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A continuous-time control model on production planning network

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Abstract

In this paper, we give a slightly detailed review of Graves and Hollywood model on constant inventory tactical planning model for a job shop. The limitations of this model are pointed out and a continuous time production model that allows work to travel through more than one station within a single time period is derived. With the relaxation of the period size limitation, we were able to match the time period within the production time frame. This is our major contribution. Unlike Graves and Hollywood model where job visit at most one workstation in each period.

1.0 Introduction

We examine a continuous time control model on production planning network under the following assumptions:

(a) Arrival occurs continuously throughout the period.

(b) Work can arrive at the start of each sub-period s, where $s = 1, 2,\ldots,m$.

(c) The queue length at the start of the period t is Q_t and $m = 1/\lambda$, where λ is the length of each sub-period and m is the number of subintervals.

(d) Arrivals are identical with mean μ and variance σ^2 the arrival at period t is A_t and the arrival at the start of each sub-period is A_t/m

(e) Production in each sub-period is set to a linear control rule; P_t is the production in period t.

The problem of optimal production planning and scheduling is not now, however, due to its increasing relevance, this issue has attracted the attention of diverse researchers. Giffer and Thompson [5] described the solution of production scheduling problems from an algorithmic point of view.

Considerable theoretical advances have been made since that time, leading to the research area of job shop scheduling problems (JSSP). French [3] provide the fundamental theory and mathematical methodology for production planning and scheduling problems in general. According to Garey and Johnson [4] the JSSP and similar scheduling problems are combinatorial optimization problems and commonly classified as NP-hard ordering problems. Due to the NP-hardness it is almost impossible to solve these problems exactly, even for small

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problem instance. Exact methods exist, like the branch and bound approach by Carlier and Pinson [2] but they are only of theoretical relevance due to the exponential runtime complexity.

In real-word production environments, provable optimality is not the criteria for a solution to be satisfied. Instead it suffices to compute results close to the optimum but in a reasonable amount of time. The later is achieved by heuristic methods that are dominating in the field of JSSP. The most popular and important ones are local search approaches like Tabu search (Barnes and Chambers, [1]) and evolutionary algorithms, especially generic algorithms (Yamada and Nakano, [10]).

Graves and Hollywood [7] and [8] proposed a constant – inventory tactical planning model (TPM) for a Job shop. However, the work of Graves and Hollywood [8] has been identified to have the following limitations:

(i) In Graves and Hollywood (TPM) model, a job visits at most one workstation in each period. It is not possible for one job to travel through more than one station in each period.

(ii) The smoothing parameter α_i in the method is constrained to be $0 < \alpha_t \leq 1,$ which entails that the planned lead time must be at least one time period. This is restrictive in the multi – station setting, as the planned lead time has to be set less than the minimum planned lead time of all the stations in the network.

(iii) The setting of the period length is short relative to the frequency of job movements between each pair of stations.

This paper addresses the above limitations of Graves and Hollywood [8] by the continuous time control productive planning model. For better understanding of the work presented in this paper, a slightly detailed treatment of the standard shop scheduling problem and the Graves and Hollywood [8] model are considered in the next two sections.

2.0 The standard job shop problem

In order to provide a common basis for the following sections, we describe the most elementary concepts and formalisms with respect to the standard job shop scheduling problem (JSSP).

The classic $n \times m$ minimum-makespan job shop scheduling problem is given by a finite set J of n jobs $\{J_t\}$ $1 \leq i \leq$ n and a set M of m machines $\{M_i\}$ $1 \leq i \geq$ m (Jain and Meeran, [9]). Each job J_i has to be processed on every machine. For this purpose, a job is subdivided into a set of m_i operations $\{0_{ij}\}$; $1 \leq i \leq n,\ I \leq j \leq m_i$ which have to be scheduled in a strictly sequential way according to a given technological order. This is also referred to as the precedence constraint. Thus O_{ik} denotes the operation of Job J_i that has to be processed on machine m_k for a certain uninterrupted processing time T_{ik} where each machine can process one job at a time (capacity constraint). The time span needed to complete all operations of all jobs is known as the makespan C_{max}. Given the starting time $t_{ik} \geq 0$ for each operation the definition of the minimum make span C^*_{max} with respect to all feasible schedules can be simply written as C^* = Min ($C_{\max}$) = Min (max ($t_{ik} + T_{ik}$), for all Feasible Schedules and $\forall\ j_i \in J,\quad m_k \in M$ for $i, k = 1,2,3$.

