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Computational result of integral quadratic objective functional with wave-diffusion effect

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Abstract

This work is on computational result of integral quadratic objective functional with wave-diffusion effect, which is in the form:

$$\left.\begin{aligned} & Min J[u,z] = \int_0^1\int_0^1 \left(u^2 + z^2\right) dxdt \\ & \text{Subject to} \\ & \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} + \frac{1}{d}\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2} \\ & z(0,t) = z(1,t), 0 \le x \le 1 \\ & z(x,0) = z(x,1) 0 \le t \le 1 \end{aligned}\right\} \qquad *$$

where u(x,t) is the control and z(x,t) is the states of the system. On the application of the Hamiltonian function we derived the states and controls which satisfy optimality condition. A suitable Fourier solution is applied to obtain the states and controls in the form of a series solution. Also, the states and controls in the form a series solution can also be obtained by use of EXTENDED CONJUGATED GRADIENT METHOD (ECGM) proposed by Ibiejugba [7] and Reju [8]. The work also consists of numerical solutions which are optimal.

Keywords
Optimal solution, Integral Quadratic Objective Functional, Fourier Solution, Diffusion,

1.0 Introduction

An integral quadratic functional with wave-diffusion effect is being studied. We aim at determining the optimal states and controls of the integral quadratic objective functional with wave-diffusion effect which is in the form

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$$\left.\begin{array}{l}\text{MinJ}\,[\text{u, z}] = \int_0^1\int_0^1 \left(u^2 + z^2\right)dxdt \\ \text{Subject to} \\ \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} + \frac{1}{d}\frac{\partial z}{\partial t} = \frac{\partial^2 z}{\partial x^2} \\ z(0,t) = z(1,t), 0 \le x \le 1 \\ z(x,0) = z(x,1), 0 \le t \le 1 \end{array}\right\} \tag{1.1}$$

where

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} + \frac{1}{d}\frac{\partial u(x,t)}{\partial t} \qquad \text{\textbackslash(1.2\}}$$

is the wave-diffusion equation part of (1.1). In (1.2), $u(x,t)$ is called the source, term that add heat to a unit bar at a rate u(x,t) per unit time per unit length. The objective functional depend on a multiplicity of variable called the controls. Controls which take values in "s*uitable*" subspaces of the space of definition of the system under study are said to be *admissible* [1]. Here, "*suitable"* means well behaved spaces in E^n, the n-dimensional Euclidean space. For example, compact, covex sets are such "*suitable"* spaces.

The integral quadratic functional

$$\int_0^1\int_0^1 \left(z^2(x,t) + u^2(x,t)\right)dxdt$$

can also be written as

$$= \int_0^1\int_0^1 [z(x,t) + iu(x,t)][z(x,t) - iu(x,t)]dxdt = \int_0^1\int_0^1 w(x,t)\overline{w}(x,t)dxdt = \int_0^1\int_0^1 w^2(x,t)dxdt \qquad (1.3)$$

where *i* is a complex constant [see Awar [2] and Tejumola [3]].

On the application of the Hamiltonian function to (1.1), we obtained the necessary condition for optimality which expresses the relationship between the states and controls of the wave-diffusion problem. We also apply a suitable Fourier solution to the optimal condition to obtained the states and controls in the form of a series solution.

Before we set out to establish the result of this paper, we state the following proposition.

***Proposition* 1.1** (Krasnov [4])

$J[u,z] = \int_0^1\int_0^1 \left(z^2(x,t) + u^2(x,t)\right)dxdt$ has a strict minimum on the surface u(z) = 0

Proof

Let $\Delta J = J[u,z] = \int_0^1\int_0^1 [u^2(x,t) + z^2(x,t)]dxdt = \int_0^1\int_0^1 [u^2(x,t) + z^2(x,t)]dxdt - \int_0^1\int_0^1 z^2(x,t)dxdt$

$= \int_0^1\int_0^1 u^2(x,t)dxdt \ge 0$ which means that, $\Delta J = J[u,z] \ge 0$, where $u(z) = 0$ is a minimum. This implies that, the parabolic surface $z^2(x,t) + u^2(x,t)$ has the origin as the minimum point.

With the above conditions, we are now in a position to establish the main result of this paper.

