1\ \forall m,\ w=\sum_{i=1}^{\infty}w^{(i)}(x)\in^i\quad(3.2)$$

On substituting (3.2) into (3.1), we get the following equations

$$Lw^{(1)}\equiv w_{,xxxx}^{(1)}+2\lambda w_{,xx}^{(1)}+w^{(1)}=\ -\ 2\overline{a}_m\ \lambda m^2\sin mx\quad(3.3)$$

$$Lw^{(2)}=0\quad(3.4)$$

$$Lw^{(3)}=\alpha w^{(1)^3}\quad(3.5)$$

For the solution, we let

$$w^{(i)}(x)=\sum_{n=1}^{\infty}w_n^{(i)}\sin nx\quad(3.6)$$

On substituting (3.6) into (3.3) we have

$$w_m^{(i)}=\frac{2m^2\lambda\overline{a}_m}{\varphi_m^2},\varphi_m^2=\left(m^4-2m^2\lambda+1\right);\ w^{(1)}=w_m^{(i)}\sin mx\quad(3.7)$$ N

ext we substitute (3.6) and (3.4),for $i=2$, and get

$$w^2(x)=0\quad(3.8)$$

On simplifying (3.5), we have

$$Lw^{(3)}=\frac{\alpha}{4}w_m^{(1)^3}\left(3\sin mx-\sin 3mx\right)\quad(3.9)$$

When $n=m\ \ and\ \ n=3m$, we solve (3.9) to get the following results

$$w_m^{(3)}=\frac{3\alpha\ w_m^{(1)^3}}{4\phi_m^2}\ \ and\ \ w_m^{(3)}=-\frac{\alpha\ w_m^{(1)^3}}{4\phi_m^2};\phi_{3m}^2=\left(81m^4-18\lambda m^2+1\right)\quad(3.10)$$

where $w_{3m}^{(3)}$ is the value of $w^{(3)}$ *when* $n = 3m$

Thus we write

$$w_a = \in w_m^{(1)} + \in^3 \left(w_m^{(3)} \sin mx + w_m^{(3)} \sin 3x \right) + \text{L} \qquad (3.11)$$

We easily observe that at $x = \dfrac{\pi}{2m}$, $w(x)$ attains the maximum w_a, given by

$$w_a = \in w_m^{(1)} + \in^3 \left(w_m^{(3)} - w_{3m}^{(3)} \right) + \text{L} \qquad (3.12)$$

The buckling load λ_s, [7] is obtained from the maximization $\dfrac{d\lambda}{dw_a} = 0$. This is however preceded by a reversal of the series (3.12) in the form

$$\in = C_1 w_a + C_3 w_a^3 + \text{L} \qquad (3.13)$$

By substituting in (3.13) for w_a and equating the coefficients of

$\in$ *and* $\in^3$, we have

$$C_1 = \frac{1}{w_m^{(1)}}, C_3 = -\left(\frac{w_m^{(3)} - w_{3m}^{(3)}}{w_m^{(1)4}} \right) \qquad (3.14)$$

The maximization $\dfrac{d\lambda}{dw_a} = 0$ easily follows from (3.13) to yield, after some simplification

$$\in = \frac{2}{3\sqrt{3}} \sqrt{\left(\frac{w_m^{(1)}}{w_m^{(3)} - w_{3m}^{(3)}} \right)} \qquad (3.15)$$

which is evaluated at $\lambda = \lambda_S$. On simplifying (3.15), we get

$$\left(m^4 - 2m^2 \lambda_S + 1\right)^{\frac{3}{2}} = \frac{9\alpha^{\frac{1}{2}} m^2 \lambda_S \in \bar{a}_m}{2} \left[1 - \frac{1}{3} \left(\frac{\varphi_m(\lambda_S)}{\varphi_m(\lambda_S)} \right)^2 \right]^{\frac{1}{2}} \qquad (3.16)\text{F}$$

or $m = 1$, we get

$$(1 - \lambda_S)^{\frac{3}{2}} = \frac{9\alpha^{\frac{1}{2}} \lambda_S \bar{a}_1 \in}{4\sqrt{2}} \left[1 - \frac{1}{3} \left(\frac{1 - \lambda_S}{41 - 9\lambda_S} \right) \right]^{\frac{1}{2}} \qquad (3.17)$$