In order to illustrate the formal specification we will present now on example of a 3 x 3 JSSP (3 jobs, 3 machines). The technological order of operation (machine sequences) is shown in table 1. The first number in each column represents the machine M_i on which the operation has to be processed whereas the number within the parentheses denotes the required processing time t_{ik}.

Table 2.1: A 3*3 JSSP illustration table

Job	Machine sequence		
1	1(3)	2(3)	3(3)
2	1(2)	3(3)	2(4)
3	2(3)	1(2)	3(1)

3.0 Graves and Hollywood model

In this section of this paper, we give a slightly detailed review of the work of Graves and Hollywood [8]. The key assumption of the model is the linear control rule, which is stated as

$$P_{it} = \alpha_i \; Q_{it} \qquad (3.1)$$

where P_{it} is the production at station i in time period t, Q_{it} is the Queue level or work in progress (WIP) level at the start of the period t, the parameter $\alpha_{i,}$ $0 \le \alpha_i \le 1$, is a smoothing parameter. Both P_{it} and Q_{it} are measured in units of the workload at station i eg hour of work. This rule states that the production P_{it} at station i is a fixed portion α_i of the Queue Q_{it}, where $1/\alpha_i$ is the planned lead time. In particular station i must process α_i of the work in queue on average in each period in order to realize the planned lead time. This is approximated by (3.1) in which the production requirement is assumed to be precisely α_i of the queue.

The Queue level Q_{it} satisfies the standard inventory balance equation

$$Q_{it} = Q_{i,t-1} - P_{i,t-l} + A_{it} \qquad (3.2)$$

where A_{it} is the amount of work that arrives at station i at the start period t. by putting (3.1) into (3.2), we have

$$P_{it} = (1-\alpha_i) - P_{i,t-l} + \alpha_i \; A_{it} \qquad (3.3)$$

which is a first order smoothing equation with α_i as the smoothing parameter and the arrivals model to a station from another station is

$$A_{ijt} = \Phi_{ij} \; P_{j,t-1} + \xi_{ij} \qquad (3.4)$$

A_{ijt} is the amount of work arriving station *i* form station j at the start of period *t*, Φ_{ij} is a positive scalar and ξ_{ijt} is a random variable. The model assumes that one unit (eg hour) of work at station *j* generates Φ_{ij} time units of work at station *i*, on average. ξ_{ijt} is a term that introduces uncertainty into the relationship between production at j and arrivals to *i*. It is assumed that the term is an identical random variable with zero mean and a known variance. Then, the arrival stream to a station *i* is given by

$$A_{it} = \Sigma \; P_{ijt} + N_{it} \qquad (3.5)$$

where N_{it} is identical random variable for the work load from new jobs that enter a shop at station *i* at time *t*. Substituting for A_{ijt}, we find:

$$A_{it} = \Sigma\Phi_{ij} \; P_{j,\;t-l\;+}\; \xi_{it,} \qquad (3.6)$$

where $\xi_{it} = N_{it} + \Sigma \; \xi_{ijt,}$

Note that ξ_{it} represents arrivals that are not predictable from the production levels of the previous period, and consists of work from new jobs and noise in the flow. By assumption, the time series ξ_{ijt} is independent and identically distributed over time.

To present the analysis for the model, we rewrite the equations for production (3.3) and for arriving work (3.6) in matrix form.

$$P_t = (1-D)\ P_{t-1} + DA_t \qquad (3.7)$$

$$A_t = \Phi P_{\ t-1} + \xi_{it} \qquad (3.8)$$

where $P_t = \{P_{1t},\ldots,P_{nt}\}$; $A_t = \{A_{1t},\ldots, A_{nt}\}^1$;
and $\xi_{it} = \{\xi_{it},\cdots,\xi_{nt}\}^1$ are column vectors of random variables, n is the number of stations, I is the identity matrix, D is a diagonal matrix with $\{\alpha_1,\cdots,\alpha_n\}$ on the diagonal, and Φ is an n-by-n matrix with element Φ_{ij}. By putting (3.8) into (3.7) we have the recursion:

$$P_t = (1-D + D\Phi)\ P_{t-1} + D\xi_{it}. \qquad (3.9)$$

By iterating this equation and assuming the system to be infinite, we re-write P_t as an infinite series

$$P_t = \sum_{s=0}^{\alpha} (1 - D + \Phi)^s D\xi_{i,t-s} \qquad (3.10)$$

We denote the mean and the covariance for the noise vector ξ_{it} by $\mu = \{\mu_1,\cdots,\mu_n\}$, and $\Sigma = \{\sigma_{ij}\}$, respectively. The first two moments of P_t are given by:

$$E[P_t] = \sum_{s=0}^{\alpha} (1 - D + \Phi)^s D\mu = (1-\Phi)^{-1}\mu \qquad (3.11)$$

and $S = var(P_t) = \sum_{s=0}^{\alpha} B^s D \Sigma D B^{1s}$, where

$$B = I - D + D\Phi \qquad (3.12)$$

and provided that $P(\Phi) < 1$, where $P(\Phi)$ denotes the spectral radius of Φ (see Graves [6]). We note that S provides the production variance for each station as well as the covariance for each pair of work stations.