2.0 Main result

Consider the wave-diffusion problem (1.1), in the form

$$\frac{\partial^2 u(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2}+\frac{1}{d}\frac{\partial u(x,t)}{\partial t}$$

as

$$\text{Min } J[u,z]=Min\int_0^1\int_0^1 u^2(x,t)+z^2(x,t).]dxdt \quad \text{subject to}$$

$$\begin{aligned} &z_{tt}(x,t)+z_1(x,t)=z_{xx}(x,t)+u(x,t)\\ &z(0,t)=z(1,t)=0, z(x,0)=z(x,1)\end{aligned} \tag{2.1}$$

$0 \le x \le 1$, $0 \le t \le 1$.

The Hamiltionian for (2.1) to that of Singh and Titli [5] is

$$H = z^2(x,t)+u^2(x,t)+\lambda^T\left[\frac{\partial^2 z(x,t)}{\partial x^2}+u(x,t)\right] \tag{2.2}$$

where $\lambda^T=\lambda^T(t)$. By setting

$$f(z(x,t),u(x,t))=\frac{\partial^2 z(x,t)}{\partial x^2}+u(x,t)$$

is equivalent to $z_{xx}(x,t) + u(x,t)$ in (2.1)` and $g(z(x,t),u(x,t))=z^2(x,t)+u^2(x,t)$ is equivalent to $u^2(x,t)$ + $z^2(x,t)$ in (2.1) also. We then have the first order necessary conditions for optimality as

$$\begin{aligned} &\frac{\partial z(x,t)}{\partial t}=\frac{\partial H(x,t)}{\partial \lambda}=\frac{\partial^2 z(x,t)}{\partial x^2}+u(x,t)= f(z(x,t),u(x,t))\\ &\frac{\partial \lambda}{\partial t}=\frac{\partial H}{\partial z}=\left[\frac{\partial f}{\partial z}\right]^T\lambda\end{aligned} \tag{2.3}$$

$$\frac{\partial H}{\partial u}=\left(\frac{\partial f}{\partial u}\right)^T\lambda+\frac{\partial g}{\partial u}=0 \tag{2.4}$$

Equation (2.4) gives

$$\lambda+2u(x,t)=0 \text{ or } \lambda=-2u(x,t)=0 \tag{2.5}$$

By virtue of (2.4) and (2.5), we have

$$\frac{\partial \lambda}{\partial t}=-2\frac{\partial u(x,t)}{\partial t}= -\ 2z\ (x,t) \tag{2.6}$$

It follows that

$$z(x,t)=\frac{\partial u(x,t)}{\partial t} \tag{2.7}$$

Equation (2.7) is of physical significance under the conditions for optimality which expresses the relationship between the temperature, and the heat source at any point x of the unit conducting rod of our diffusion problem. We can treat (2.7) as a differential transform of any previously known solution of the diffusion equation.

On the application of a suitable Fourier transform there exist optimal states and controls of the system (2.1) which can be evaluated explicitly.

If $$z(x,t)=\sum_{1=T}^{\infty}\alpha_i(t)\sin \pi ix \text{ and } u(x,t)=\sum_{1-1}^{\infty}u_1(t)\sin \pi ix \tag{2.8}$$

we have our new solution as

$$Z(x,t)\frac{\partial}{\partial t} = \sum_{1=1}^{\infty} \alpha_i(t)\sin \pi ix = \sum_{1-1}^{\infty} u_{i1}(t)\sin \pi ix \qquad (2.9)$$

This implies that,

$$\left.\begin{aligned} &\alpha_1(t) = u_{it}(t) \\ &z_1(x,t) = \sum_{1=1}^{\infty} u_{itt}(t)\sin \pi ix \\ &z_{xx}(x,t) = \sum_{1=1}^{\infty} - i^2\pi^2 u_{it}(t)\sin \pi ix \\ &z_{tt}(t) = \sum_{1=1}^{\infty} u_{ittt}(t)\sin \pi ix \end{aligned}\right\} \qquad (2.10)$$

From the constrained equation (2.1), we have that,

$$z_{tt}(x,t) + z_t(x,t) = z_{xx}(x,t) + u(x,t).$$

This implies that,

$$\sum_{1=1}^{\infty} u_{ittt}(t)\sin \pi ix + \sum_{1=1}^{\infty} u_{itt}(t)\sin \pi ix = \sum_{1=1}^{\infty} - i^2\pi^2 u_{it}(t)\sin \pi ix + \sum_{1=1}^{\infty} u_i(t)\sin \pi ix$$

which means that $\quad u_{ittt}(t) + u_{itt}(t) = -i^2\pi^2 u_{it} + u_i \qquad (2.11)$

and Equation (2.1) becomes

$$Min\int_0^1 \left[u_1^2 + u_2^2 + \ldots + u_n^2\right] + \int_0 \left[u_{1t}^2 + u_{2t} + \ldots u_{nt}\right]dt \qquad (2.12)$$

