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Effect of a magnetic field on a rotating fluid flow

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Abstract

In this paper, we investigate the effect of a magnetic field on a rotating fluid flow in a rotating frame. A system of equations of motion was considered for some components of velocity and magnetic fields. Under some mathematical conditions and assumptions, the system of equations of motion give rise to a differential equation whose result in a graphical representation shows that the velocity of rotation for the rotating fluid flow increases as the product of the imposed magnetic field increases but as the z-component becomes very large or tends to a very large value, the velocity of rotation for the rotating fluid flow is the same or constant for all the products of the imposed magnetic field.

1.0 Introduction

In a paper by Owen [5], rotating cavities have been employed to model many practical rotating flows between co-rotating turbines or compressor discs. The operational rotation of rotating machinery experiences both the starting and stopping process.

In another paper by Kawashima and Yang [2], unsteady transport phenomena in hollow drum subjected to a sudden acceleration and deceleration were experimentally studied by means of laser droppler velocimetry. The corresponding theoretical study was performed by [7].

Later [8] observed the flow pattern and temperature distribution in a rotating drum subject to a sudden acceleration and inner surface heating by using the laser light-sheet method and iron constantan thermocouples respectively.

In order to evaluate quantitatively the temperature distribution in the rotating drum, [3] employed temperature sensitive liquid crystals as a non invasive method. A series of studies was focused on visualization of two dimensional thermal fields.

In another paper by Torii and Yang [6], they studied the unsteady fluid flow transport phenomenon in axially rotating drum under a spin-down process. Emphasy was placed on the effect of aspect ratio, A, on the velocity and temperature profiles. The governing differential equations for three dimensional unsteady fluid flows were discretized by means of a finite difference technique. Theoretical predictions, particularly the velocity fields were compared with experimental results by Ohue *et al* [4].

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In another paper by Ghosh *et al* [1], they worked on hydromagnetic rotating flow of a dusty fluid near a pulsating plate. In this work, an initial value problem was solved for a motion of an incompressible viscous conducting fluid with small inert spherical particles bounded by infinite rigid non conducting plate and fluid are in a state of solid body rotating with constant angular velocity about an axis normal to the plate. The general solution for the fluid velocity and the wall shear stress are examined numerically and the simultaneous effects of rotation, the magnetic field and the particles on them are determined. The result for the fluid velocity was compared numerically with that of an impulsively moved plate in a particular case when time was large.

2.0 Mathematical formulation

The equations of balance of linear momentum for the Navier-stokes equations are:

$$\left.\begin{aligned}
\frac{\partial v_1}{\partial t}+v_1\frac{\partial v_1}{\partial x}+v_2\frac{\partial v_1}{\partial y}+v_3\frac{\partial v_1}{\partial z}-v_2 f &= -\frac{1}{\rho}\frac{\partial p}{\partial x}+F_1\\
\frac{\partial v_2}{\partial t}+v_1\frac{\partial v_2}{\partial x}+v_2\frac{\partial v_2}{\partial y}+v_3\frac{\partial v_2}{\partial z}+v_1 f &= -\frac{1}{\rho}\frac{\partial p}{\partial y}+F_2\\
\frac{\partial v_3}{\partial t}+v_1\frac{\partial v_3}{\partial x}+v_2\frac{\partial v_3}{\partial y}+v_3\frac{\partial v_3}{\partial z} &= -\frac{1}{\rho}\frac{\partial p}{\partial z}-g+F_3
\end{aligned}\right\} \qquad (2.1)$$

where v_1, v_2 and v_3 are the components of the velocity of the fluid; F_1, F_2 and F_3 are the representations of the viscous force in the medium; *f* is the coriolis parameter; t, *x*, *y*, and z are the independent variables.

