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On the multiplicity effect of an *m*-fold moving point load on the dynamic response of an Euler beam resting on a Kelvin foundation.

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Abstract

In this work the response of an elastic beam supported by viscoelastic foundation (Winkler Model) to an external excitation (force) is investigated with particular attention to the efect of the excitation by a multipple cyclic moving load. The effect of the multiplicity of the cyclic - moving load with respect to the amplitude of vibration of the structure is examined. It was. It was observed that the multipple load system has a multiplicative effect on the condition of the resonance of the beam – moving load on the condition of the resonance of the beam – moving load system.

Keywords
Multibeam, visco-elastic foundation, winkler model

1.0 Introduction

With the ever increasing velocities anticipated for future, sea, land and air vehichles one can expect corresponding increase in the number o situations in which the suitability of a structure depends on its ability to withstand the effect of a moving load. A large variety of structural configurations in which such dynamic load can be applied includes beams, plates, shells etc. These structures may either be elastic, viscoelastic or inelastic and the moving load may be of constant or variable magnitude. In addition to the edge constraints the structure may or may not be continuosly supported by the foundation.

Associated with this dynamic system are the phenomena of resonance with it ambivalent properties. This is a technical term that discribes the sudden amplifiction of a vibrating body when the frequency of the driving (external excitation) force approaches the natural frequency of the body. This is a fequent occurrence in the field of electronics, accoustics, highway and structural engineering etc. while this phenomenon is desirable in some fields such as comminication engineering it is a nuisance in the field of strutural and high way engineering. The Tacoma narrow bridge disaster in Washington is a clear example of the undesirability of resonance in structures which is actually the motivation for this research work.

In this reseach work therefore an elastic beam of length ρ is transverse by multiple cyclic moving point loads at position χ_i. It is assumed that the ith load is moving with transverse and longitudinal frequencies of $\Box_i$ and β_i respectively. The position of the *i*th load is defined parametrically according to Aiyesimi [1] as;

$$x_i = \tilde{x}_i + \chi_i Sin\beta_i t$$

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The beam is futher assumed to be continuously supported by a viscoelastic foundation with a viscosity constant ε. This model is adopted because of the practical importance of Viscoelastic foundations in those viscoelastic materials are very useful for their high energy dissipation capability. Very often attempts are made in practical situations exploiting their energy dissipation properties in damping undesirable vibrations.

Hence for the aforementioned reason we have considered the model integrating the desirable property of viscosity into the moving load – beam system.

2.0 Mathematical formulation

The uniform bean of length ρ, with uniform mass per unit length m, resting on a Kelving foundation of viscosity ε_0, is assumed to be simply-supported, at both ends. At time $t = 0$, the load $F(x,t)$ is dropped on the beam at the point x_0. Consequently, the external excitation by the moving load results in the transverse displacement $z(x,t)$ of the bean from its equilibrium position. According to Aiyesimi [1], the governing equation of motion of the beam is given as

$$EI\frac{\partial^4 z}{\partial x^4} - N\frac{\partial^2 z}{\partial x^2} - m\frac{\partial^2 z}{\partial t^2} + \varepsilon_0\frac{\partial z}{\partial t}KZ = F(x,t) \tag{2.1}$$

In particular, considering a multiple cyclic moving load, then $F(x,t)$, according to Oni [2] takes the form:

$$F(x,t) = \sum P_j \cos\omega_j t\delta\left[x - \left(x_j + \chi_j \sin\beta_j t\right)\right] \tag{2.2}$$