The corresponding unconstrained problem is given by

$$\begin{aligned} u_{ittt} &= -u_{itt} - 1^2\pi^2 u_{1t} + u_1 \\ u_{2ttt} &= -u_{2tt} 2^2\pi^2 u_{2t} + u_2 \\ &\vdots \\ u_{nttt} &= -u_{ntt} - n^2\pi^2 u_{nt} + u_n \end{aligned} \qquad (2.13)$$

The problem (2.13) now becomes

$$\begin{aligned} &Min\int_0^1 \left[u_1^2 + u_2^2 + \ldots + u_n^2\right]dt + \int_0^1 \left[u_{it}^2 + \ldots u_{nt}^2\right]dt \\ &+ \mu\int\left[\left\|u_{ittt} + u_{itt} + \pi^2 u_{it}\right\|^2 + \left\|u_{2ttt} + u_{2tt} + 2^2\pi^2 u_{2t} - u_2\right\|^2\right. \\ &+ .. + \left.\left\|u_{nttt} + u_{ntt} + n^2\pi^2 u_{nt} - u_n\right\|^2\right]dt \end{aligned} \qquad (2.14)$$

We now solve (2.13). However, since the sequence of equation (2.13) only differ by constant multiplicants, it suffices to solve just only one of the n-third order equations. [see Anthony and Jairo [6], Ibiejugba [7]]. By choosing the first equation we have

$$\frac{\partial^3 u(x,t)}{\partial t^3} + \frac{\partial^2 u(x,t)}{\partial t^2} + \pi^2\,\frac{\partial u(x,t)}{\partial t} - u(x,t) \qquad (2.15)$$

Equation (2.15) can be put in the alternative form, that is, the auxiliary equation for (2.15) as

$$m^3 + m^2 + \pi^2 m - 1 = 0 \qquad (2.16)$$

On solving (2.16) we find that the roots are 0.1002, - 0.5501 + 3.1108i and – 0.5501 – 3.1108i. If we take the norm of the complex conjugate roots, we find that in each case it is equal to 1.8030, which implies coincident root. Thus $m^3 + m^2 + \pi^2 m - 1 = 0$ has roots equal to 0.1002 and 1.8030 twice. By suppressing x we write the general equation of $u(x,t)$ in the form

$$U(\bullet,t) = Ae^{m_1 t} + (B+ct)e^{m_1 t} \tag{2.17}$$

where $m_1 = 0.1002$ and $m_2 = 1.8030$

$$u(\cdot,t) = u(0) = A + B = \sum_{I=1}^{\infty} u_I(0)\sin \pi ix \tag{2.18}$$

We take the partial derivative of (2.17) with respect to t and obtain

$$\begin{aligned} u(\cdot,t) &= m_1 Ae^{m_1 t} + m_2 Be^{m_2 t} + Ce^{m_1 t} + Cte^{m_1 t} = m_1 Ae^{m_1 t} + \left(m_2 B + C + Ct^2\right)e^{m_1 t} \\ u(0) &= m_1 A + M_2 B + C \end{aligned} \tag{2.19}$$

$$= \sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix \tag{2.20}$$

Solving equation (2.18) and (2.20) we have that, from (2.18),

$$A = \sum_{1=1}^{\infty} u_1(0)\sin \pi ix + B - m_2 B\,.$$

Then (2.19) becomes

$$m_1 \sum_{i=1}^{\infty} u_i(0)\sin \pi ix + B - m_2 B\,.$$

By (2.19), we have that,

$$\sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix = m_1 \sum_{1=1}^{\infty} u_i(0)\sin \pi ix - B + m_2 - m_1 \sum_{1=1}^{\infty} u_i(0)\sin \pi ix + B - m_2 B + m_2 B$$

$$\sum_{i=1}^{\infty} u_{it}(0)\sin \pi ix = m_2 B,\; B = \frac{\sum_{i=1}^{\infty} u_{it}(0)\sin \pi ix}{m_2}$$

But

$$A + B = \sum_{i=1}^{\infty} u_i(0)\sin \pi ix,$$

$$A = \sum_{i=1}^{\infty} u_i(0)\sin \pi ix - \frac{1}{m_2}\sum_{i=1}^{\infty} u_{it}(0)\sin \pi ix\,.$$