4.0 Step loading analysis

The step loading analysis is obtained by setting $f(t) = 1$ in (2.4) and insetting homogeneous initial conditions thus

$$w,_{tt} + w,_{xxxx} + 2\lambda w,_{xx} + w - \alpha w^3 = -2\lambda \in \frac{d^2 \bar{w}}{dx^2}, t > 0 \qquad (4.1)$$

$$0 < x < \pi ; w = w,_{xx} \; at \quad x = 0, \pi \qquad (4.2)$$

$$w(x,0) = w,_{xx}(x,0) = 0, \quad 0 < x < \pi \qquad (4.3)$$

We let $\tau = \in^2 t$, $w(x,t) = U(x,t,\tau,\in)$ so that

$$w,_t = U,_t + \in^2 U,_\tau \; ; \; w,_{tt} = U,_{tt} + \in^4 U,_{\tau\tau} ; \qquad (4.4)$$

By letting

$$U(x,t,\tau,\in)=\sum_{i=i}^{\infty}U^{(i)}(x,t,\tau)\in^{i} \qquad (4.5)$$

and substituting (4.4) and (4.5) into (4.1) and equating the coefficients of $\in^{i}$, $i=1,2,3\Lambda$, we have the following equations

$$MU^{(1)}\equiv U_{,tt}^{(1)}+U,_{xxxx}^{(1)}+2\lambda U,_{xx}^{(1)}+U^{(1)}=-2\lambda\frac{d^{2}\overline{w}}{dx^{2}} \qquad (4.6)$$

$$MU^{(2)}=0 \qquad (4.7)$$

$$MU^{(3)}=\alpha U^{(3)^{3}} \qquad (4.8)$$

$$U^{(i)}(x,0,0)=0\ ,i=1,2,3,\Lambda\ ;\ U,_{,t}^{(1)}(x,0,0)=0\ U,_{,t}^{(2)}(x,0,0)=0,\ \ U,_{,t}^{(3)}(x,0,0)+U,_{,t}^{(1)}(x,0,0)=0 \qquad (4.9)$$

$$U^{(i)}=U,_{xx}^{(i)}=0\ \ at\ \ \ x=0,\pi;\ i=1,2,3,\Lambda \qquad (4.10)$$

We still assume the first equation in (3.2) and now let

$$U^{(i)}(x,t,\tau)=\sum_{n=1}^{\infty}U_{n}^{(i)}(t,\tau)\sin nx \qquad (4.11)$$

If we substitute (4.11) into (4.6), using the first equation in (3.2),we have ,for $n=m$,

$$U_{m,tt}^{(1)}+\varphi_{m}^{2}U_{m}^{(1)}=2m^{2}\lambda\overline{a}_{m};\ U_{m}^{(1)}(0,0)=U_{m,\ t}^{(1)}(0,0)=0 \qquad (4.12)$$

where φ_{m}^{2} is an in (3.7) and is such that $\varphi_{m}^{2}>0\ \forall\, m=1,2,3,\Lambda$. On solving (4.12) we get

$$U_{m}^{(1)}(t,\tau)=\alpha_{m}^{(1)}(\tau)\cos\varphi_{m}t+\beta_{m}^{(1)}(\tau)sin\varphi_{m}t+B_{m}\ ;\ B_{m}=\frac{2m^{2}\overline{a}_{m}}{\varphi_{m}^{2}},\alpha_{m}^{(1)}(0)=-B_{m},\beta_{m}^{(1)}(0)=0 \qquad (4.13)$$

We now substitute (4.11) into (4.7) and get

$$U_{m}^{(2)},_{tt}+\varphi_{m}^{2}U_{m}^{(2)}=0,\ U_{m}^{(2)}(0,0)=U_{m}^{(2)},_{t}(0,0)=0 \qquad (4.14)$$