We therefore have as the governing differential equation for our beam-load dynamic system as

$$EI\frac{\partial^4 z}{\partial x^4} - N\frac{\partial^2 z}{\partial x^2} - m\frac{\partial^2 z}{\partial t^2} + \varepsilon_0\frac{\partial z}{\partial t}KZ = \sum P_j \cos\omega_j t\delta\left[x - \left(x_j + \chi_j \sin\beta_j t\right)\right] \tag{2.3}$$

where EI = bending stiffness of beam

N = prestressed axial force on beam

K = elastic coefficient of beam

P_i = constant amplitude of the kth load

□i = transverse frequency of the kth load

β_i = longitudinal frequency of the kth load

$x_i = \chi_i sin\beta_i t$ = position of the kth load at time t

δ = direct delta function

We recall that for a beam with simple ends the corresponding boundary conditions for (2.3) is given as

$$\left[z_i z^n\right]_\beta = 0 \tag{2.4}$$

In the case of a drop load the corresponding initial conditions are

$$\left[z_i z^n\right]_{t=0} = 0 \tag{2.5}$$

3.0 Solution technique

In this section we discuss the solution of the initial-boundary-value problem (2.3) through (2.5). Assuming a solution of the form

$$z(x,t) = \sum_{r=1}^{\infty} u_r(t)\sin\frac{r\pi}{\rho}x \tag{3.1}$$

we thus have

$$\sum_{r=1}^{\infty}\left[m\frac{d^2u_r}{dt^2} + \varepsilon_0\frac{du_r}{dt} + \left(EI\left(\frac{r\pi}{\rho}\right)^4 + N\left(\frac{r\pi}{\rho}\right)^2 + K\right)u,(t)\right]\sin\frac{r\pi}{\rho}x$$

$$= \sum_{j=1}^{N} P_j \cos\omega_j t\delta\left[x - \left(x_j + \chi_j \sin\beta_j t\right)\right] \tag{3.2}$$

Multiplying through (3.2) by $\sin\frac{r\pi}{\rho}x$ and integrating through the length of the beam yields,

$$\frac{d^2u_r}{dt^2}+2\gamma\frac{du_r}{dt}+\xi^2u_r(t)=\frac{2}{m\rho}\sum_{j=1}^{N}P_j\cos\omega_j t\sin jj\left(+A_j+B_j\sin\beta_j t\right) \quad (3.3)$$

where
$$2\gamma=\frac{\varepsilon_0}{2},\xi^2=\left(EI\left(\frac{r\pi}{\rho}\right)^4+N\left(\frac{r\pi}{\rho}\right)^2+K\right),A_j=\frac{r\pi}{\rho}x_j,B=\frac{r\pi}{\rho}\chi_j \quad (3.4)$$

Defining the following parameters:

$$\Lambda^2=\xi^2-\gamma^2,a_{0j}=\Lambda+\omega_j,a_{1j}=\Lambda-\omega_j,a_{2j}=a_{0j}+2m\beta,$$
$$a_{3j}=a_{oj}-2m\beta_j,a_{4j}=a_{1j}+2m\beta_j a_{5j}=a_{1j}-2m\beta_j,a_{6j}=a_{0j}+(2m+1)\beta_j, \quad (3.5)$$
$$a_{7j}=a_{0j}-(2m+1)\beta_j,a_{8j}=a_{1j}+(2m+1)\beta_j,a_{9j}=a_{1j}-(2m+1)\beta_j$$

and invoking the following Bessel function identities as found in Watson [4].

$$\cos(a\cos bt)=J_0(a)+2\sum_{m=1}^{\infty}(-1)^m J_{2m}(a)\cos 2mbt$$
$$\cos(a\sin bt)=J_0(a)+2\sum_{m=1}^{\infty}J_{2m}(a)\sin 2mbt$$
$$\sin(a\cos bt)=J_0(a)+\sum_{m=1}^{\infty}(-1)^m J_{2m+1}(a)\cos(2m-1)bt \quad (3.6)$$
$$\sin(a\sin bt)=J_0(a)+\sum_{m=1}^{\infty}J_{2m+1}(a)\sin(2m+1)bt$$