Equations (2.1) are augmented by the equation of Conservation of mass when in the case of incompressible fluid states

$$\frac{\partial v_1}{\partial x}+\frac{\partial v_2}{\partial y}+\frac{\partial v_3}{\partial z}=0$$

Based on the scale arguments, in many oceanic flows, the following representations are used.

$$F_1 = A_v\frac{\partial^2 v_1}{\partial z^2}, F_2 = A_v\frac{\partial^2 v_2}{\partial z^2}, F_3 = 0$$

When the flow is steady and the acceleration terms are small compared with the Coriolis force, the pressure gradient and the viscous forces; we include the magnetic effects and then end up with the viscous Geostrophic equations:

$$\left.\begin{aligned}
-v_2 f &= \beta_1 v_1 - \frac{1}{\rho}\frac{\partial p}{\partial x} + A_v + \frac{\partial^2 v_1}{\partial z^2}\\
v_1 f &= \beta_2 v_2 - \frac{1}{\rho}\frac{\partial p}{\partial y} + A_v\frac{\partial^2 v_2}{\partial z^2}\\
0 &= \beta_3 v_3 - \frac{1}{\rho}\frac{\partial p}{\partial z} - g
\end{aligned}\right\} \qquad (2.2)$$

where β_1, β_2 *and* β_3 are the components of the imposed magnetic field and *g* is the acceleration due to gravity.

3.0 Method of Solution

We now solve (2.2) subject to the following conditions

$$\left.\begin{array}{l} v_1 = u(z), v_2 = v(z), \text{ and } v_3 = w(z) \\ u = U \text{ and } v = 0 \text{ when } z \to \infty \\ u = v = 0 \text{ when } z = 0 \\ \rho \text{ and } A_v \text{ are constant} \end{array}\right\} \quad (3.1)$$

With the conditions in (3.1), equations (2.2) become

$$\left.\begin{array}{l} -fv = \beta_1 u - \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} + A_v \dfrac{\partial^2 u_1}{\partial z^2} \\ fu = \beta_2 v - \dfrac{1}{\rho}\dfrac{\partial p}{\partial y} + A_v \dfrac{\partial^2 v_2}{\partial z^2} \\ 0 = \beta_3 w - \dfrac{1}{\rho}\dfrac{\partial p}{\partial z} - g \end{array}\right\} \quad (3.2)$$

Here *u*, *v* and *w* are components of the velocity and g is the gravitational force.

Now, if we differentiate $\frac{\partial p}{\partial z}$ in the third equation of (3.2) with respect to x and y we obtain $\frac{\partial^2 p}{\partial z \partial x} = \frac{\partial^2 p}{\partial z \partial y} = 0$ since the horizontal pressure gradient $\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right)$ is independent of z. From the first two equations of (3.2), $\frac{\partial p}{\partial x}$ *and* $\frac{\partial p}{\partial y}$ are independent of *x* and *y* since *u* and *v* only depend on *z*. Hence $\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right)$ is a constant vector and so (3.2) becomes.

$$\left.\begin{array}{l} -fv = \beta_1 u + A_v \dfrac{\partial^2 u}{\partial z^2} \\ fu = \beta_2 v + A_v \dfrac{\partial^2 v}{\partial z^2} \\ 0 = \beta_3 w - g \end{array}\right\} \quad (3.3)$$

Now, eliminating *v* from the first equation of (3.3) and taking the first and second partial derivatives with respect to z we obtain

$$-f\frac{\partial v}{\partial z} = \beta_1 \frac{\partial u}{\partial z} + A_v \frac{\partial^3 u}{\partial z^3} \quad (3.4)$$

And

$$-f\frac{\partial^2 v}{\partial z} = \beta_1 \frac{\partial^2 u}{\partial z^2} + A_v \frac{\partial^4 u}{\partial z^4} \quad (3.5)$$

From equation (3.5) we obtain

$$\frac{\partial^2 v}{\partial z^2} = -\frac{1}{f}\left[\beta_1 \frac{\partial^2 u}{\partial z^2} + A_v \frac{\partial^4 u}{\partial z^4}\right] \quad (3.6)$$

Also, from the first equation of (3.3) we have

$$v = -\frac{1}{f}\left[\beta_1 u + A_v \frac{\partial^2 u}{\partial z^2}\right] \quad (3.7)$$