4.0 Continuous time control model

In this section we derive a continuous time production function that allows work to travel through more than one station within a single time period.

We rewrite the control rule (3.1) for each sub-period s as.

$$V(\lambda,s) = \alpha\lambda u\ (\lambda,s);\ s = 1,2 \ldots m \qquad (4.1)$$

where $V(\lambda,s)$ is the production in sub-period S of length $\lambda, u(\lambda,s)$ is the queue length at start of sub-period S and α is the smoothing parameter. Similarly to Graves and Hollywood (2006), we interpret $^1/\alpha$ as the planned lead-time; however, we now permit α_1 to assume any positive value, thus, we permit the planned lead-time to be less than one time period.

We proceed to develop an expression for P_t in terms Q_t and A_t. These variables have the same definition as Graves and Hollywood [8]; P_t is the production in period t, Q_t is the queue length at the start of the period t, A_t is the arrival of work to the station period t. We assume that A_t does not arrive at the start of the period, but arrives uniformly over period t. In particular, we assume that the arrival amount at the start of each sub-period is equal to A_t/m

We began with the following boundary condition for the queue length at the start of the first sub-period within each time period.

$$U((\lambda, S=1) = Q_t = A_t/m \text{ for } S=2,\ldots,m \qquad (4.2)$$

We model the queue length at the start of sub-period S by the standard balance equation

$$U(\lambda, s-1) - v\ (\lambda, s-1)\ A_t/m \qquad (4.3)$$

Substituting (4.1) into (4.3) we have.

$$\text{U }(\lambda, s) = (1-\alpha\ \lambda)\, U(\lambda,\ s-1) + A_t / m \qquad (4.4)$$

for S = 2,…,m. Then, the production function is given as

$$P_t = \sum_{s=1}^{M} V\ (\lambda, s) = \alpha \underset{s=1}{\overset{M}{X}} \sum U\ (\lambda, s) \qquad (4.5)$$

Using equation (4.2) and (4.4) we find:

$$\alpha \underset{s=1}{\overset{M}{\lambda}} \sum U\ (\lambda, s) = Q_t + A_t - (1-\alpha\ \lambda)\, U\ (\lambda, m) \qquad (4.6)$$

Putting equation (4.6) into (4.5) we have.

$$P_t = Q_t + A_t - (1-\alpha, \lambda)\, U\ (\lambda, m) \qquad (4.7)$$

But we need to find U (λ, m) from (4.7) above.

We can write from equation (4.2) that

$$\text{U}(\lambda, m) = (1-\ \alpha\lambda)^{\ m-1}\ x\ Q_t\ +\ (1+(1-\alpha\ \lambda)+\cdots+(1-\alpha\ \lambda)^{m-1} x\ A_t / m \qquad (4.8)$$

Using equation (4.8), we can rewrite equation (4.7) as

$$\text{P}_t = \ \beta\ (\lambda)\ Q_t\ +\tau\ (\lambda)\ A_t \qquad (4.9)$$

where

$$\beta(\lambda) = 1-(1-\alpha\ \lambda)^m \qquad (4.10)$$

and

$$\tau\ (\lambda) = \left(\frac{1-1-\alpha\lambda}{\alpha}\right)(1-(1-\alpha\lambda)^m)$$

$$1-\beta\ (\lambda)\ (\tfrac{1-\alpha\lambda}{\alpha}) \qquad (4.11)$$

Thus, we have a linear control for the production in each time period, where the production depends on both the work queue at the start of the period plus the arrivals during the period. The coefficients for the linear control rule depend on the size of the subintervals.

