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Thermosolutal MHD flow and radiative heat transfer with viscous work and heat source over a vertical porous plate

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Abstract

This paper investigates double diffusive convection MHD flow past a vertical porous plate in a chemically active fluid with radiative heat transfer in the presence of viscous work and heat source. The resulting nonlinear dimensionless equations are solved by asymptotic analysis technique giving approximate analytic solutions for the steady velocity, temperature and concentration. The parameters involved are used to give pictorial illustrations of the distributions of the flow variables and are discussed. Also the shear stress, heat and mass transfer characteristics in terms of the flux rates at the plate wall are discussed.

Keywords
Heat source; Optically thin incompressible fluid; thermosolutal MHD flow

AMS Subject Classification: 76W05, 76R10, 76R50, 76V05, 76S05

1 0 Introduction

Thermosolutal or otherwise known as double diffusive convection [1] occurs in many transport processes and industrial applications. Its occurrence is as results of the spatial variations in density, which are known as buoyancy effects, caused by the nonuniform distribution or diffusion of heat and chemical species. The study of such processes in porous media have several applications in the design of chemical processing equipments, catalytic reactors, geophysics, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing and pollution of the environment. Other applications are found in reservoir engineering, where thermal recovery processes are encountered and in the study of dynamics of hot and salty springs of a sea. In particular and specifically, these processes are useful in investigating the movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes [2].

Nomenclature

u - flow along plate	α_d - thermal diffusivity
v - flow across plate	D - effective diffusion coefficient
U_∞ - free stream velocity	k - permeability of porous medium
ρ - fluid density	ν - kinematic viscosity

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c - concentration	σ_c - electric conductivity of the fluid
c_∞ - free stream concentration	k_r - first-order chemical reaction rate
T - temperature of fluid	g - acceleration due to gravity
T_∞ - free stream temperature	g_m - modified gravitational acceleration
T_w - wall temperature	c_w - wall concentration
U - dimensionless velocity	Θ - dimensionless temperature
C - dimensionless concentration	c_p - specific heat at constant pressure
y - co-ordinate across flow	Q - heat generation constant
q - radiative flux	B_0 - applied magnetic field strength
σ - Stefan-Boltzmann constant	α - absorption coefficient
M - magnetic parameter	G_r - thermal Grashof number
G_m - mass Grashof number	χ - porosity parameter
P_r - Prandtl number	N - radiation parameter
δ - heat source/sink parameter	E_c - Eckert number
S_c - Schmidt number	R - reaction parameter

Investigations of heat generation in fluids are important in problems dealing with chemical reactions. The consequence of heat generation is the likely alteration of temperature, thereby affecting the particle deposition rate in nuclear reactors, electric chips and semiconductor wafers [3]. It is emphasized here that the study of chemical reaction, heat and mass transfer with heat radiation is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes [2].

The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated [4]. In most of cases of chemical reaction, the reaction rate depends on the concentration of the species itself.

The applications of MHD theory to technology and the effects of magnetic field on the performance of many systems have long been studied [5, 6]. The uses of magnetic field to control the flow and heat transfer processes in fluids near different types of boundary layer flows are now well known. For example, the particular case of liquid metal MHD, magnetic fields are used to levitate samples of liquid metal, to control their shape and to induce internal stirring for the purpose of homogenisation of the final product [7]. The advent of studies of thermonuclear fusion reaction enhanced problems associated with the behaviour of high-temperature plasma in a magnetic field. In power engineering and metallurgy, the thermal physics of hydromagnetic problems with mass transfer has considerable interest. It is also pertinent to note here that due to industrialization, the environment continues to experience the impact of oil, gas flaring, toxic chemicals and biological wastes with attendant heat generation [8]. Therefore, the inclusion of viscous work or dissipation and heat generation source terms in the energy equation, except of the theoretical interest, has applications in glaciology, in granular material, in the infall of molten iron during gravitational differentiation of terrestrial planets, in the interaction between the crust and mantle during continental convergence and in the separation of oceanic crust from the descending oceanic lithosphere. The viscous dissipation term is always positive and represents also a source of heat due to friction between the fluid particles. A variety of expressions are used in the literature for this term like viscous heating, shear-stress heating and viscous work [9].