On solving (4.14), we have

$$U_{m}^{(2)}(t,\tau)=\alpha_{m}^{(2)}(\tau)\cos\varphi_{m}t+\beta_{m}^{(2)}(\tau)\sin\varphi_{m}t,\alpha_{m}^{(2)}(0)=\beta_{m}^{(2)}(0)=0 \qquad (4.15)$$

We next substitute (4.11) into (4.8),and, for $n=m$ *and* $n=3m$ we get the following equations

$$U_{m}^{(3)},_{tt}+\varphi_{m}^{2}U_{m}^{(3)}=\frac{3\alpha U_{m}^{(1)3}}{4};U_{m}^{(3)}(0,0)=0,\ U_{m\ ,\ t}^{(3)}(0,0)+U_{m,\tau}^{(1)}(0,0)=0 \qquad (4.16)$$

and

$$U_{m}^{(3)},_{tt}+\varphi_{m}^{2}U_{m}^{(3)}=-\frac{\alpha U_{m}^{(1)3}}{4};U_{m}^{(3)}(0,0)=0,\ U_{3m,\ t}^{(3)}(0,0)=0,U_{3m,\ t}^{(3)}(0,0)=0,\ \varphi_{m}^{2}=(81m^{4}-18m^{2}\lambda+1) \qquad (4.17)$$

We consider $\varphi_{m}^{2}>0\ \ \forall m$. On simplifying the equation in (4.16), we have

$$U_{m}^{(3)},_{tt}+\phi_{m}^{2}U_{m}^{(3)}=-\frac{3\alpha}{4}\left[\ r_{0}+r_{1}\sin\phi_{m}t+r_{2}\cos\phi_{m}t+r_{3}\sin2\phi_{m}t+r_{4}\cos2\phi_{m}t+r_{5}\sin3\phi_{m}t+r_{6}\cos3\phi_{m}t\right]$$
$$+2\phi_{m}\left(\ \alpha_{m}^{(1)\prime}\ \sin\phi_{m}t-\beta_{m}^{(1)\prime}\ \cos\phi_{m}t\ \right) \qquad (4.18)$$

where $(\ )'=\dfrac{d(\ )}{d\tau}$ and

$$r_0=\left[\frac{3b_m\alpha_m^{(1)2}}{2}+B_m^3+\frac{3b_m\beta_m^{(1)2}}{2}\right], r_1=\beta_m^{(1)}\left[\frac{3\alpha_m^{(1)2}}{4}+3B_m^2+\frac{3\beta_m^{(1)2}}{4}\right] \quad (3.19)$$

$$r_2=3\alpha_m^{(1)}\left[\frac{\alpha^{(1)2}}{4}+B_m^2+\frac{\beta_m^{(1)2}}{4}\right],\ r_3=3\alpha_m^{(1)}\beta_m^{(1)}B_m\ ,\ r_4=\frac{3B_m}{2}\left[\alpha_m^{(1)2}-\beta_m^{(1)2}\right] \quad (3.20a)$$

$$r_5=\frac{\beta_m^{(1)}}{4}\left[3\alpha_m^{(1)2}-\beta_m^{(1)2}\right],\ r_6=\frac{\alpha_m^{(1)}}{4}\left[\alpha_m^{(1)2}-3\beta_m^{(1)2}\right], \quad (4.20b)$$

where

$$r_0(0)=\frac{5B_m^3}{2}, r_1(0)=0,\ r_2(0)=-\frac{9B_m^3}{2}, r_3(0)=0,\ r_4(0)=\frac{3B_m^3}{2},$$
$$r_5(0)=0, r_6(0)=-\frac{B_m^3}{4} \quad (4.21)$$