we therefore have,

$$u_r(t)=[a\sin\Lambda_r t+b\cos\Lambda_r t]e^{-\gamma t}+\frac{1}{2\Lambda_r\rho}\sum_{j=1}^{N}P_j\left[\left[\left\{J_0(B_j)\left(\frac{\gamma\sin(\Lambda_r-a_{0j})t+a_{0j}\cos(\Lambda_r-a_{0j})t}{\gamma^2+a_{0j}^2}\right.\right.\right.\right.$$
$$\left.\left.+\frac{\gamma\sin(\Lambda_r-a_{1j})t+a_{1j}\cos(\Lambda_r-a_{1j})t}{\gamma^2+a_{1j}^2}\right)\right]+\sum_{j=1}^{\infty}J_{2m}(B_j)\left(\frac{\gamma\sin(\Lambda_r-a_{2j})t+a_{2j}\cos(\Lambda_r-a_{2j})t}{\gamma^2+a_{2j}^2}\right.$$
$$+\frac{\gamma\sin(\Lambda_r-a_{3j})t+a_{3j}\cos(\Lambda_r-a_{3j})t}{\gamma^2+a_{3j}^2}+\frac{\gamma\sin(\Lambda_r-a_{4j})t+a_{4j}\cos(\Lambda_r-a_{4j})t}{\gamma^2+a_{4j}^2}$$
$$\left.\left.+\frac{\gamma\sin(\Lambda_r-a_{5j})t+a_{5j}\cos(\Lambda_r-a_{5j})t}{\gamma^2+a_{5j}^2}\right)\right\}\sin A_j+\sum_{j=1}^{\infty}J_{2m-1}(B_j)\left(\frac{\gamma\sin(\Lambda_r-a_{6j})t+a_{6j}\cos(\Lambda_r-a_{6j})t}{\gamma^2+a_{6j}^2}\right.$$
$$-\frac{\gamma\sin(\Lambda_r-a_{7j})t+a_{7j}\cos(\Lambda_r-a_{7j})t}{\gamma^2+a_{7j}^2}+\frac{\gamma\sin(\Lambda_r-a_{8j})t+a_{8j}\cos(\Lambda_r-a_{8j})t}{\gamma^2+a_{8j}^2}$$
$$\left.-\frac{\gamma\sin(\Lambda_r-a_{9j})t+a_{9j}\cos(\Lambda_r-a_{9j})t}{\gamma^2+a_{9j}^2}\right)\cos A_j$$

On applying the initial conditions (2.5) and substituting into (3.1) we thus have,

$$z(x,t)=$$

$$\sum_{r=1}^{\infty}\sum_{j=1}^{N}\chi_j J_0\left(B_j\right)\left(\begin{array}{c}\dfrac{a_{0j}\{Cos(\Lambda_r-a_{1j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{0j})t-a_{0j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{0j}^{2}}\\ +\dfrac{a_{1j}\{Cos(\Lambda_r-a_{1j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{1j})t-a_{1j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{1j}^{2}}\end{array}\right)$$

$$+\sum J_{2m}\left(B_j\right)\left(\begin{array}{c}\dfrac{a_{2j}\{Cos(\Lambda_r-a_{3j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{2j})t-a_{2j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{2j}^{2}}\\ +\dfrac{a_{3j}\{Cos(\Lambda_r-a_{3j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{3j})t-a_{3j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{3j}^{2}}\\ +\dfrac{a_{4j}\{Cos(\Lambda_r-a_{4j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{4j})t-a_{4j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{4j}^{2}}\\ +\dfrac{a_{5j}\{Cos(\Lambda_r-a_{5j})t-e^{-\gamma t}Cos\Lambda_r t\}+\gamma\{Sin(\Lambda_r-a_{5j})t-a_{5j}^{*}e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{5j}^{2}}\end{array}\right)SinA_j$$