Studies of combined heat and mass transfer effects on MHD free convection flow [10, 11], indicates that radiative heat transfer and chemical reaction effects in the presence of viscous work and a heat source are rarely reported. This paper, therefore, incorporates radiative heat transfer and chemical reaction effects to the study of thermosolutal MHD flow past a vertical porous plate embedded in a porous medium in the presence of heat source. Regular perturbation or asymptotic expansion method is herein employed to advance approximate analytic results, which could serve as kits to testing numerical codes.

In Section 2, the mathematical formulations of the problem and dimensionless forms of the governing equations are established. Solutions to these equations for the flow variables are obtained in Section 3. The shear stress, the heat and mass transfer rates are obtained in section 4. The results of the previous sections are discussed in section 5. In section, 6 general concluding remarks of the results of the previous sections are given.

2.0 Mathematical formulations

The equation governing the radiative flux is generally nonlinear. However, under the condition that the radiative flux reflects the notion of an optically thin environment such as one would find in the intergalactic layers where the plasma gas is assumed to be of low density and α<<1 [12], the nonlinearity may be eliminated. In this case there will be no measurable change in temperature elsewhere within the plate and the equilibrium temperature. Thus, in a one space coordinate *y*, the linear differential approximation of Cogley-Vincenti-Gilles equilibrium model [13] of the radiative flux *q*

$$\frac{dq}{dy} = 16\sigma\alpha T_\infty^3 (T - T_\infty) \tag{2.1}$$

becomes significant. The subscript ∞ denotes conditions in the undisturbed fluid.

The equation of state for a binary incompressible fluid, which satisfies Boussinesq approximation, is considered here. It is given by

$$\rho = \rho_\infty[1 - \beta(T - T_\infty) - \beta_m(c - c_\infty)]. \tag{2.2}$$

A steady two-dimensional vertical flow of electrically conducting, incompressible, viscous and thermally radiating binary fluid past an infinite vertical porous plate with constant suction in the presence of heat source is assumed. It is further considered that the plate is embedded in a porous medium. A magnetic field of strength B_0 is assumed to be applied transversely to the plate, so that there exists free convection currents in the vicinity of the plate. The *x*-axis is taken along the plate in the vertically upward direction and the *y*-axis is taken normal to the plate in the direction of the applied magnetic field. Then, invoking Boussinesq approximation (2.2) of an incompressible fluid model for negligible small pressure gradients along the direction of motion, and the flow is such that the induced magnetic fields are neglected, the governing equations of a chemically active fluid with a first-order chemical reaction and radiative heat transfer (2.1) with viscous work are given by

$$v\frac{du}{dy} = \nu\frac{d^2u}{dy^2} - \frac{\sigma_c B_0^2}{\rho}(u - U_\infty) + g\beta(T - T_\infty) + g_m\beta_m(c - c_\infty) - \frac{\nu}{k}(u - U_\infty), \tag{2.3}$$

$$v\frac{dT}{dy} = \alpha_d \frac{d^2T}{dy^2} - \frac{1}{\rho c_p}\frac{dq}{dy} + Q(T_\infty - T) + \frac{\mu}{\rho c_p}\left(\frac{du}{dy}\right)^2, \tag{2.4}$$

$$v\frac{dc}{dy} = D\frac{d^2c}{dy^2} - k_r(c - c_\infty). \tag{2.5}$$

The magnetic and porosity terms $\frac{\sigma_c B_0^2}{\rho}(u - U_\infty)$ and $\frac{\nu}{k}(u - U_\infty)$, respectively, signify that the magnetic field and the porous medium are fixed relative to the plate. The term $Q(T - T_\infty)$ is assumed to be the amount of heat generated or absorbed per unit volume. Q is considered a constant, which may take on either positive or negative values.

When the wall temperature T_w exceeds the free stream temperature T_∞, the source term represents the heat source when $Q < 0$ and heat sink when $Q > 0$. For the condition that $T_w < T_\infty$, the opposite relationship is true.

The boundary conditions associated to equations (2.3 – 2.5) are

$$u = 0, v = -v_0, T = T_w, c = c_w \text{ at } y = 0, \tag{2.6}$$

$$u = U_\infty, T = T_\infty, c = c_\infty \text{ as } y \to \infty, \tag{2.7}$$

where v_0 is a constant, accounts for suction or injection depending on whether v_0 is positive or negative.