To ensure a uniformly valid solution in *t*, we equate to zero the coefficients of $\cos\phi_m t$ and $\sin\phi_m t$ (4.18), and get

$$\alpha_m^{(1)\prime}+\frac{9\alpha\beta_m^{(1)}\psi}{8\varphi_m}=0\ \ and\ \ \beta_m^{(1)\prime}-\frac{9\alpha\alpha_m^{(1)}\psi}{8\varphi_m}=,\psi(\tau)=\frac{\alpha_m^{(1)2}}{4}+B_m^2+\frac{\beta_m^{(1)2}}{4} \quad (4.22)$$ B

y multiplying the first of (4.22) by $\alpha_m^{(1)}$ and the second by $\beta_m^{(1)}$ and integrating we have

$$\alpha_m^{(1)2}+\beta_m^{(1)2}=B_m^2 \quad (4.23)$$ O

n substituting (4.23) in (4.22), as in $\psi(\tau)$, and satisfying we get

$$\alpha_m^{(1)\prime}+\frac{45\alpha\beta_m^2\beta_m^{(1)}}{32\phi_m}=0 \text{ and } \beta_m^{(1)\prime}-\frac{45\alpha\beta_m^2\alpha_m^{(1)}}{32\phi_m}=0, \quad (4.24)$$

To solve (4.24), we let $\alpha_m^{(1)}=\eta\, e^{\theta\ (\tau)},\ \beta_m^{(1)}=\zeta\, e^{\theta\ (\tau)}$ (4.25)

On substituting (4.25) into (4.24) and solving, we get

$$\theta(\tau)\pm i\mu_m\tau,\ \mu_m=\frac{45\alpha B_m^2}{32\varphi_m},\alpha_m^{(1)}=-B_m\cos(\mu_m\tau)\ ,\ \beta_m^{(1)}=-B_m\sin(\mu_m\tau) \quad (4.26a)$$ T

$$U_m^{(1)}(t,\tau)=B_m[\ 1-\cos(\varphi_m t-\mu_m\tau)] \quad (4.26b)$$

he remaining equation in (4.18) is now solved to get

$$U_m^{(3)}(t,\tau)=\alpha_m^{(3)}(\tau)\cos\phi_m t+\beta_m^{(3)}(\tau)\sin\phi_m t+$$
$$\frac{3\alpha}{4\phi_m^2}\left[r_o-\frac{r_3\sin 2\phi_m t}{3}-\frac{r_4\cos 2\phi_m t}{3}-\frac{r_5\sin 3\phi_m t}{8}-\frac{r_6\cos 3\phi_m t}{8}\right],$$
$$\alpha_m^{(3)}(0)=-\frac{195\alpha B_m^3}{128\phi_m^2},\ \beta_m^{(3)}(0)=0 \quad (4.27)$$

If we simplify the equation in (4.17), we have

$$U_{3m}^{(3)},_{tt} + \phi_3^2 U_{3m}^{(3)} = -\frac{\alpha}{4}\Big[r_0 + r_1 \sin\phi_m t + r_2 \cos\phi_m t + r_3 \sin\phi_m t + r_4 \cos 2\phi_m t + r_5 \sin 3\phi_m t + r_6 \cos 3\phi_m t \Big] \quad (4.28)$$

We solve (4.28) ,using the initial conditions in (4.17), to get

$$U_{3m}^{(3)}(t,\tau) = \alpha_{3m}^{(3)}(\tau)\cos\phi_{3m}t + \beta_{3m}^{(3)}(\tau)\sin\phi_{3m}t - \frac{\alpha}{4}\left[\frac{r_0}{\phi_{3m}^2} + \frac{1}{\phi_{3m}^2 - \phi_m^3}\{r_1 \sin\phi_m t + r_2 \cos\phi_m t\} + \frac{1}{\phi_{3m}^2 - 4\phi_m^2}\{r_3 \sin 2\phi_m t + r_4 \cos 2\phi_m t\} + \frac{1}{\phi_{3m}^2 - 9\phi_m^2}\{r_5 \sin 3\phi_m t + \cos 3\phi_m t\}\right] \quad (4.29a)$$

$$\alpha_{3m}^2(0) = \frac{\alpha B_m^3}{8}\left[\frac{5}{\phi_{3m}^2} - \frac{9}{\phi_{3m}^2 - \phi_m^3} + \frac{3}{\phi_{3m}^2 - 4\phi_m^2} - \frac{1}{2\left(\phi_{3m}^2 - 9\phi_m^2\right)}\right], \quad \beta_{3m}^{(3)}(0) = 0 \quad (4.29b)$$

