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Comparative performance of autoregressive order determination criteria for subset modelling

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Abstract

The efficiencies of eight autoregressive model order determination criteria: AIC, BIC, SIC, S, Φ, FPE4, CAT_2, CAT_3, for the selection of subset models are compared using artificial and real series. Our observation is that BIC, S and FPE4 performed well in a wide range of models.

1.0 Introduction

A stationary mean corrected time series $\{X_t\}$ is said to follow an autoregressive model of order p (denoted by $AR(p)$) if it is a solution of the following difference equation

$$X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \cdots + \alpha_p X_{t-p} = \varepsilon_t \qquad (1.1)$$

where the α_i's are constants such that the characteristic equation $1 + \alpha_1 z + \alpha_2 z^2 + \cdots + \alpha_p z^p =$
0
has roots outside the unit circle, and $\{\varepsilon_t\}$ is a white noise process of variance σ^2.

Any stationary time series $\{X_t\}$ can be expressed as an infinite-order autoregressive model [3]. In practice, a stationary time series is modeled by an $AR(p)$ which is an approximation to the theoretical model. The determination of the order represents a major obstacle since either underestimation or overestimation makes the model unreliable.

Contemporary autoregressive modeling techniques use automatic criteria for order determination like Akaike's information criterion (AIC) [1], Bayesian information criterion (BIC) [2], etc. Interest is often in the comparative performance of the order determination criteria under certain conditions (see, e.g. [4], [14] and [16]).

Constraining some of the α_i's in equation (1.1) to zero makes it a subset order model. This work is a study of eight automatic order determination criteria: AIC, BIC, S, SIC, Φ, CAT_2, CAT_3, and FPE4 in subset order model selection. Etuk [9] has shown that for full order modeling AIC and CAT_3 excel while the rest underestimate the order. He has also demonstrated that their comparative performance depends on the method of estimation of the α_i's in (1.1) Here, we shall use the least squares method.

We shall use simulated as well as real series. For the artificial series we shall study the effect of sample size on the comparative performance. Interest shall also be in determining if

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model distance from the non-stationarity boundary affects their performance. We shall use the least-square-method based subset selection algorithm of Haggan and Oyetunji [12].

2.0 Model selection

For the realization $X_1, X_2 z^2 \cdots X_N$ of the time series $\{X_t\}$ we use the subset algorithm of Furnival [11] after specifying a maximum lag. We compare the efficiencies of the criteria defined by

$$FPE\alpha(p) = \left(1 + \frac{\alpha p}{n}\right)\left(1 - \frac{\alpha p}{N}\right)\sigma^2,\ \alpha = 0,1,2,\cdots$$ (Bhansali and Downham, [5])

$$AIC(p) = N \ln \sigma_p^2 + 2p,\ p = 0,1,2,\cdots$$ (Akaike, [1])

$$BIC(p) = N \ln \sigma_p^2 - (N-p)\ln\left(1 - \frac{p}{N}\right) + p \ln N + p \ln p^{-1}\left(\frac{\sigma_0^2}{\sigma_p^2}\right) 2p,\ p = 0,1,2,\cdots$$ (Akaike, [2]

$$SIC(p) = N \ln \sigma_p^2 + p \ln N,\ p = 0,1,2,\cdots$$ (Schwarz, [19])

$$CAT_2(p) = \frac{1}{N}\left(\sum \frac{1}{\sigma^2}\right) - \frac{1}{\sigma_p^2},\ p = 0,1,2,\cdots$$

(Parzen, [17])

$$= -\left(1 + \frac{1}{N}\right),\ p = 0$$

$$CAT_3(p) = \frac{1}{N}\left(\sum \frac{1}{\sigma^2}\right) - \frac{1}{\sigma_p^2},\ p = 0,1,2,\cdots$$ (Tong, [21])

$$\Phi(p) = \ln \sigma_p^2 + N^{-1} 2pc \ln \ln N, c \succ 1,\ p = 0,1,2,\cdots$$ (Hannan and Quinn, [13])

$$S_N(p) = (N + 2p)\sigma_p^2,\ p = 0,1,2,\cdots$$ (Shibata, [20])

where σ^2_p and σ^2_p are respectively the least squares and the maximum likelihood estimates of the residual variance. Each of them chooses the model corresponding to its minimum.

3.0 Simulation results

Four *AR*(2) series with (α_1, α_2) equal to (0.00, -0.78), (0.00, -0.15), (0.00, 0.89) and (0.00, 0.10) are simulated twenty independent times. We shall refer to them as series I, II, III and IV respectively. The white noise process for each simulation is a sequence of pseudorandom numbers generated using the RAN function of the FORTRAN 77 language. The sequence was made normally distributed. To minimize the transient effect of starting values we ignored the first 100 values. We use N = 50, 150 and 250, for each model, giving a total of 60 series for each model. We note that series II and IV are nearly white noise being far from the non-stationary boundary. Series I and III on the other hand are close to the boundary. Table 3.1 gives the frequencies of correct selection by the criteria of the models.

As evident from Table 3.1, BIC outperforms the rest followed by S. The performances of the criteria do not depend on the sample size.