In order to facilitate the analysis, the following dimensionless variables and parameters are employed:

$$Y=\frac{yv_0}{\nu},\ U=\frac{u}{U_\infty},\Theta=\frac{T-T_\infty}{T_w-T_\infty},\ C=\frac{c-c_\infty}{c_w-c_\infty},\ M=\frac{\nu\sigma_c B_0^2}{\rho v_0},$$
$$G_r=\frac{\nu\beta g(T_w-T_\infty)}{v_0^2U_\infty},\ G_m=\frac{\nu\beta_m g_m(c_w-c_\infty)}{v_0^2U_\infty},\ \chi=\frac{\nu}{kv_0^2},\ \Pr=\frac{\nu}{\alpha_d}, \tag{2.8}$$
$$N=\frac{16\nu\alpha\sigma T_\infty^3}{\rho c_p v_0},\ \delta=\frac{Q\nu}{v_0^2},\ E_c=\frac{U_\infty}{c_p(T_w-T_\infty)},\ Sc=\frac{\nu}{D},\ R=\frac{\nu k_r}{Dv_0^2}.$$

Therefore, the dimensionless governing equations are

$$\frac{d^2U}{dY^2}+\frac{dU}{dY}-M(U-1)+G_r\Theta+G_mC-\chi(U-1)=0, \tag{2.9}$$
$$\frac{d^2\Theta}{dY^2}+P_r\frac{d\Theta}{dY}-P_r(N+\delta)\Theta+P_rE_c\left(\frac{d\Theta}{dY}\right)^2=0, \tag{2.10}$$
$$\frac{d^2C}{dY^2}+S_c\frac{dC}{dY}-S_cRC=0, \tag{2.11}$$

with the associated boundary conditions as

$$U=0,\ \Theta=1,\ C=1 \text{ at } Y=0, \tag{2.12}$$
$$U=1,\ \Theta=0,\ C=0 \text{ as } Y\to\infty. \tag{2.13}$$

The parameters entering the problem are M, magnetic parameter; G_r, thermal Grashof number; G_m, mass or modified Grashof number; χ, porosity parameter; P_r, Prandtl number; N, radiation parameter; δ, source/sink parameter; E_c, Eckert number; S_c, Schmidt number, and R, reaction parameter. The mathematical statement of the problem embodies the solution of equations (2.9 – 2.11) subject to equations (2.12 – 2.13).

3.0 Method of Solution

The equations (2.9 – 2.11) are coupled nonlinear differential equations and the heat due to viscous work is superimposed on the motion. In seeking analytic solutions to such coupled nonlinear equations, it is usual practice to expand the field variables in series about a small parameter. Fortunately, the Eckert number, which is a measure of the rigor of heat convection, is always small (i. e. $E_c \ll 1$) for most incompressible fluids. Therefore, approximate solutions are adopted as follows:

$$\Phi(Y)=\Phi^0(Y)+E_c\Phi^1(Y)+0(E_c{}^2), \tag{3.1}$$

where Φ stands for any of U,Θ or C.

Using equation (3.1) in equations (2.9 – 2.11) together with the associated boundary conditions (2.12 – 2.13), straightforward algebra gives the respective solution for U,Θ and C as follows:

$$U(Y)=1+\beta_1(e^{-\alpha_3Y}-e^{-\alpha_1Y})+\beta_2(e^{-\alpha_3Y}-e^{-\alpha_2Y})-e^{-\alpha_3Y}-G_rE_c[B_1e^{-2\alpha_3Y}$$
$$+B_2e^{-2\alpha_2Y}+B_3e^{-2\alpha_1Y}+B_4e^{-(\alpha_1+\alpha_3)Y}+B_5e^{-(\alpha_2+\alpha_3)Y}+B_6e^{-(\alpha_1+\alpha_2)Y}], \tag{3.2}$$
$$\Theta(Y)=e^{-\alpha_1Y}+P_rE_c[A_1e^{-2\alpha_3Y}+A_2e^{-2\alpha_2Y}+A_3e^{-2\alpha_1Y}+A_4e^{-(\alpha_1+\alpha_3)Y}$$
$$+A_5e^{-(\alpha_2+\alpha_3)Y}+A_6e^{-(\alpha_1+\alpha_2)Y}], \tag{3.3}$$
$$C(Y)=e^{-\alpha_2Y}, \tag{3.4}$$

where

$$\alpha_1 = \frac{1}{2}P_r + \frac{1}{2}\sqrt{P_r^2 + 4P_r(N+\delta)}, \alpha_2 = \frac{1}{2}S_c + \frac{1}{2}\sqrt{S_c^2 + 4S_cR}, \alpha_3 = \frac{1}{2} + \frac{1}{2}\sqrt{1+4(M+\chi)},$$