We now write

$$U(x,t,\tau,\in) = \in U_m^{(1)} \sin mx + \in^3 \left(U_m^{(3)} \sin mx + U_{3m}^{(3)} \sin 3mx \right) + \Lambda \quad (4.30)$$

where $U_2^{(m)}(t,\tau) = 0$.The maximum of $U(x,t,\tau,\in)$ is obtained in space and time and the conditions for this are

$$U,_x = 0; \quad U,_t + \in^2 U,_\tau = 0 \quad (4.31)$$

and these are evaluated at x_a, t_a *and* τ_a, which are the values, at maximum displacement, of x, t *and* τ respectively. On substituting (4.30) into the first (4.31) we get

$$x_a = \frac{\pi}{2m} \quad (4.32a)$$

We now let

$$t_a = t_0 + \in^2 t_2 + \mathrm{L} \;;\; \tau_a = \in^2 t_a = \in^2\left(t_0 + \in^2 t_2 + \mathrm{L}\right) \quad (4.32b)$$

and now substitute(4.30) into the second equation in (4.31), using (4.32a), and , for terms of 0(1), we obtain

$$t_0 = \frac{\pi}{\phi_m} \quad (4.32c)$$

where we have taken the last nontrivial value of t_0. We next evaluate (4.30) at maximum values of the variables to get $U_a = U\left(x_a, t_a, \tau_a, \in\right) = 2B_m \in + \in^3\left(U_m^{(3)} - U_{3m}^{(3)}\right) + \mathrm{L}$ (4.33a)

This on simplification, gives $U_a = 2B_m \in + \frac{9\alpha B_m^3}{2\phi_m^2}\left(1 + \frac{\phi_m \Omega_m}{36}\right)\in^3 + \mathrm{L}$ (4.33b)

$$\Omega_m = \left[\frac{3\left\{\cos\left(\frac{\pi\phi_{3m}}{\phi_m}\right) - 1\right\}}{\phi_{3m}^2 - 4\phi_m^2} + 5\left\{\cos\left(\frac{\pi\phi_{3m}}{\phi_m}\right) - 1\right\} - \frac{9\left\{\cos\left(\frac{\pi\phi_{3m}}{\phi_m}\right) + 1\right\}}{\phi_{3m}^2 - \phi_m^2} + \frac{\left\{\cos\left(\frac{\pi\phi_{3m}}{\phi_m}\right) + 1\right\}}{2\left(\phi_{3m}^2 - 9\phi_m^2\right)}\right] \quad (4.33c)$$

To determine the dynamic buckling load $\lambda_D^{(1)}$ for the case of a step load without pre-loading, we follow a similar process that led from (3.13) to (3.16),and eventually get

$$\left(m^4-2m^2\lambda_D^{(1)}+1\right)^{\frac{3}{2}}=\frac{9\sqrt{3}}{2}m^2\lambda_D^{(1)}\in\bar{a}_m\alpha^{\frac{1}{2}}A_m^{\frac{1}{2}}\left(\lambda_D^{(1)}\right),\ A_m\left(\lambda_D^{(1)}\right)=\left\{1+\frac{\phi_m\left(\lambda_D^{(1)}\right)\Omega_m\left(\lambda_D^{(1)}\right)}{36}\right\} \tag{4.34}$$

For the case $m=1$, we have

$$\left(1-\lambda_D^{(1)}\right)^{\frac{3}{2}}=\frac{9\sqrt{6}}{8}\lambda_D^{(1)}\alpha^{\frac{1}{2}}\in\bar{a}_1A_1^{\frac{1}{2}}\left(\lambda_D^{(1)}\right);\ A_1^{\frac{1}{2}}\left(\lambda_D^{(1)}\right)=\left\{1+\frac{\phi_1\left(\lambda_D^{(1)}\right)\Omega_m\left(\lambda_D^{(1)}\right)}{36}\right\} \tag{4.35a}$$