Then substitute equations (3.6) and (3.7) in the second equation of (3.3) to obtain

$$fu = \beta_2\left[-\frac{1}{f}\left(\beta_1 u + A_v \frac{\partial^2 u}{\partial z^2}\right)\right] + A_v\left[-\frac{1}{f}\left(\beta_1 \frac{\partial^2 u}{\partial z^2} + A_v \frac{\partial^4 u}{\partial z^4}\right)\right]$$

$$fu = -\frac{\beta_1\beta_2}{f} - \frac{\beta_2 A_v}{f}\frac{\partial^2 u}{\partial z^2} - \frac{\beta_1 A_v}{f}\frac{\partial^2 u}{\partial z^2} - \frac{A_v^2}{f}\frac{\partial^4 u}{\partial z^4}$$

$$\therefore \qquad u = -\frac{\beta_1\beta_2 u}{f^2} - \frac{\beta_2 A_v}{f^2}\frac{\partial^2 u}{\partial z^2} - \frac{\beta_1 A_v}{f^2}\frac{\partial^2 u}{\partial z^2} - \frac{A_v^2}{f^2}\frac{\partial^4 u}{\partial z^4} \qquad (3.8)$$

Let $a^2 = \frac{A_v^2}{f^2}, b^2 = \frac{\beta_1 A_v}{f^2},\ c^2 = \frac{\beta_2 A_v}{f^2}$ and $d^2 = \frac{\beta_1\beta_2}{f^2}$ in (3.8); so (3.8) becomes

$$a^2 u''''(z) + \left(b^2 + c^2\right)u''(z) + \left(d^2 + 1\right)u(z) = 0 \qquad (3.9)$$

As $z \to \infty,\quad u''(z) \to 0\ \ \sin ce\ u(z)$ have asymptotic value 0; hence (3.9) becomes

$$a^2 u''''(z) + \left(d^2 + 1\right)u(z) = 0 \qquad (3.10)$$

Suppose we seek a solution of the form $u(z) = e^{mz}$ for (3.10), we obtain

$$a^2 m^4 + \left(d^2 + 1\right) = 0$$

implies $$m^4 = -\frac{\left(d^2+1\right)}{a^2} = -\left[\frac{d^2+1}{a^2}\right]$$

So that $m^2 = \pm\sqrt{-1}\sqrt{\frac{d^2+1}{a^2}} = \pm\frac{i\sqrt{d^2+1}}{a}$ and $m = \pm\sqrt{i}\ \frac{\left(d^2+1\right)^{1/4}}{\sqrt{a}}$. But $i = \left[\frac{1}{\sqrt{2}}(1+i)\right]^2$,

so $$m = \frac{1}{\sqrt{2a}}\left(d^2+1\right)^{1/4}(1+i) \quad or \quad m = \frac{1}{\sqrt{2a}}\left(d^2+1\right)^{1/4}(-1+i) \qquad (3.11)$$

Let $\eta = \sqrt{2a}$ and $\xi = \left(d^2+1\right)^{1/4}$, so (3.11) becomes $m = \frac{\xi}{\eta}(1+i)$ or $m = \frac{\xi}{\eta}(-1+i)$.

Hence the general solution of (3.10) is

$$u(z) = U + c_1 e^{-\frac{\xi}{\eta}z}\cos\frac{\xi}{\eta}z + c_2 e^{-\frac{\xi}{\eta}z}\sin\frac{\xi}{\eta}z + c_3 e^{\frac{\xi}{\eta}z}\cos\frac{\xi}{\eta}z + c_4 e^{\frac{\xi}{\eta}z}\sin\frac{\xi}{\eta}z \qquad (3.12)$$

Because we are only interested in a solution that is bounded as $z \to \infty,$ we choose $c_3 = c_4 = 0$ and so solution (3.12) becomes

$$u(z) = U + c_1 e^{-\frac{\xi}{\eta}z}\cos\frac{\xi}{\eta}z + c_2 e^{-\frac{\xi}{\tau}z}\sin\frac{\xi}{\eta}z \qquad (3.13)$$

The boundary condition at z = 0 requires that $u(0) = 0$ and so from (3.13) $u(0) = U + c_1 = 0$ and so $c_1 = -U$, then (3.13) becomes

$$u(z) = U - Ue^{-\frac{\xi}{\eta}z}\cos\frac{\xi}{\eta}z + c_2 e^{-\frac{\xi}{\eta}z}\sin\frac{\xi}{\eta}z \qquad (3.14)$$