$$\beta_1 = \frac{G_r}{\alpha_1^2 - \alpha_1 - M - \chi}, \beta_2 = \frac{G_m}{\alpha_2^2 - \alpha_3 - M - \chi}, A_1 = -\frac{\alpha_3^2(1+\beta_1^2 + 2\beta_1\beta_2 + \beta_2^2 - 2\beta_1 - 2\beta_2)}{4\alpha_3^2 - 2P_r\alpha_3 - P_r(N+\delta)},$$

$$A_2 = -\frac{\beta_2^2\alpha_2^2}{4\alpha_2^2 - 2P_r\alpha_2 - P_r(N+\delta)}, A_3 = -\frac{\beta_1^2\alpha_1^2}{4\alpha_1^2 - 2P_r\alpha_1 - P_r(N+\delta)},$$

$$A_4 = \frac{2\alpha_1\alpha_2(\beta_1^2 + \beta_1\beta_2 - \beta_1)}{(\alpha_1+\alpha_3)^2 - P_r(\alpha_1+\alpha_3) - P_r(N+\delta)}, A_5 = \frac{2\alpha_2\alpha_3(\beta_2^2 + \beta_1\beta_2 - \beta_2)}{(\alpha_2+\alpha_3)^2 - P_r(\alpha_2+\alpha_3) - P_r(N+\delta)}, \tag{3.5}$$

$$A_6 = -\frac{2\alpha_1\alpha_2\beta_1\beta_2}{(\alpha_1+\alpha_2)^2 - P_r(\alpha_1+\alpha_2) - P_r(N+\delta)}, B_1 = \frac{A_1}{4\alpha_3^2 - 2\alpha_3 - M - \chi}, B_2 = \frac{A_2}{4\alpha_2^2 - 2\alpha_2 - M - \chi},$$

$$B_3 = \frac{A_3}{4\alpha_1^2 - 2\alpha_1 - M - \chi}, B_4 = \frac{A_4}{(\alpha_1+\alpha_3)^2 - (\alpha_1+\alpha_3) - (M+\chi)}, B_5 = \frac{A_5}{(\alpha_2+\alpha_3)^2 - (\alpha_2+\alpha_3) - (M+\chi)},$$

$$B_6 = \frac{A_6}{(\alpha_1+\alpha_2)^2 - (\alpha_1+\alpha_2) - (M+\chi)}.$$

4.0 Shear stress, heat and mass transfer fluxes

From the physical point of view, it is necessary to know the shear on the plate wall. It is given by

$$\tau = \mu \frac{du}{dy}\Big|_{y=0}, \tag{4.6}$$

which by virtue of equations (2.8) can be written as

$$\tau_s = \frac{\tau}{\rho U_\infty v_0} = \frac{dU}{dY}\Big|_{Y=0} \tag{4.7}$$

$$\therefore \quad \begin{aligned} \tau_s &= \beta_1(\alpha_1 - \alpha_2) + \beta_2(\alpha_2 - \alpha_3) + \alpha_3 + G_rE_c[2B_1\alpha_3 + 2B_2\alpha_2 + 2B_3\alpha_1 \\ &\quad + B_4(\alpha_1+\alpha_3) + B_5(\alpha_2+\alpha_3) + B_6(\alpha_1+\alpha_2)]. \end{aligned} \tag{4.8}$$

Knowing the temperature distribution we can calculate the rate of heat transfer, q_T, between the fluid and the wall of the plate from the relation

$$q_T = -K_T \frac{dT}{dy}\Big|_{y=0}, \tag{4.9}$$

which by virtue of equations (2.8) reduces to the following non-dimensional form:

$$q_h = \frac{hq_T}{K_T(T_w - T_\infty)} = \frac{d\Theta}{dY}\Big|_{Y=0} \tag{4.10}$$

$$\therefore \quad \begin{aligned} q_h &= -\alpha_1 - P_rE_c[2A_1\alpha_3 + 2A_2\alpha_2 + 2A_3\alpha_1 + A_4(\alpha_1+\alpha_3) \\ &\quad + A_5(\alpha_2+\alpha_3) + A_6(\alpha_1+\alpha_2)]. \end{aligned} \tag{4.11}$$

Also, knowing the concentration distribution, the non-dimensional form of the rate of mass transfer is simply,

$$q_m = \frac{h_m q_c}{D(c_w - c_\infty)} = \frac{dC}{dY}\Big|_{Y=0}, = -\alpha_2, \tag{4.12}$$

where h and h_m expresses respectively the heat and mass transfer coefficients.