$$\Omega_m\left(\lambda_D^{(1)}\right)=\left[5\left\{\cos\left(\frac{\pi\left(41-9\lambda_D^{(1)}\right)}{1-\lambda_D^{(1)}}\right)-1\right\}-\frac{9}{16\left(5-\lambda_D^{(1)}\right)}\left\{\cos\left(\frac{\pi\left(41-9\lambda_D^{(1)}\right)}{1-\lambda_D^{(1)}}\right)+1\right\}\right.$$
$$\left.+\frac{3}{2\left(37-5\lambda_D^{(1)}\right)}\left\{\cos\left(\frac{\pi\left(41-9\lambda_D^{(1)}\right)}{1-\lambda_D^{(1)}}\right)-1\right\}-\frac{1}{128}\left\{\cos\left(\frac{\pi\left(41-9\lambda_D^{(1)}\right)}{1-\lambda_D^{(1)}}\right)+1\right\}\right] \tag{4.35b}$$

5.0 Step load superposed on pre-static load.

In this case the maximum lateral displacement w_c, is the sum of (3.12) and (4.33a) and this gives

$$w_c=\in\left(w_m^{(1)}+2B_m\right)+\in^3\left[w_m^{(3)}-w_{3m}^{(3)}+\frac{9\alpha B_m^3A_m(\lambda)}{2\phi_m^2}\right]+\ldots=3B_m\in+\frac{3\alpha B_m^3\in^3}{4\phi_m^2}\left[1-\frac{1}{3}\left(\frac{\phi_m}{\phi_{3m}}\right)^2+6A_m(\lambda)\right]+\ldots \quad 5.1)$$

te that (5.1) is similar to (3.12) hence the same procedure that led from (3.12) to (3.16), now gives the dynamic buckling load $\lambda_D^{(2)}$ for this particular case as

$$\left(m^4-2m^2\lambda_D^{(2)}+1\right)^{\frac{3}{2}}=\frac{9}{2\sqrt{3}}m^2\lambda_D^{(2)}\in\bar{a}_m\alpha^{\frac{1}{2}}\left[1-\frac{1}{3}\left(\frac{\phi_m\left(\lambda_D^{(2)}\right)}{\phi_{3m}\left(\lambda_D^{(2)}\right)}\right)^2+6A_m\left(\lambda_D^{(2)}\right)\right]^{\frac{1}{2}} \tag{5.2}$$

6.0 Analysis of results and conclusion

The results (3.16), (3.17),(4.34), (4.35a) and (5.2) are implicit in the load parameter $\lambda_D^{(i)}, i=1,2$ and valid for small values of $\bar{a}_m\in, m=1,2,3\ldots$ They are strictly asymptotic in nature. If we set $m=1$ in (5.2), we have

$$\left(1-\lambda_D^{(2)}\right)^{\frac{3}{2}}=\frac{9\alpha^{\frac{1}{2}}\lambda_D^{(2)}\bar{a}_1\in}{4\sqrt{6}}\left[1+A_1\left(\lambda_D^{(2)}\right)-\frac{1}{3}\left(\frac{1-\lambda_D^{(2)}}{41-9\lambda_D^{(2)}}\right)\right]^{\frac{1}{2}} \tag{6.1}$$

Using (3.16) and (4.34), (3.16) and (5.2), (3.17) and (4.35a) ,(3.17) and (6.1) we easily eliminate the imperfection amplitude in each pair and obtain the following respective equations

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$$\left(\frac{m^4-2m^2\lambda_D^{(1)}+1}{m^4-2m^2\lambda_S+1}\right)^{\frac{3}{2}}=\sqrt{3}\left(\frac{\lambda_D^{(1)}}{\lambda_S}\right)\left[\frac{A_m\left(\lambda_D^{(1)}\right)}{1-\frac{1}{3}\left(\frac{\phi_m\left(\lambda_S\right)}{\phi_{3m}\left(\lambda_S\right)}\right)^2}\right]^{\frac{1}{2}} \tag{6.2a}$$

