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A mathematical model of combustion kinetics of municipal solid waste (MSW)

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Abstract

Municipal Solid Waste has become a serious environmental problem troubling many cities. In this paper, a mathematical model of combustion kinetics of municipal solid waste with focus on plastic waste was studied. An analytical solution is obtained for the model. From the numerical simulation, it is observed that the heating rate β is proportional to the heating temperature and the conversion rate of the system. It is also observed that the conversion rate increases as the pre-exponential factor increases while it decreases as the activation energy and reaction order increases.

Keywords
Combustion kinetics, municipal solid waste, pseudo-part, waste plastic

1.0 Introduction

The study of the combustion characteristics of Municipal Solid Waste (MSW) is interesting because of its advantage of maximum volume/quantity reduction and energy recovery. The typical combustible components of MSW include wood, paper, food, Polyethylene (PE) and Polyvinyl Chloride (PVC).

The entire weight loss process of typical MSW components consists of one to three distinct combustion stages, two stages for biowaste (waste paper, wood, food), one stage for plastic waste (PE) and three stages for PVC. The first stage of biowaste involves volatile release and combustion; the second stage involves char combustion. Because plastic does not contain fixed carbon, it has only one stage. The weight loss of PVC is more complicated, the first stage mainly involves release of Hydrogen Chloride; the second stage involves release of benzene, toluene and other hydrocarbons (Wu et al [7]); the third stage may involve char combustion and decomposition of inorganic material.

The combustion of typical components of MSW could be modeled by one to three independent reactions. The corresponding parameters of typical components of MSW are the activation energy, pre-exponential factor, and reaction order.

Thermogravimetric analysis is one of the most used techniques to study the combustion kinetics of fuel. Most kinetic studies use simple reaction models in different temperature range, which will inevitably lead to a breakpoint in the calculation in order to obtain satisfactory fits to the experimental results and usually could not yield good results for complex and diverse MSW (Li et al [5];Cozzani et al [2]; Fritsky et al [3] and Narukawa et al [6]).

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Jin et al [4] proposed a new consecutive kinetic model for the combustion of MSW based on the hypothesis that each stage of weight loss stands for a reaction of a component of the sample and that the kinetic parameters of each reaction remains constant over the whole temperature range of the experiment. The corresponding parameters of typical components of MSW such as activation energy, pre-exponential factor, and reaction order were determined experimentally.

In this present study, we will focus on the plastic waste, $i = 1$ and provide an analytical solution of the problem. We investigate the combined effects of heating rate, activation energy parameter, pre-exponential factor and reaction order on the temperature rise and conversion rate of the system.

2.0 Mathematical model

Following [1], the equation for each pseudo-part is given by

$$\frac{d\alpha_i}{dt} = A_i(1-\alpha_i)^{n_i}\exp\left(-\frac{E_i}{RT}\right) \tag{2.1}$$

where for PVC, $i = 1,2,3$; for biowaste, $i = 1, 2$; for plastic waste (PE), $i = 1$, α_i is the degree of transformation of the pseudo-part i, n_i is the reaction order of pseudo-part i, A_i is the pre-exponential factor of pseudo-part i, E_i is the apparent activation energy of pseudo-part i, R is the gas constant, T is heating temperature.

Heating rate β is a constant since the temperature is raised linearly, that is

$$\beta = \frac{dT}{dt} \tag{2.2}$$

The combustion of each sample can be written as
$$\alpha = \sum_{i=1}^{I} z_{i0}\alpha_i \tag{2.3}$$

α is the general conversion rate of the sample at time t, i is the number of pseudo-parts, z_{i0} is the ratio of the weight of pseudo-part i to the global loss weight of the sample.