5.0 Discussion of results

The problem of thermosolutal MHD flow past a vertical porous plate with radiative heat transfer in the presence of viscous work and heat source for a chemically active incompressible and optically thin fluid has been solved. Approximate analytic solutions are derived using the technique of regular perturbation. The discussions are based on the parameters entering the problem, and these are M, magnetic parameter; G_r, Grashof number or free convection parameter; G_m, mass or modified Grashof number; χ, suction parameter; P_r, Prandtl number; N, radiation parameter; δ, heat source; E_c, Eckert number; S_c, Schmidt number, and R, chemical reaction parameter.

The value of the Schmidt number, S_c, which is chosen as 0.62 corresponds to water vapour that represents a diffusion chemical species of most common interest in air. The Prandtl number P_r is set equal to a fixed value of 0.71 throughout the investigations, which physically corresponds to an astrophysical body (air) at 20^0C. Air is chosen because it is weakly electrically conducting under certain circumstances. Whereas the Grashof number $G_r < 0$ depicts external heating of the plate by free convection currents, $G_r > 0$ corresponds to external cooling of the plate and $Gr = 0$, corresponds to absence of free convection currents. Only values of $G_r = 2.0$ and $G_m = 1.0$ are considered in the discussions. Typical values of the other parameters used are indicated on the graphs.

Figures 5.1 are due to the velocity solution (3.2) for various flow parameters. It is observed from Figures 5.1a, 5.1b, 5.1c, 5.1d that the velocity exhibit overshoots for Magnetic, radiation, reaction and heat source parameters, respectively, and steadily approaches the profile shape for which $U(\infty) = 1$. The velocity is zero at the plate and non-zero at the edge of the boundary layer. The velocity reaches a maximum in the boundary layer. We note from Figures 1 that although the boundary layer thickness is $\delta_U \approx 4$, the maximum velocity occurs at $Y \approx 1.4$. In Figure 5.1, the velocity decreases with increasing magnetic, radiation, reaction and heat source parameters, respectively. In particular, the decreasing effect of the magnetic parameter on the velocity indicates that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. This result qualitatively agrees with the expectations of the effects of magnetic strength.

Figure 5.2 gives temperature profiles for different values of the Magnetic, radiation, reaction and heat source parameters, respectively. It is observed that the temperature reduces with increasing radiative heat transfer and heat source. Consequently, the radiative heat transfer and heat source reduces the thermal boundary layer thereby increasing the rate of heat transfer to the plate. Increasing magnetic and heat source parameters produces no significant difference in the temperature.

The effect of chemical reaction on the concentration is shown in the Figure 5.3. It is observed that the concentration of the fluid reduces with increase of the chemical reaction. This implies reduction in the mass boundary layer.