$$+\sum_{m=0}^{\infty}J_{2m+1}(B_j)\left(\begin{array}{c}\dfrac{\gamma\{Cos(\Lambda_r-a_{6j})t+e^{-\gamma t}Cos\Lambda_r t\}-a_{6j}\{Sin(\Lambda_r-a_{6j})t+a_{6j}'e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{6j}^{2}}\\ -\dfrac{\gamma\{Cos(\Lambda_r-a_{7j})t+e^{-\gamma t}Cos\Lambda_r t\}-a_{7j}\{Sin(\Lambda_r-a_{7j})t+a_{7j}'e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{7j}^{2}}\\ -\dfrac{\gamma\{Cos(\Lambda_r-a_{9j})t+e^{-\gamma t}Cos\Lambda_r t\}-a_{9j}\{Sin(\Lambda_r-a_{9j})t+a_{9j}'e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{9j}^{2}}\\ -\dfrac{\gamma\{Cos(\Lambda_r-a_{8j})t+e^{-\gamma t}Cos\Lambda_r t\}-a_{8j}\{Sin(\Lambda_r-a_{8j})t+a_{8j}'e^{-\lambda t}Sin\Lambda_r t\}}{\gamma^2+a_{8j}^{2}}\end{array}\right)CosA_j]$$

$$\times Sin\left(\frac{r\pi x}{\rho}\right)$$

where,

$$a_{kj}^{*}=\frac{a_{kj}}{\Lambda_r}-\Lambda_r+a_{kj}\ \text{and}\ a_{kj}'=\Lambda_r-a_{kj}-\frac{\gamma^2}{\Lambda_r a_{kj}}\ ,\ \chi_j=\left(\frac{P_j}{2\rho\Lambda}\right)$$

4.0 Numerical simulation

In what follows, the deflection profiles along the length ρ of the beam at a time is studied for various values of the parameters of the beam-load system such as the position x_1, x_2 the amplitude p_1, p_2 and the symmetry of the positioning of the load relative to the midpoint of the beam are considered. The physical constants are adapted from Timoshenko [3].

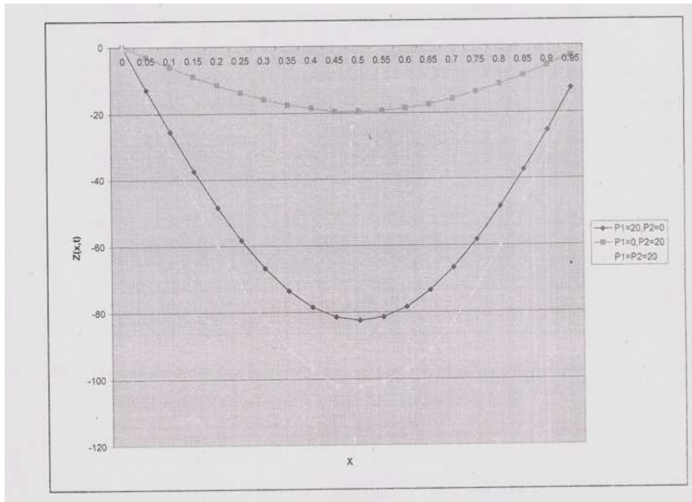

Figure 4.1: Displacement along the length with varying load magnitude with the load positioned symmetrically about the midpoint of the beam.

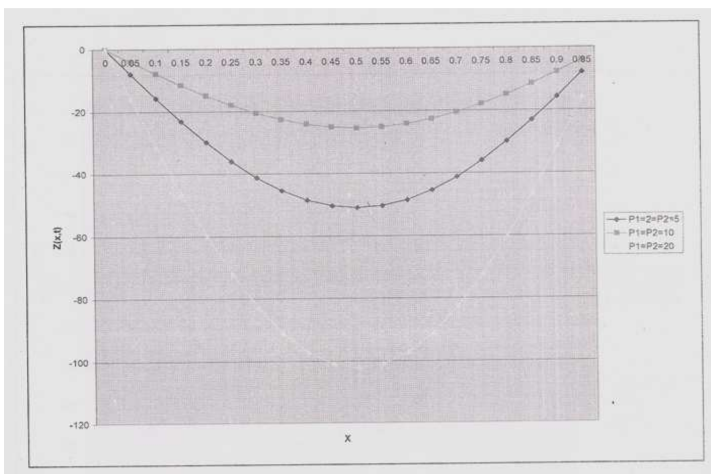

Figure 4.2: Displacement along the length with varying load amplitude with the load positioned symmetrically about the midpoint of the beam.