Combining (2.1) and (2.3) and for plastic waste $i = 1$, we can write

$$\frac{d\alpha}{dt} = A_1 z_{10}(1-\alpha_1)^{n_1}\exp\left(-\frac{E_1}{RT}\right) \tag{2.4}$$

satisfying the initial conditions $T(0) = T_0 \;, \alpha(0) = 0$ (2.5)

2.1 Non-dimensionalisation analysis

Let
$$\theta = \frac{E_1}{RT_0^2}(T - T_0), t' = \frac{t}{\tau}, \in = \frac{RT_0}{E_1} \tag{2.6}$$

Using the dimensionless variables (2.6) in (2.2) and (2.4), we obtain the dimensionless equation (after dropping of prime) as
$$\frac{d\alpha}{dt} = Az_{10}(1-\alpha_1)^{n_1}\exp\left(\frac{\theta}{1+\in\theta}\right) \tag{2.7}$$

$$\frac{d\theta}{dt} = \beta_1 \tag{2.8}$$

$$\theta(0) = 0,\ \alpha(0) = 0 \tag{2.9}$$

where, $\beta_1 = \dfrac{\beta}{\in T_0}$, $A = A_1\exp\left(-\dfrac{E_1}{RT_0}\right)$

3.0 Method of solution

Combining (2.7) and (2.8) we have

$$\frac{d\alpha}{d\theta} = \frac{Az_{10}}{\beta_1}(1-\alpha_1)^{n_1}\exp\left(\frac{\theta}{1+\in\theta}\right) \tag{3.1}$$

$$\alpha(0) = 0 \tag{3.2}$$

The solution of (3.1) is obtained as

$$\alpha(\theta)=\frac{Az_{10}(1-\alpha_1)^{n_1}}{\beta_1\in^2}\left(\in(1+\in\theta)\exp\left(\frac{1}{\in}-\frac{1}{\in(1+\in\theta)}\right)-Ei\left(1,\frac{1}{\in(1+\in\theta)}\right)\exp\left(\frac{1}{\in}\right)\right)-$$
$$\frac{Az_{10}(1-\alpha_1)^{n_1}}{\beta_1\in^2}\left(\in-Ei\left(1,\frac{1}{\in}\right)\exp\left(\frac{1}{\in}\right)\right) \qquad (3.3)$$

where Ei is the exponential integral. Solving (2.8), we obtain, $\theta(t)=\beta_1 t$ (3.4)

The solution of (2.7) now becomes

$$\alpha(t)=\frac{Az_{10}(1-\alpha_1)^{n_1}}{\beta_1\in^2}\left(\in(1+\in\beta_1 t)\exp\left(\frac{1}{\in}-\frac{1}{\in(1+\in\beta_1 t)}\right)-Ei\left(1,\frac{1}{\in(1+\in\beta_1 t)}\right)\exp\left(\frac{1}{\in}\right)\right)-$$
$$\frac{Az_{10}(1-\alpha_1)^{n_1}}{\beta_1\in^2}\left(\in-Ei\left(1,\frac{1}{\in}\right)\exp\left(\frac{1}{\in}\right)\right) \qquad (3.5)$$

The results presented in figures 4.1 to 4.6 demonstrate the effect of heating rate, pre-exponential factor, activation energy and reaction order on the heating temperature and conversion rate profile.

4.0 Result and discussion

Figure 4.1 shows the heating temperature profile against time *t* for different values of heating rate β_1 and figures 4.2 – 4.5 show the conversion rate profiles against time *t* for different values of heating rate β_1, pre-exponential factor *A*, activation energy $\in$ and reaction order n_1 while figure 4.6 shows the conversion rate profiles against temperature θ for different values of activation energy $\in$.

Figure 4.1 displays the heating temperature profile for the different values of β_1 at $\in=0.01,\ A=2.5,\ n_1=1.2$. It is shown that the heating rate is proportional to the heating temperature this shows that temperature raises linearly.

Figure 4.2 reveals the conversion rate profile at a fixed $\in=0.01,\ A=2.5,\ n_1=1.2$ for varying values of β_1. It is shown that the heating rate is proportional to the conversion rate. This is as a result of increase of heating temperature which tends to lead to the entire weight loss of the process.

Figure 4.3 displays the conversion rate profile for the different values of *A* at $\in=0.01$, $\beta_1 = 1$, $n_1 = 1, 2$... It is shown that the conversion rate increases with increase in pre-exponential factor. This shows that as the pre-exponential factor increases the conversion rate increases.

Figure 4.4 reveals the conversion rate profile at a fixed $\beta_1 = 1$, $A = 2.5$, $n_1 = 1, 2$ for varying values of $\in$. It is shown that the conversion rate increases with decrease in activation energy, that is, as the $\in\rightarrow 0$ the maximum conversion rate is achieved.