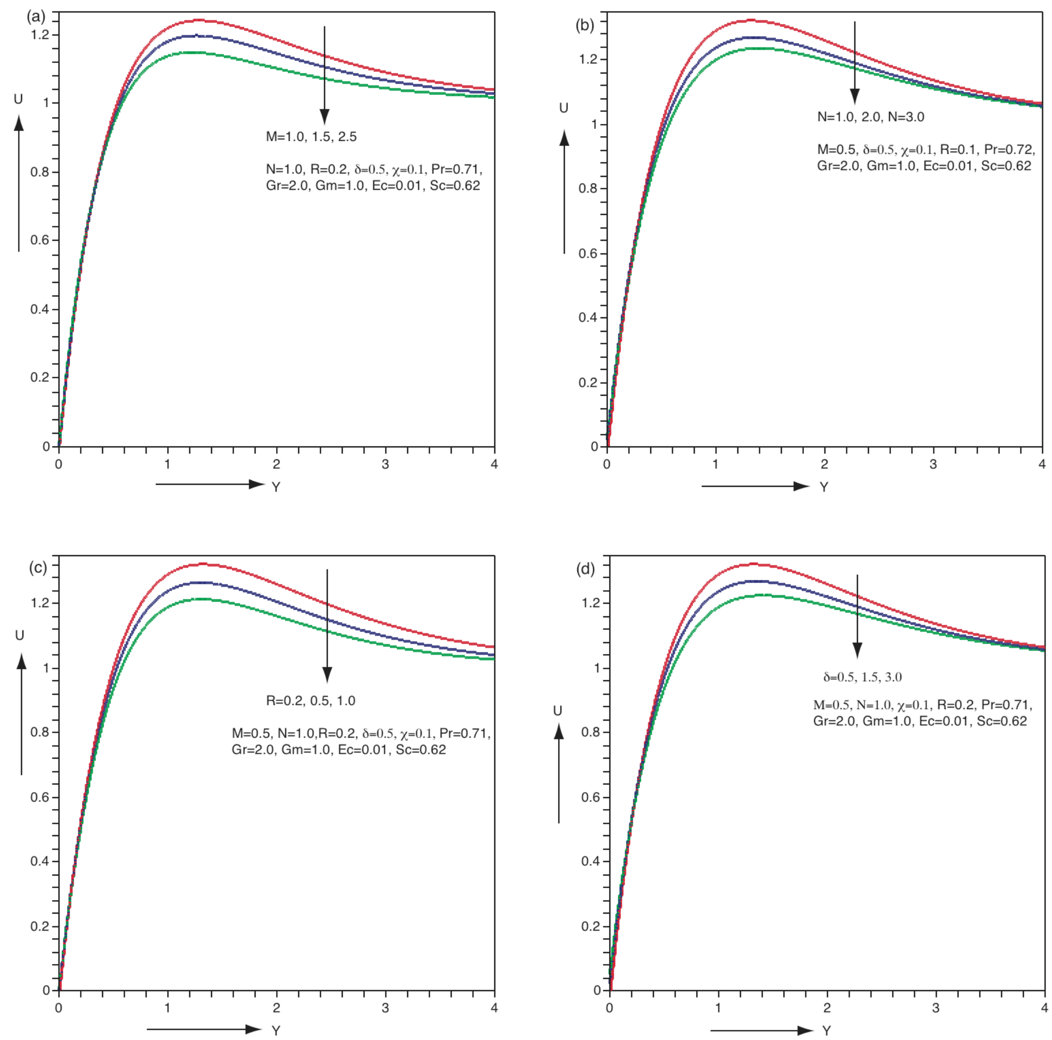

Figure 5.1: Velocity profiles as a function of Y for variations in the parameters (a) Magnetic, M; (b) Radiation, N; (c) Chemical Reaction, R; (d) Heat Source, δ.

6.0 Concluding remarks

The problem of thermosolutal MHD flow past a vertical porous plate with radiative heat transfer in the presence of viscous work and heat source for a chemically active incompressible and optically thin fluid has been examined. Approximate analytic solutions of the flow variables are presented. Some physical parameters were identified entering the problem. It is generally observed that the flow variables are significantly influenced by these parameters. The features shown by the present results are consistent and has an excellent agreement with those previously reported in literature [2, 3].

The primary findings are summarized as follows:

- Increase in the magnetic strength decreases the velocity, while it shows no significant effect on the temperature.
- The velocity and temperature of the fluid decreases with increasing radiation. While increasing radiation signifies reduction in the maximum velocity, for the temperature it decreases the thermal boundary layer thickness, physically implying higher heat transfer to the plate.