Table 3.1: Frequency out of 20 of correct picking of model by each criterion

Series	Sample size	AIC	BIC	Φ	SIC	S	CAT_2	CAT_3	FPE4
I	50	16	19	20	20	20	5	17	20
	150	13	20	20	20	19	6	13	20
	250	17	20	20	20	18	5	17	19

Series	Sample size	AIC	BIC	Φ	SIC	S	CAT_2	CAT_3	FPE4
II	50 150 250	6 9 12	7 8 8	0 6 5	0 6 4	5 9 11	1 5 8	6 9 12	0 7 9
III	50 150 250	17 14 17	18 19 20	18 18 20	18 19 20	18 17 19	3 3 5	17 14 17	18 16 20
IV	50 150 250	3 6 6	3 6 3	0 0 0	0 1 0	1 5 5	3 4 3	3 6 6	0 3 3
Total		136	151	127	128	147	51	137	135

4.0 Real series results

We use a maximum lag of 15 here. Each model selected was subjected to diagnostic checks which include Box-Pierce [7] portmanteau statistic R test. The parametric spectrum was compared with the raw one for each of the models. Moreover the diagnostic aids: inverse autocorrelation function (IACF) and partial autocorrelation function (PACF) were used.

4.1. Series A (Box and Jenkins, [6]) pp. 525

IACF and *PACF* recommend an *AR*(7) model (Etuk, [10]). *BIC*, Φ, *SIC*, *S* and *FPE*4 pick the *AR*(3) model

$$X_t - 0\cdot 381X_{t-1} - 0\cdot 216X_{t-2} - 0\cdot 188X_{t-3} = \varepsilon_t \sigma^2 = 0\cdot 0955, R = 24\cdot 31 \qquad (4.1)$$

CAT_2 and CAT_3 choose the *AR*(2)

$$X_t - 0\cdot 427X_{t-1} - 0\cdot 252X_{t-2} = \varepsilon_t \sigma^2 = 0\cdot 1002, R = 100\cdot 03 \qquad (4.2)$$

AIC chooses the *AR*(15) with significant lags 1, 2, 7, 14 and, of course, 15

$$X_t - 0\cdot 388X_{t-1} - 0\cdot 220X_{t-2} - 0\cdot 174X_{t-7} - 0\cdot 126X_{t-14} - 0\cdot 122X_{t-15} = \varepsilon_t \sigma^2 = 0\cdot 0934, R = 20\cdot 15 \qquad (4.3)$$

Figure 4.1 shows the spectra of (4.1) and (4.2) superimposed on the non-parametric spectrum. That of (4.1) clearly agrees with the non-parametric one while that of (4.2) does not as much. In support, the R test is significant for (4.2) but not for the rest.

We then infer that (4.1) fits the data well. The model (4.3) therefore over-fits it and (4.2) under-fits it. Evidence here is therefore in favour of *BIC,Φ, SIC, S and FPE*4.

4.2 Canadian Lynx numbers (1821 – 1934) (Campbell and Walker, [8] pp. 430)

The logarithmic transformation was used. *BIC, Φ, AIC, SIC, S, FPE*4 recommend

$$X_t - 1\cdot 094X_{t-1} + 0\cdot 357X_{t-2} + 0\cdot 127X_{t-4} - 0\cdot 324X_{t-10} + 0\cdot 325X_{t-11} = \varepsilon_t \sigma^2 = 0\cdot 2355, R = 19\cdot 28$$

which is shown adequate by both the spectrum and *R*-tests (Etuk, [9]). CAT_2 and CAT_3 select a subset *AR*(4) model with significant lags 1, 2 and 4, which is discredited by both the *R*- and the spectrum tests. We therefore infer that CAT_2 and CAT_3 underestimate while the rest fare well.

4.3 Wolfer's sunspot series (1700 – 1955) (Waldmeier, [22])

*FPE*4, *BIC,Φ* and *SIC* recommend the *AR*(9)

$$X_t - 1\cdot 228X_{t-1} + 0\cdot 529X_{t-2} - 0\cdot 157X_{t-9} = \varepsilon_t \sigma^2 = 216\cdot 32, R = 29\cdot 87 \qquad (4.4)$$

whose *R*-value is not significant and whose spectrum agrees closely with that of the series (Etuk, 1987). It is noteworthy that Morris [15] and Schaerf [18] each fitted a model with lags 1, 2 and 9 significant as in (4.4). *AIC* selects lags 1, 2, 3, 4, 5 and 9; CAT_3 selects lags 1, 2 and 3; *S* selects lags 1, 2, 3 and 9; CAT_2 selects lag zero meaning that it chooses the raw data in preference to any autoregressive model. Inference here is therefore in favour of *FPE*4, *BIC*, Φ and *SIC*.

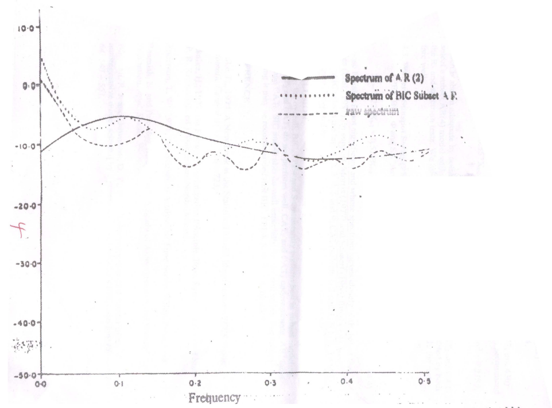

Figure 4.1: Some series A s[ectra)in db)

5.0 Conclusion

On the overall, *BIC*, *FPE*4 and *S* consistently perform best in subset modelling. *AIC*, CAT_2 do not perform well. CAT_2 and CAT_3 underestimate; *AIC* fares better than CAT_2. *SIC* and Φ also do well especially for the real series. For subset modeling it may be recommended that *BIC*, *FPE*4, *S*, Φ or *SIC* be used.

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