Figure 4.5 displays the conversion rate profile for the different values of n_1 at $\in=0.01$, $\beta_1 = 1$, $A = 2.5$. It is shown that the reaction order is inversely proportional to the conversion rate. This shows that as the reaction order of the pseudo-part *i* increases the conversion rate increases.

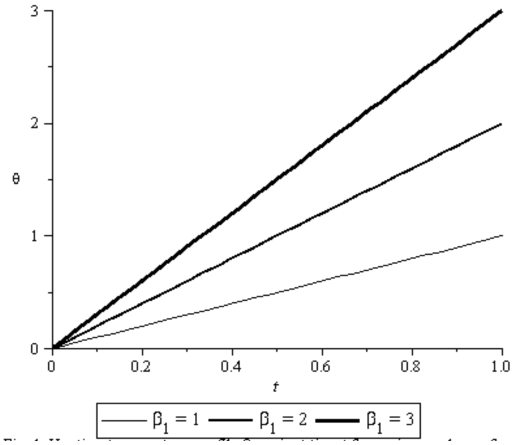

Figure 4.1: Heating temperature profile θ against time *t* for various values of β₁.

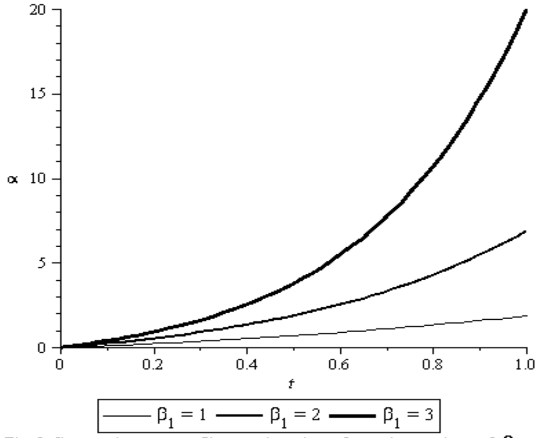

Figure 4.2: Conversion rate profile θ against time *t* for various values of β₁.

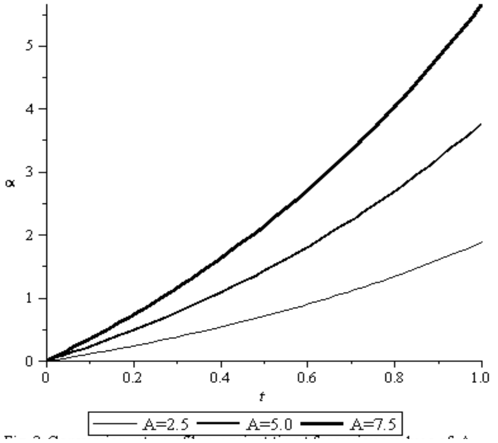

Figure 4.3: Conversion rate profile α against time *t* for various values of *A*

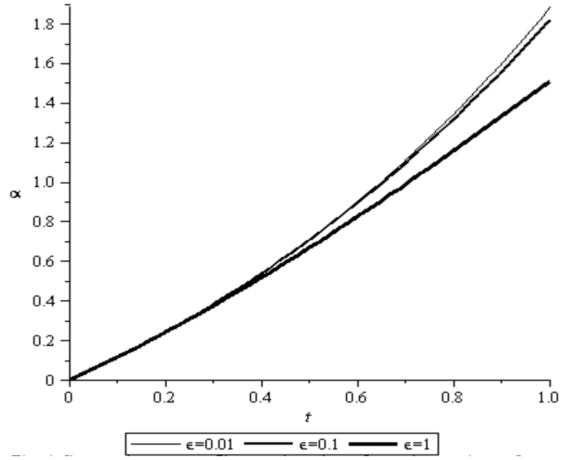

Figure 4.4: Conversion rate profile α against time *t* for various values of ∈ .

