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Horizontal and vertical projectile motion in a resistant medium subject to varying path angles and speed

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Abstract

A study of projectile motion in a resistant medium subject to varying path angles and speed was carried out. Solutions to the governing equations of motion is developed employing double integration. Analysis of the results shows that the path of the particle in a resistant medium is affected by both increase in path angle and increase in velocity though the effect is more prominent in the horizontal direction.

1.0 Introduction

In projectile motions, we assumed that air resistant is negligible, or that the projectile is moving in a vacuum. In practice the frictional effects of air can make significant changes from an ideal parabolic trajectory but we assumed that, air is steady and its effect minimal. In his paper, Hayen [4] considered the projectile motion of a particle under the influence of a uniform gravitational field. In the study, he used drag force to act on the particle which he assumed is proportional to the square of the particle's velocity. Ayeni and Ayandokun [1] included electric field with a general power n in the earlier mentioned study of Hayen with the assumption that the initial motion is vertical and the resulting unsteady motion investigated. Also Fenuga and Ayeni [2] made some important and necessary remarks on the projectile motion of a particle in a resistant medium under the influence of a magnetic field and showed that both velocity and path angle increase as magnetic field increases and decrease as magnetic field decreases. They [3] also compared the impact of electric and magnetic fields in a resistant medium and showed that the magnetic field has much impact on the velocity than the electric field. Recently, Ngiangia et al [5] critically examined the effect of magnetic field by considering explicitly the vertical and horizontal motions of a projectile where the resistant medium is air and assumed that the acceleration due to gravity is constant over the range of motion and directed downwards and the rotation of the earth does not affect the motion. Fenuga and Ayeni [3] study was in free space and ignored varying the velocity of projection because the effect of the resistant medium was perhaps not explicitly determined. Ngiangia et al [5] considered the motion of a particle in vertical and horizontal direction but neglected the effect of varying path angles and speed. In this paper, our aim is to critically examine the effect of variation of path angles and speed in the horizontal and vertical motions of the projectile which is an extension of the study of Ngiangia et al [5] and Fenuga and Ayeni [3]

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2.0 Model formulation

A particle of mass m moving with an initial velocity u is projected from 0 (Figure 2.1) at an angle θ with the horizontal in a medium whose resistance *R* to the motion of the particle is mvk, where v is the velocity of the particle and k is constant. It is assumed that, resistance acts instantaneously in the direction opposite to that of the particle's motion. The mathematical model describing the resolved parts of the motion, parallel and perpendicular to the x and y axes respectively are;

$$\frac{md^2x}{dt^2} = -mkv\cos\theta \tag{2.1}$$

$$\frac{md^2y}{dt^2} = -mkv\sin\theta - mg \tag{2.2}$$

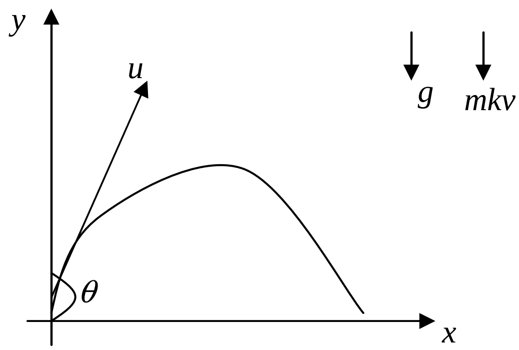

Figure 2.1: Physical model of the problem

2.1 Non-dimensional analysis

For dimensional homogeneity of the governing equations, we substitute the following dimensionless parameters having employed the Rayleigh's technique

$$x' = \frac{x}{y}, y' = \frac{y}{x}, v' = \frac{v}{u}, t' = \frac{gt}{u}, g' = \frac{gx}{v^2}, \theta' = \frac{\theta}{vt}$$

and rewrite (2.1) and (2.2) as

$$\frac{d^2x'}{dt'^2} = -kv'\cos\theta' \tag{2.3}$$

$$\frac{d^2y'}{dt'^2} = -kv'\sin\theta' - g \tag{2.4}$$

With the boundary conditions

$$\begin{aligned} &x'(0) = 0, y'(0) = 0 \\ &x^1(0) = v'\cos\theta' \\ &y^1(0) = v'\sin\theta \end{aligned} \tag{2.5}$$

3.0 Method of solution

Integrating (2.3) and (2.4) twice, respectively results in

$$x' = -\frac{1}{2}kv'(\cos\theta')t^2 + C_1t' + C_2 \tag{3.1}$$

and

$$y' = -\frac{1}{2}kv'(\sin\theta')t^2 - \frac{1}{2}g't^2 + C_3t' + C_4 \tag{3.2}$$

where C_1, C_2, C_3, C_4 are constants. Imposing the boundary conditions (2.5) into (3.1) and (3.2), we get

$$C_2 = C_4 = 0,$$
$$C_1 = v'\cos\theta', C_3 = v'\sin\theta'$$

Equations (3.1) and (3.2) can therefore be written as

$$x' = -\frac{1}{2}kv'(\cos\theta')t'^2 + (v'\cos\theta')t' \qquad (3.3)$$

and

$$y' = -\frac{1}{2}kv'(\sin\theta')t' - \frac{1}{2}g't'^2 + (v'\sin\theta')t' \qquad (3.4)$$