Then (2.19) becomes

$$m_1 \sum_{1=1}^{\infty} u_1(0)\sin \pi ix - \frac{m_1}{m_2}\sum_{i=1}^{\infty} u_{it}(0)\sin \pi ix + \sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix + C = \sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix$$

$$C = \sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix - m\sum_{1=1}^{\infty} u_i(0)\sin ix + \frac{m_1}{m_2}\sum_{1=1}^{\infty} u_i(0)\sin \pi ix$$

Then (2.17) becomes

$$\left[\sum_{1=1}^{\infty} u_i(0)\sin \pi ix - \frac{1}{m}\sum_{1=1}^{\infty} u_{it}(0)\sin \pi ix\right]e^{m_2 t} \tag{2.21}$$

$$u(\cdot,t)=\left[\sum_{1=1}^{\infty}u_i(0)\sin\pi ix-\frac{1}{1.8030}\sum_{1=1}^{\infty}u_1(0)\sin\pi ix\right]te^{1.8030t}$$

$$=\sum_{1=1}^{\infty}u_i(0)\sin\pi ix-0.5873\left[\sum_{1=1}^{\infty}u_{it}(0)\sin\pi ix+0.0588\sum_{1=1}^{\infty}u_{it}(0)\sin\pi ix\right]\exp(0.1002t)+$$

$$0.0588\left[\sum_{1=1}^{\infty}u_{it}(0)\sin\pi ix+0.0588\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(1.8030t)$$

$$u(x,t)=\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-\left[0.05873\sum_{I=1}^{\infty}u_{ti}(0)\sin\pi ix-0.5873(0.0588)\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(0.1002t)$$

$$+0.58773\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix.\exp(1.8030t)+0.0588\sum_{I=1}^{\infty}u_i(0)\sin\pi ix.\exp(1.8030t)$$

$$u(x,t)=\left[1.0588\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-0.5873\sum_{I=1}^{\infty}u_{ti}(0)\sin\pi ix\right]\exp(0.1002t)$$

$$+\left[0.5873\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix+0.0588\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right].\exp(1.8030t) \tag{2.22}$$

Also we have that,

$$z(x,t)=u_{it}(x,t)= m_1 A \exp(m_1 t) + m_2 (B+ct) \exp(m_2 t)$$

$$=\left[m_1\sum_{I=1}^{\infty}u_1(0)\sin\pi ix-\left(\frac{m_1}{m_2-m_1}\right)\sum_{I=1}^{\infty}u_{It}(0)\sin\pi ix\right]\exp(m_1t)+\left(\frac{m_2}{m_2-m_1}\right)$$

$$\times\left[\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix-m_1\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(m_2t)=0.1002\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-\left(\frac{0.1002}{1.8030-0.1002}\right)$$

$$\times\left\{\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix-0.1002\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right\}\exp(0.1002t)-\left(\frac{1.8030}{1.8030-0.1002}\right)$$

$$\left[\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix-0.1002\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(1.8030t)=\left[0.1002\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-0.3884\right.$$

$$\left.\times\sum_{I=1}^{\infty}u_i(0)\sin\pi ix+0.0058\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(0.1002t)+\left[1.0588\sum_{I=1}^{\infty}u_{it}(0)\sin\pi ix-0.31061\right.$$

$$\left.\times\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(1.8030t)$$

$$z(x,t)=\left[0.1061\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-0.05884\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(0.1002t)+$$

$$\left[1.05884\sum_{I=1}^{\infty}u_i(0)\sin\pi ix-0.1061\sum_{I=1}^{\infty}u_i(0)\sin\pi ix\right]\exp(1.8030t) \tag{2.23}$$

Here $z(x,t)$ is the optimal state of temperature at any point x of the rod, while $u(x,t)$ is the optimal control rate of flow of heat through the rod at any position .

We now apply a computational approach which is similar to that of Reju[8]. By using the following parameters: $u_i(0) = 0.1$, $u_{it}(0) = 0.1$, $i = 1, 2, \ldots, N$. In (2.22) and (2.23), we have the following results for the states and controls running the program in the Appendix for various x and t for the one dimensional wave-diffusion problem with N as presented in tables (2.1) and (2.2).