Also from the boundary condition, $u''(0) = 0$ gives $c_2 = 0$. Hence (3.14) becomes

$$u(z) = U\left(1 - e^{-\frac{\xi}{\eta}z}\cos\frac{\xi}{\eta}z\right) \qquad (3.15)$$

Recall that $\xi = \left(d^2+1\right)^{1/4}$ and $\eta = \sqrt{2a}$. So $\xi = \left(\frac{\beta_1\beta_2 + f^2}{f^2}\right)^{1/4}$ and $\eta = \left(\frac{4A_v^2}{f^2}\right)^{1/4}$.

Then substituting for $\xi\ \ and\ \ \eta$ in (3.15) we obtain

$$u(z) = U\left[1 - e^{-\left(\frac{\beta_1\beta_2 + f^2}{4A_v^2}\right)^{1/4} z}\quad \cos\left(\frac{\beta_1\beta_2 + f^2}{4A_v^2}\right)^{1/4} z\right] \qquad (3.16)$$

If $\beta_1 = 0 \;\; or \;\; \beta_2 = 0$ in (3.16) we obtain

$$u(z) = U\left[1 - e^{-\left(\sqrt{\frac{f}{2A_v}}\right)z}\cos\left(\sqrt{\frac{f}{2A_v}}\right)z\right] \tag{3.17}$$

Then using a unit value for *f*, A_y and *U* for $z = 0,1,2,3,\cdots$ in solution (3.16), we obtain a graphical representation of solution (3.16) for $\beta_1\beta_2 = 0, \beta_1\beta_2 = 2, \beta_1\beta_2 = 4 \;\; and \;\; \beta_1\beta_2 = 6$

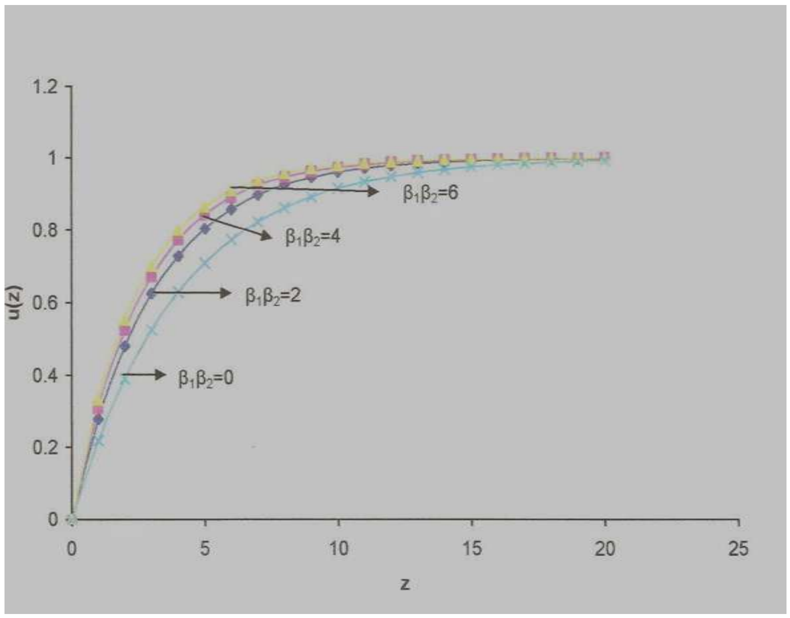

Figure 3.1: The graphical representation of solution (3.16)

4.0 Analysis of result

The graphical representation of the solution (3.16) shows that the velocity of rotation for the rotating fluid flow increases as the product of the imposed magnetic field increases but as the z-component becomes very large or tends to a very large value, then the velocity of rotation for the rotating fluid flow is the same or constant for all the products of imposed magnetic field.

5.0 Conclusion

This result shows that the velocity in the boundary layer increases as the magnitude of the magnetic components increases .This is not so outside the boundary layer when z-component is large.Thus,it is possible to increase the velocity by increasing the magnetic effects at the surface.

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