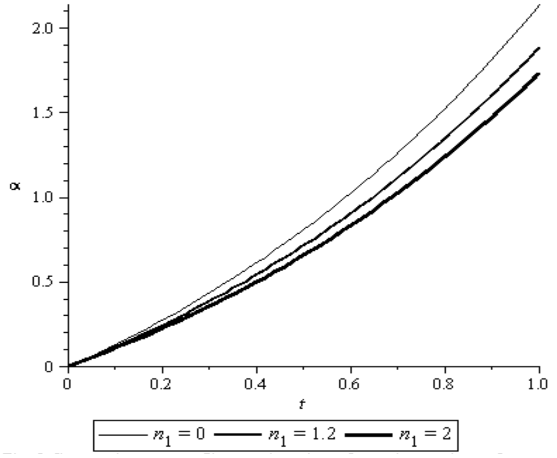

Figure 4.5: Conversion rate profile α against time *t* for various values of n_1.

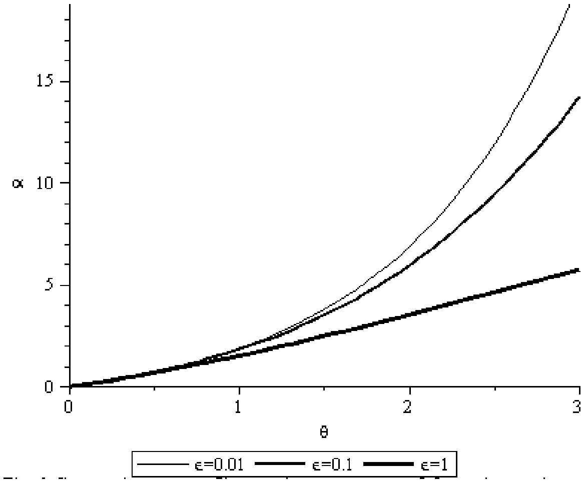

Figure 4.6: Conversion rate profile α against temperature θ for various values of ∈ .

Figure 4.6 reveals the conversion rate profile at a fixed $\beta_1 = 1$, $A = 2.5$, $n_1 = 1, 2$ for varying values of $\in$. It is shown that the conversion rate increases with decrease in activation energy, that is, as the $\in \to 0$ the maximum conversion rate is achieved.

5.0 Conclusion

Mathematical model of combustion kinetics of municipal solid waste with focus on plastic waste was studied. The analytical solutions is provided, the effect of varying the values of heating rate parameter, activation energy parameter, pre-exponential factor and reaction order on the heating temperature profile and conversion rate profile of MSW was investigated. The result in [4] indicated the entire weight loss process of each typical component of MSW consist of one to three distinct combustion stages. From our findings, it is observed that the heating temperature and conversion rate rise with increase in the heating rate. Also the conversion rate increases with decrease in activation energy and reaction order while it is increases with increase in pre-exponential factor.

References

[1] Coats, A.W. and Redfern, J.P. (1964) “Kinetic parameters from thermogravimetric data,’’ Nature, 201 (4914): 68-69.

[2] Cozzani, V., Petarca, L. and Tognotti, L. (1995) “Devolatilization and pyrolysis of refuse derived fuels: characterization and kinetic modeling by a thermogravimetric and calorimetric approach,’’ Fuel, 74: 903-912.

[3] Fritsky, K.J., Miller, D.L. and Cernansky, N.P. (1994) “Methodology for Modeling the devolatilization of refuse-derived fuel from thermogravimetric Analysis of municipal solid waste components,’’ J. Air and Waste Manage. Assoc., 44: 1116-1123.

[4] Jin, Y.Q., Yan, J.H. and Cen, K.F. (2004) “Study on the comprehensive combustion Kinetics of MSW,’’ J. Zhejiang Univ. SCI, 5 (3): 283-289.

[5] Li, B, Yan, J.H., Shang, N., Chi, Y. and Cen, K.F. (1998) “Study on the combustion characteristics of MSW,” Journal of Fuel Chemistry and Technology, 26 (6): 564-568.

[6] Narukawa, K., Goto, H., Chen, Y., Yamazaki, R. and Mori, S. (1997) “Japanese RDFFire Power Generation System and Fundamental Research on RDF Combustion,’’ Proc.14th Fluidized Bed Combustion, 1: 633-639.

[7] Wu, C.H., Chang, C.Y., Hor, J.L., Shih, S.M. and Chen, L.W. (1994) “Two-stage Pyrolysis model of PVC,’’ Can. J. Chem. Eng., 72 (4): 644-650.