Table 2.1: Optimal sssssssparabolic states and controls for the wave equation with diffusion effect

	X	***T***	**Z* (*X*,*T*)**	***U** (*X*,*T*)**
N = 1 :	.1	.1	3.848228E-02	3.427516E-02
N = 2 :	.1	.1	7.319763E-02	6.519523E-02
N = 1 :	.1	.2	4.580888E-02	3.828762E-02
N = 2 :	.1	.2	8.713367E-02	7.282738E-02
N = 1 ;	.1	.3	5.458026E-02	4.306506E-02
N = 2 :	.1	.3	1.1038178	8.191462E-02
N = 1 :	.1	.4	6.508185E-02	4.875835E-02
N = 2 :	.1	.4	.123793	9.274389E-02
N = 1 :	.1	.5	7.765546E-02	5.554814E-02
N = 2 :	.1	.5	.1477095	.1056588
N = 1 :	.1	.6	9.271046E-02	.0636508
N = 2 :	.1	.6	.1763458	.121071
N = 1 :	.1	.7	.1107371	7.332543E-02
N = 2 ;	.1	.7	.2106344	.1394733
N = 1 :	.1	.80000001	.1323224	8.488232E-02
N = 2 :	.1	.80000001	.2516921	.1614558
N = 1 :	.1	.90000001	.1581695	9.869307E-02
N = 2 :	.1	.90000001	.3008563	.1877254
N = 1 :	.2	.1	7.319763E-02	6.519523E-02
N = 2 :	.2	.1	.1184363	.1054881
N = 1 :	.2	.2	8.713367E-02	7.282738E-02
N = 2 ;	.2	.2	.1409852	.1178372
N = 1 :	.2	.3	.1038178	8.191462E-02
N = 2 :	.2	.3	.1679808	.13256406
N = 1 :	.2	.4	.123793	9.274389E-02
N = 2 :	.2	.4	.2003013	.1500628
N = 1 :	.2	.5	.1477095	.1056588
N = 2 :	.2	.5	.2389989	.1709596
N = 1 :	.2	.6	.1763458	.121071
N = 2 :	.2	.6	.2853335	.195897
N = 1 :	.2	.7	.2106344	.1394733
N = 2 ;	.2	.7	.3408137	.2256725
N = 1 :	.2	.80000001	.2516921	1614558
N = 2 :	.2	.80000001	.4072465	.2612409
N = 1 :	.2	.90000001	.3008563	.1877254
N = 2 ;	.2	.90000001	.4867957	.303746
N = 1 :	.2	.1	.1007479	8.973353E-02
N = 2 :	.3	.1	..1184363	.1054881
N = 1 :	.3	.2	.1199292	..1002383
N = 2 :	.3	.2	.1409852	.1178372
N = 1 :	.3	.3	.142893	.1127458
N = 2 :	.3	.3	.1679808	.1325406
N = 1 :	.3	.4	.1703865	.127651
N = 2 :	.3	.4	.2003013	.1500628
N = 1 :	.3	.5	.2033046	.1454269

	X	T	Z* (X,T)	U* (X,T)
N = 2 ;	.3	.5	.2389989	.1709596
N = 1 :	.3	.6	.2427191	.6664
N = 2 :	.3	.6	.2853334	.195897
N = 1 :	.3	.7	.2899134	.1919685
N = 2 :	.3	.7	.3408136	.2256725
N = 1 :	.3	.80000001	.3464245	.2222248
N = 2 :	.3	.80000001	.4072464	.2612409
N = 1 :	.3	.90000001	.4140932	.2583818
N = 2 :	.3	.90000001	.4867957	.303746
N = 1 :	.4	.1	.1184363	.1054881
N = 2 :	.4	.1	7.319763E-02	6.519522E-02
N = 1 :	.4	.2	.1409852	.1178372
N = 2 :	.4	.2	8.713365E-02	7.282737E-02
N = 1 :	.4	.3	.1679808	.1325406
N = 2 :	.4	.3	.1038178	.0819146
N = 1 :	.4	.4	.2003013	.1500628
N = 2 :	.4	.4	.123793	9.274387E-92
N = 1 :	.4	.5	.2389989	.1709596
N = 2 :	.4	.5	.1477094	.1056588
N = 1 :	.4	.6	.2853335	.195897
N = 2 :	.4	.6	.1763457	.121071
N = 1 :	.4	.7	.3408137	.2256725
N = 2 :	.4	.7	.2106344	.1394732
N = 1 :	.4	.80000001	.4072465	.2612409
N = 2 :	.4	.80000001	.2516921	..1614557
N = 1 :	.4	.90000001	.4867957	.303746
N = 2 :	.4	.90000001	.3008562	.1877254
N = 1 :	.5	.1	.1245313	.1109167
N = 2 :	.5	.1	-1.088687E-08	-9.69665E-09