To get some insight into the structure and behaviour of the model, we examine the continuous-time limits for $\beta(\lambda)$ and $\tau(\lambda)$ as the length of the sub-period goes to zero. This corresponds to a continuous-time control that is independent of λ, and as a result we assume that the control rule (4.1) holds of every instant in time. We obtain the continuous-time limit of $\beta(\lambda)$:

$$\beta = \lim_{x\to o} \beta(\lambda) = \lim_{x\to 0} [1-(1-\alpha\lambda)^{\ m}] = \lim_{x\to 0} [1-(1-\alpha\lambda)^{1/\lambda}] = 1-e^{-\alpha}$$

For $\tau(x)$, we find that

$$\tau = \lim_{x\to 0} \tau(\lambda) = \lim_{x\to 0} \left\{1-(\tfrac{1-\alpha\lambda}{\alpha})\,(1-(1-\alpha\lambda)^m)\right\} = 1-\frac{1}{\alpha}+\frac{1}{\alpha}\ e^{-\alpha} = 1-\frac{1}{\alpha}\,(1-e^{-\alpha}) = 1-\frac{\beta}{\alpha}$$

We now restate (4.9) for a continuous-time control as:

$$P_t = \beta Q_t + \tau A_t \qquad (4.12)$$

where β and τ are given above. The balance equation for the queue length for a single station is now given by:

$$Q_t = Q_t - 1 - P_t - 1 + A_t - 1 \qquad (4.13)$$

Equation (4.13) differs from the balance equation in (3.2) of Graves and Hollywood [8] because of the new assumption that arrivals occur continuously throughout a period. Hence, we define Q_t to be the queue length at the start of the period t, prior to any arrivals in that period. By substituting (4.12) into (4.13), we have:

$$Q_t = (1-\tau) \sum_{\iota=1}^{\iota-1} (1-\beta)\ A_t - \iota \qquad (4.14)$$

If we assume that the arrivals are $\iota\iota d$ with mean μ and variance σ^2, we find the two moments for the queue length for (4.14)

$$\text{E}\ (Q_t) = \frac{(1-\tau)}{\beta}\ \mu = \frac{\mu}{\alpha} \tag{4.15}$$

$$Var\ (Q_t) = \frac{(1-\tau)^2 \sigma^2)}{2\beta - \beta^2} \tag{4.16}$$

In the same vein, we obtain the two moments for the production variable:

$$E\ [P_t] = \mu \tag{4.17}$$

$$Var\ (P_t) = (\tfrac{\beta}{2-\beta}\ (1-\tau)^2 + \tau^2)\ \sigma^2 \tag{4.18}$$

Note that the expected queue length (4.15) and expected production length (4.17) for our continuous-time model are identical to Graves and Hollywood [8] on equations (3.5) and (3.7). We can see that the expected lead-time corresponds to the planned lead-time $1/\alpha$. The variance (4.16 and 4.17). However, is different from that of Graves and Hollywood [8].

In (3.6) and (3.8). The production variables differ from Graves and Hollywood [8] due to the uniform workflow assumption of our continuous-time model.

5.0 Conclusion

In this paper, we derived a continuous-time production model that allows work to travel through more than one station within a single time period. With the relaxation of the period size limitation, we are able to match the time period within the production time frame, which is our major contribution to Graves and Hollywood [8] in which work flows between stations at the start of each time period; thus visit at most workstation in each period.

References

[1] Barnes, J.W and chambers, J.B (1995) "Solving the Job Shop Scheduling Problem with Tabu Search" *11E Transactions,* 27:257-263

[2] Carlier, J. and Pinson, E. (1989) "An Algorithm for Solving the Job- Shop Problem." *Management Science,* 35:164-176.

[3] French, S. (1982) "An Introduction to the Mathematics of the Job Shop." Chi Chester, Horwood.

[4] Garey, M.R and Johnson, D.J. (1979) "Computers and Intractability a Guide to the Theory of NP-Completeness" Freeman Company San Francisco, C.A.

[5] Giffer, B. and Thompson, J.L. (1960), "Algorithms for Solving Production Scheduling Problems." *Operations Research,* 8:487-503.

[6] Graves, S.C (1986) "A Tactical Planning Model for a Job Shop." *Operations Research* 34 (4), 522-533.

[7] Graves, S.C and Hollywood, J.S (2001) "A constant Inventory Tactical Planning Model for a Job Shop." Working Paper, Cambridge M.A.

[8] Graves, S.C. and Hollywood, J.S (2006) "A constant – Inventory Tactical Planning Model for a Job Shop." Revised Edition, Working Paper, Cambridge M.A.

[9] Jain, A.S. and Meeran (1999) "Deterministic Job Shop Scheduling: Past, Present and Future." *European Journal of Operational Research* 133 (2), 124-132.

[10] Yamada, T. and Nakano, R. (1997) "Generic Algorithms for Job-Shop Scheduling Problems." *Proceedings of Modern Heuristic for Decision Support,* 67 – 81 *UNICOM* Seminar,