4.0 Results

We take

$g' = 9.8, k = 0.2455$

$\theta' = 10,20,30,40,45$

$v' = 10,20,40,60,80$

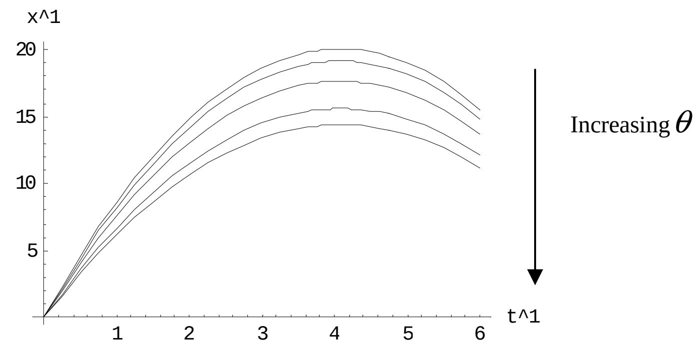

Figure 3.2: Graph of (3.3) with θ' varying

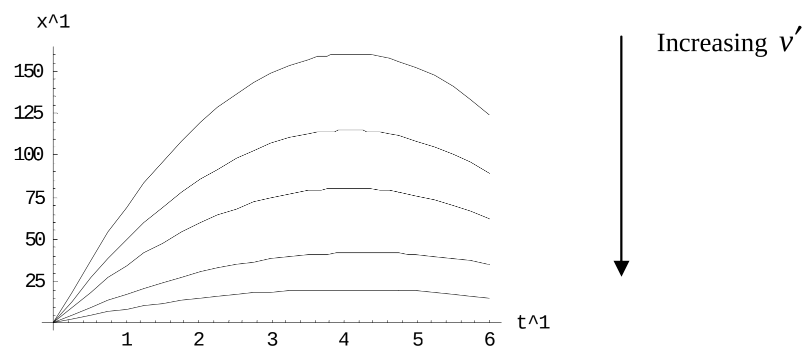

Figure 3.3: Graph of (3.3) with v' varying

5.0 Discussion

We have formulated and solved the problem of a projectile motion in a resistant medium subject to varying path angle and speed. To give physical insight and numerical validation to the problem, we have selected values that will give realistic results (values of physical relevance).

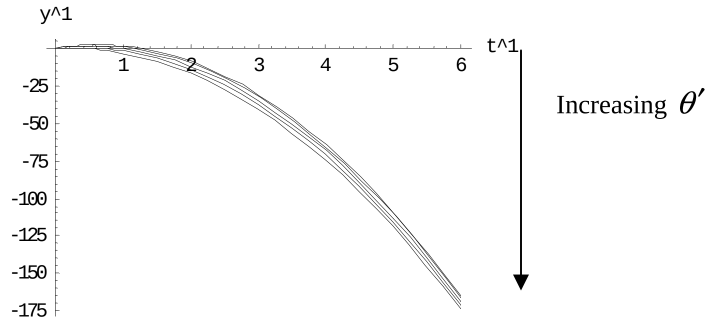

Figure 4.4:. Graph of (3.4) with θ' varying

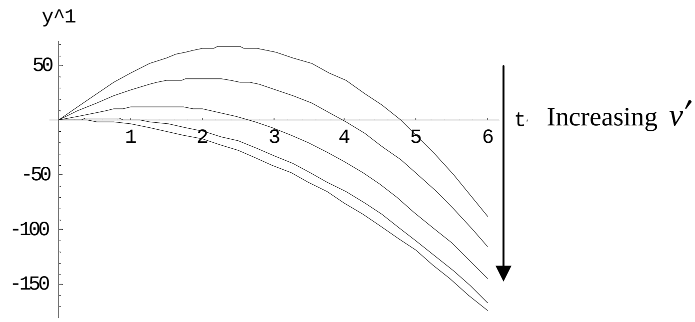

Figure 4.5: Graph of (3.4) with v' varying

From our analysis, Figure 4.2 shows that increase in path angle affects the timing of the path of an horizontal projectile motion. Figure 3 also reveals that increase in velocity, changes the path of a particle of a projectile motion in the horizontal direction. Similarly, Figure 4.4 shows that increase in path angle affects the timing of the path of a projectile in the vertical direction but not as pronounced as that of the horizontal direction. The reason may be as a result of the effect of gravity in the vertical direction. Increase in velocity is shown in Figure 4.5, and it reveals that a drastic change in the path of the particle is observed as a result of increase in velocity of the particle.

6.0 Conclusion

The study reveals that in a resistant medium where speed is a component, varying of path angle in the vertical direction delays timing of projectile motions. It is therefore imperative that similar results may not be achieved with uniform variation of path angles and speed in the vertical and horizontal projectile motions of particles which [3] and [5] neglected. Therefore, for proper timing and observation of projectile motions of particles, care should be taken to ensure that the path angle and the velocity of the particle is carefully chosen for desired results to be obtained. Although air resistance is ignored this may necessarily not be the case as its effect in some situations may affect desired results.

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