Results summary

$z^* = -1.088687E - 08,\ U^* = -9.69665E - 09,\ at\ N = 2,\ X^* = .5,\ T^* = .1$

Table 2.2: Optimal parabolic states and controls for the wave equation with diffusion effect

	X	T	Z* (X,T)	U* (X,T)
N = 1 :	.1	.1	3.848228E-02	3.427516E-02
N = 2 :	.1	.1	7.319763E-02	6.519523E-02
N = 3 :	.1	.2	.1007479	8.828762E-02
N = 4 :	.1	.2	.1184363	.1054881
N = 5 ;	.1	.2	.1245313	.1109167
N = 1 :	.1	.2	4.580888E-02	3.828762E-02
N = 2 :	.1	.2	6.508185E-02	4.875835E-02
N = 3 :	.1	.2	.1199292	.1002383
N = 4 :	.1	.2	.1409852	.1178372
N = 5 :	.1	.2	.1482406	.1239013
N = 1 :	.1	.3	5.458026E-02	4.306506E-02
N = 2 :	.1	.3	.1038179	8.191462E-02
N = 3 :	.1	.3	.142893	.1127458
N = 4 ;	.1	.3	.1679808	.1325406
N = 5 :	.1	.3	.1766254	.1393615
N = 1 :	.1	.4	6.508185E-02	4.875835E-02

	X	*T*	*Z** (*X*,*T*)	*U** (*X*,*T*)
N = 1 :	.1	.1	3.848228E-02	3.427516E-02
N = 2 :	.1	.4	.123783	9.274389E-02
N = 3 :	.1	.4	.1703865	.127651
N = 4 :	.1	.4	.2003013	.1500628
N = 5 :	.1	.4	.2106093	.1577853
N = 1 :	.1	.5	7.765546E-02	5.554814E-02
N = 2 ;	.1	.5	.1477095	.1056588
N = 3 :	.1	.5	.2033046	.1454269
N = 4 :	.1	.5	.2389989	.1709596
N = 5 :	.1	.5	.2512983	.1797575
N = 1 :	.1	.6	9.271046E-02	.0636508
N = 2 :	.1	.6	.176358	.121071
N = 3 :	.1	.6	.2427191	.16664
N = 4 :	.1	.6	.2853335	.195897
N = 5 :	.1	.6	.3000173	.2059783
N = 1 :	.1	.7	.1107371	7.332543E-02
N = 2 ;	.1	.7	.2106344	.1394733
N = 3 :	.1	.7	.2899134	.191985
N = 4 :	.1	.7	.3408137	.2256725
N = 5 :	.1	.7	.3583527	.2372861
N = 1 ;	.1	.80000001	.1323224	8.488232E-02
N = 2 :	.1	.80000001	.2516921	.1614558
N = 3 :	.1	.80000001	.3464245	.2222248
N = 4 :	.1	.80000001	.4072465	.2612409
N = 5 :	.1	.80000001	.4282043	.2746849
N = 1 :	.1	.90000001	.1581695	9.869307E-02
N = 2 :	.1	.90000001	.3008563	.1877254
N = 3 :	.1	.90000001	.4140932	.2583818
N = 4 :	.1	.90000001	.4867957	.303746
N = 5 :	.1	.90000001	.5118473	.3193775
N = 1 ;	.2	.1	7.319763E-02	6.519523E-02
N = 2 :	.2	.1	.1184363	.1054881
N = 3 :	.2	.1	.1184363	.1054881
N = 4 :	.2	.1	7.319763E-02	6.519522E-02
N = 5 :	.2	.1	-1.67166E-08	-1.488904E-02

Result summary

Z^* = -1.67166E –08, U* -1.488904E –08 at $N = 5$, $X^* = .2$, $T^* = .1$

The computational simulations for the integral quadratic functional with wave-diffusion control problem exhibit the most unique characteristics phenomena. The following remarks are in order in the analysis of the simulated solutions.

(i) A sudden change in the input $u(x,t)$ gives rise to a corresponding sudden optimal change in the state $z(x,t)$

(ii) In the parabolic control problems, we observed that the optimal solution (states and controls) are exponential in time and periodic in space.

(iii) The exponential time contribution exhibits a damping influence on the periodic space perturbations of the optima.

3.0 Conclusion

The result in (2.22) and (2.23) which is the states and controls of the system (2.1) is optimum. The computational results in table (2.1) and (2.2) buttress the truth for convergence as N increases. We can also use the EXTENDED CONJUGATED CRADIENT METHOD (ECGM) proposed by Reju [8] and others to obtain the solutions of (2.22) and (2.23) as required.

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