$$\left(\frac{m^4-2m^2\lambda_D^{(1)}+1}{m^4-2m^2\lambda_S+1}\right)^{\frac{3}{2}}=\frac{1}{\sqrt{3}}\left(\frac{\lambda_D^{(2)}}{\lambda_S}\right)\left[\frac{1+6A_m\left(\lambda_D^{(2)}\right)-\frac{1}{3}\left(\frac{\phi_m\left(\lambda_D^{(2)}\right)}{\phi_{3m}\left(\lambda_D^{(2)}\right)}\right)^2}{1-\frac{1}{3}\left(\frac{\phi_m\left(\lambda_S\right)}{\phi_{3m}\left(\lambda_S\right)}\right)^2}\right]^{\frac{1}{2}} \tag{6.2b}$$

$$\left(\frac{1-\lambda_D^{(1)}}{1-\lambda_S}\right)^{\frac{3}{2}}=\sqrt{3}\left(\frac{\lambda_D^{(1)}}{\lambda_S}\right)\left[\frac{A_1\left(\lambda_D^{(1)}\right)}{1-\frac{1}{3}\left(\frac{1-\lambda_S}{41-9\lambda_S}\right)}\right]^{\frac{1}{2}} \tag{6.2c}$$

$$\left(\frac{1-\lambda_D^{(2)}}{1-\lambda_S}\right)^{\frac{3}{2}}=\frac{1}{\sqrt{3}}\left(\frac{\lambda_D^{(2)}}{\lambda_S}\right)\left[\frac{1+6A_1\lambda_D^{(2)}-\frac{1}{3}\left(\frac{1-\lambda_D^{(2)}}{41-9\lambda_S}\right)}{1-\frac{1}{3}\left(\frac{1-\lambda_S}{41-9\lambda_S}\right)}\right]^{\frac{1}{2}} \tag{6.2d}$$

From above results, we observe that we are able to eliminate the imperfection parameter, namely $\bar{a}_m \in, m=1,2,3\Lambda$. Thus, given any value of the dynamic buckling load, say $\lambda_D^{(i)}$, i = 1,2, we can automatically evaluate the corresponding value of the associated static load λ_s (and vice versa) without necessarily performing the calculation for different imperfection amplitudes. The worst load degradation is in the case in which the buckling mode is strictly in the shape of imperfection. The corresponding results in this case are obtained by setting $w_{3m}^{(3)}=U_{3m}^{(3)}=0$, and disregarding the terms $\frac{1}{3}\left(\frac{1-\lambda_S}{41-9\lambda_S}\right)$ and $\frac{1}{3}\left(\frac{1-\lambda_D^{(i)}}{41-9\lambda_D^{(i)}}\right)$, i =1,2 where they occur. We also set $A_m\left(\lambda_D^{(i)}\right)=A_m\left(\lambda_S\right)=1.$ Such results for equations (6.2c) and (6.2d) are respectively given as

$$\left(\frac{1-\lambda_D^{((1))}}{1-\lambda_S}\right)^{\frac{3}{2}}=\sqrt{3}\left(\frac{\lambda_D^{(1)}}{\lambda_S}\right) \text{ and } \left(\frac{1-\lambda_D^{(2)}}{1-\lambda_S}\right)^{\frac{3}{2}}=\sqrt{\frac{7}{3}}\left(\frac{\lambda_D^{(2)}}{\lambda_S}\right) \tag{6.3}$$

The ratio of the second equation to the first in (5.5), expressed in the form

$$\frac{\dfrac{\left(1-\lambda_D^{(2)}\right)^{\frac{3}{2}}}{\lambda_D^{(2)}}}{\dfrac{\left(1-\lambda_D^{(1)}\right)^{\frac{3}{2}}}{\lambda_D^{(1)}}}=\frac{1}{3}\sqrt{7} \qquad (6.4)$$

gives the ratio of the load degradation of the pre-statistically loaded column per unit dynamic buckling load to the load degradation per unit dynamic buckling load of the column that is loaded by a step load without pre-loading. This means that the magnitude of the dynamic buckling load of a column under a step load is lower than that of a similar column pre-statistically loaded. Thus step loading provides a lower bound for similar loadings. Alternatively, pre-statistically loaded columns are dynamically more stable in the sense that they buckle at relatively higher dynamic loads.

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