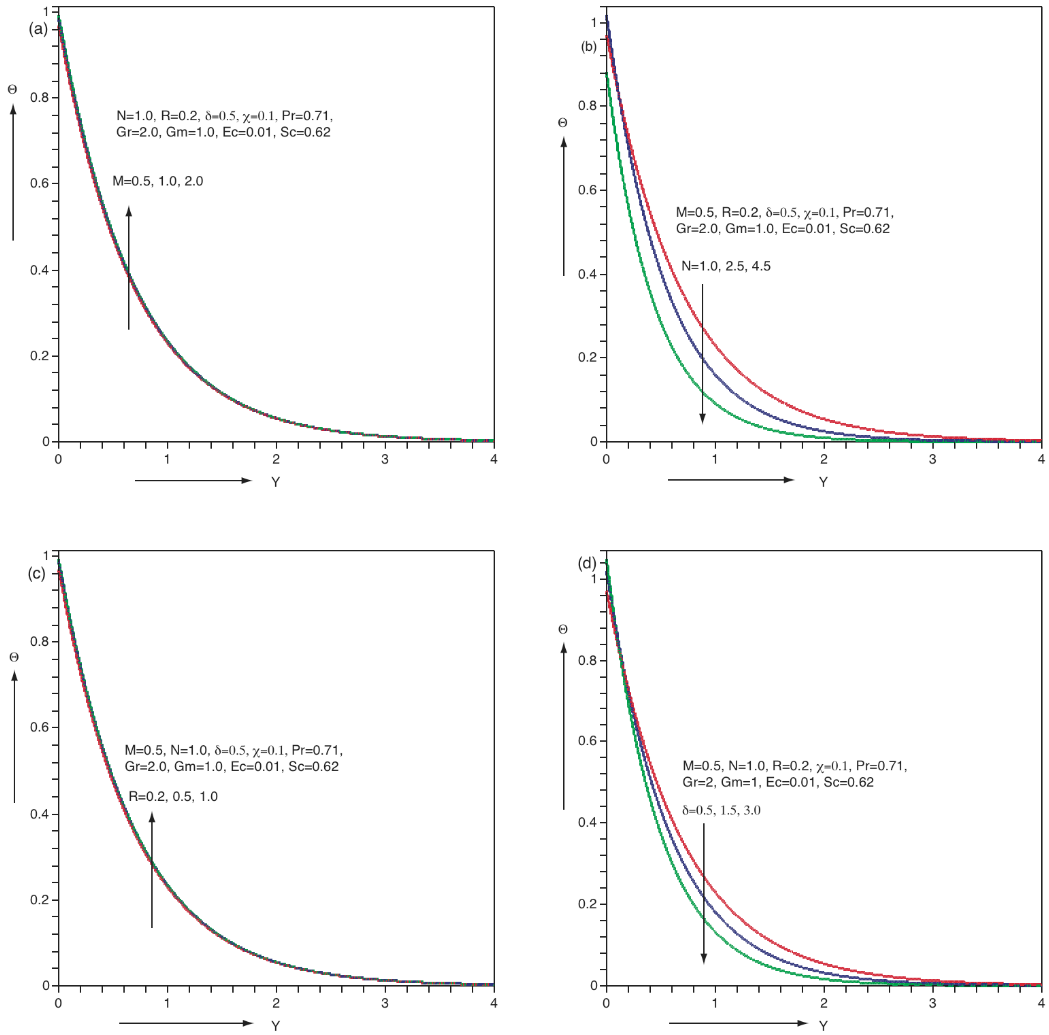

Figure 5.2: Velocity profiles as a function of Y for variations in the parameters (a) Magnetic, M; (b) Radiation, N; (c) Chemical Reaction, R; (d) Heat Source, δ.

- The velocity and concentration of the fluid decreases with increase in the chemical reaction, while chemical reaction shows minimal effects on the temperature. Physically, the effect of the chemical reaction reducing the concentration means reduction in the concentration boundary layer, thereby increasing the rate of mass transfer in the fluid.
- Increasing heat source decreases both the velocity and temperature. Significantly, the heat source plays similar role like the radiation.

Thus, it is pertinent to note that the applications of magneto-chemistry to processes of flow in geothermal, geophysical and astrophysical environments even cosmic regions whose temperatures are usually very high and which radiate a lot of heat through the media surrounding them, requires a complete understanding of the equation of state and transport properties such as diffusion, the shear stress-shear rate relationships, thermal conductivity, electrical conductivity, and radiation. Therefore, it has been shown that some of these properties are influenced by the presence of an external magnetic field that sets the plasma in hydrodynamic motion.

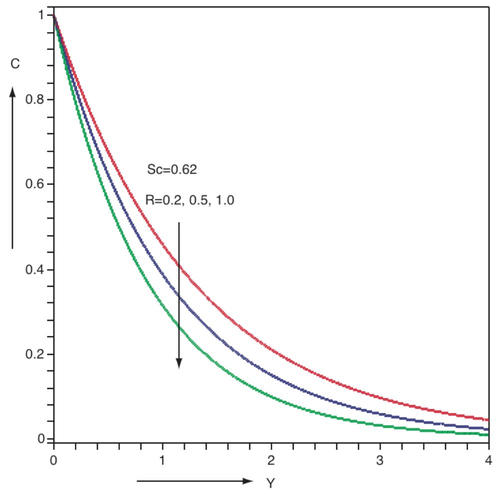

Figure 5.3: Concentration profiles as a function of Y for variations in the parameter: Chemical Reaction, R

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