5.0 Discussion of results and conclusion

Figure 4.1 above shows the deflection profiles of the beam-multiload system with the load positioned symetrically at the point 0.01 and 0.99 about the midpoint of the beam. The amplitude of vibration prove to be additive pect to the magnitudes of the load.

In Figure 4.2 the magnitude of the load are varied. The results show that the amplitude of vibration increases with increasing magnitude of the load. The positioning of the load is skewed about the midpoint of the beam in Figure 4.3. This results in a sharp increase in the amplitud of vibration of the system.

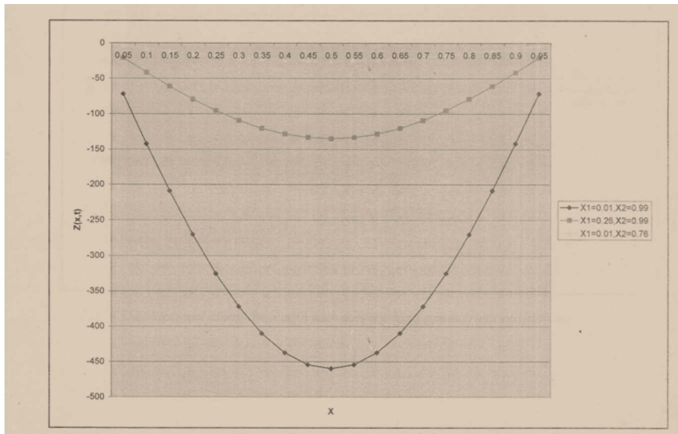

Figure 4.3: Displacement along the length with varying load positioning exhibiting skew positioning about the midpoint of the beam.

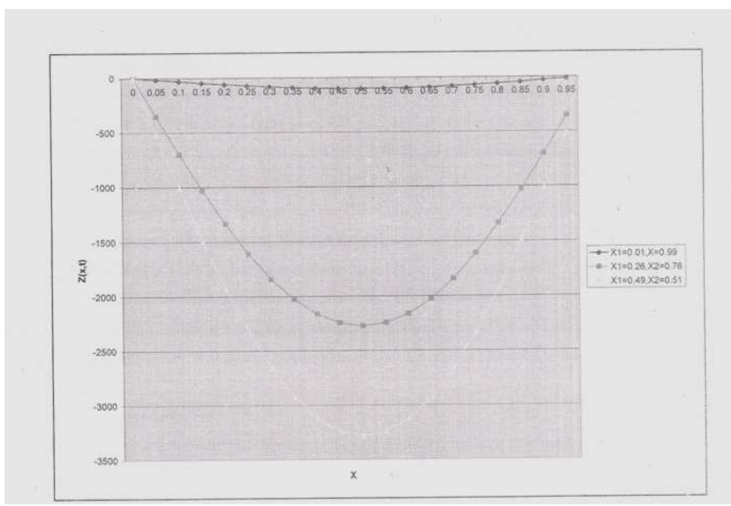

Figure 4.4: Displacement along the length with varying load symmetrical positioning

Finally, Figure 4.4 demostrates varaibility in the symmetrical positioning about the midpoint of the beam. The amplitude of vibrateion increases as the positions of the load approaches the midpoint of the beam.

From the observations above we therefore draw the following conclusion:

(i) The *m*-fold load system has a multiplying effect on the amplitude of vibration;

(ii) lack of symmetry of the positioning of the load has a distabilizing effect on the system;

(iii) The closer the positioning of the symmetrically placed load to the midpoint of the beam the higher the amplitude of vibration.

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