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Throughput capacity computation model for hybrid wireless networks

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Abstract

Throughput capacity is a critical parameter for the design and evaluation of wireless networks. We present in this paper, a computational model for obtaining throughput capacity for hybrid wireless networks. For a hybrid network with n nodes and m base stations, we observe through simulation that the throughput capacity increases linearly with the base station infrastructure connected by the wired network, provided that the number of nodes n, does not grow asymptotically slower than $\sqrt{n}$.

Keywords
TDMA, Ad-hoc Networks, Network Throughput Capacity, Bandwidth Transmission

1.0 Introduction

Throughput capacity is an important factor for evaluating the performance of wireless networks. It represents the long-term achievable data transmission rate that a network can support. The throughput capacity of a wireless network depends on various aspects of the network. Some of these aspects include the network architecture, power and bandwidth constraints, routing strategy and radio interference. A good understanding of the capacities of different network architectures allows a designer to choose an appropriate architecture for his or her specific purpose, Gastpar and Vitterli,; Lic, Blake, Couto, Lee and Morris,; Toumpis and Goldsmith, [1, 2, 3].

Gupta and Kumar [4] have shown that the uniform throughput capacity per node of an ad-hoc network with n nodes decreases with n as $\Theta\left(\frac{T}{\sqrt{n \log n}}\right)$, where T is the common transmission rate of each node over the wireless channel.

In recent studies, Oliver and Hasker [5], Negi and Rajeswaran, [6] propose a hybrid network model to improve network connectivity. In their model, a sparse network of base stations connected by a wire-lined network is placed within an ad-hoc network. The resulting network consists of normal nodes and some-well connected access points (base stations).

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In this paper, we show that under a hybrid network model, where normal nodes are well connected to well-known base stations; the uniform throughput capacity per node increases as $\Theta\left(\sqrt{\frac{n}{\log\frac{n}{m^2}}}W\right)$ where Θ is a standard order bound, *m*, the number of base stations and *W*, the transmission bandwidth.

2.0 Background of issues

The objective of this paper is to demonstrate the effect of base stations (access points) connectivity on ad-hoc network capacity. This requires a contrast of the assumptions and results in Gupta and Kumar [4] with ours. Therefore, for the sake of completeness, this section reviews the methods (relevant for useful comparison) used by Gupta and Kumar [7].

2.1 Network model

The network model in Gupta and Kumar [4] is one of the Random Networks (RNs), where the *n* static nodes on a unit area are uniformly, independent and identically distributed (*i.i.d*). The *n* nodes communicate over a wireless channel, with possible co-operation, to relay traffic. Each of the *n* nodes has an independent randomly chosen destination (chosen as the node closest to a random point). All nodes require to send traffic at a rate of *r(n)* bits per second to their corresponding destinations. A uniform throughput *r(n)* is feasible if there exists a scheduling and relaying scheme by which every source destination pair can communicate an average time of *r(n)* bits per second. The maximum feasible uniform throughput is the uniform throughput capacity and is the metric choice. The motivation for choosing this metric is a sense of fairness, since all nodes are assumed to be homogeneous in their capabilities and requirement.

2.2 Communication Model

Gupta and Kumar [4] assume that each node can transmit at a rate of *T* bits/sec. The homogenous nodes show a common range (equivalently, power) of transmission *tr(n)*. This simplistic communication model assumes that all operating links transport data at a constant rate. The transmission by a legitimate transmitter x_i to the intended receiver x_j is successful, if their distances are related as

$$|x_k - x_j| \geq (1+\Delta)\,|x_i - x_j| \qquad (2.1)$$

for every other x_k transmitting simultaneously. Here, the interference criterion models a protocol, which specifies a guard zone, Δ, around the receiver where no other node may transmit, and is termed as the ‘protocol model’. A second model, more directly related to physical layer design, works with the signal of interference Noise Ratio (NR) at the receiving node with the assumption of arbitrary large power. The order results remain the same under both models. For simplicity, this review assumes the protocol model.

2.3 Network Throughput Capacity

With these assumptions, it is proven in Gupta and Kumar [4] that the capacity of random ad-hoc network is

$$r(n) = \Theta\left(\frac{T}{\sqrt{n\log n}}\right) \qquad (2.2)$$

Thus the throughput per node decreases with increasing node density. The reason for this capacity decrease is the requirement for all nodes to share the wireless channel locally. This may be demonstrated by a contrast between the Medium Access Control (MAC) of scheduling and routing requirements. This mean source-destination distance is assumed to be *L*, and the mean number of hops taken by packets is given as $\frac{L}{tr(n)}$. The total traffic generated by all nodes due to the multi-hop relaying (routing) is $nr(n)\left(\frac{L}{tr(n)}\right)$bits/sec. This traffic is required to be served by all nodes. However, the capacity of each node is reduced by the interference (MAC), since nodes close to the receiver cannot transmit simultaneously. The tradeoff between routing requirement and the MAC restriction yields the network capacity shown in equation (2.3):

$$r(n) \leq K_2 \frac{T}{tr(n)L\,n} \tag{2.3}$$

A lower limit on *tr(n)*, due to the requirement of network connectivity, has been derived as $tr(n) \geq \sqrt{\frac{\log n}{\pi\ n}}$. This lower limit ensures that no node is isolated. The application of this limit to equation (2.3) results in the capacity upper bound as

$$r(n) \leq K_2 \frac{T}{\sqrt{n \log n}} \tag{2.4}$$

3.0 The hybrid network model

In this section, we present the hybrid network capacity model. A hybrid network consists of two components:

(i) An ad-hoc network containing only normal nodes, similar to the model defined in Gupta and Kumar [4].

(ii) The second component is a sparse network of base stations. Under this model, the following assumptions are made:

a) A population of *n* nodes is randomly located through scaling, within a disk area of *1*square meter in the plane, Oliver and Hasker, [5].

b) The nodes homogeneously employ the same transmission power. Each node is a data source and the destination of each node is independently chosen as the node nearest to a randomly located point within the unit area disk. Unlike normal nodes, the base stations are neither data sources nor data receivers. They are added as relay nodes to improve network performance and they only engage in routing and forwarding data for normal nodes.

c) The base stations are connected together by a wired network and are placed within the ad-hoc network in a regular pattern.

d) The link bandwidth in the wired network is large enough such that there are no bandwidth constraints in the wired network. We also assume here that there are no power constraints at the base stations.

3.1 Network model

In the network model, all nodes and base stations share a common wireless channel. We assume a time-division multiplexing (TDMA) scheme for data transmission over the wireless channel. Time is divided into slots of fixed durations. In each time slot, a node is scheduled to send data. A node cannot transmit and receive data simultaneously but can only receive data from one another at the same time.

The wireless transmissions in the network are assumed to be homogeneous. Nodes including the base stations employ the same transmission range, denoted by r. For the interference model, we adopt the protocol model introduced in Gupta and Kumar [4] and Oliver and Hasker [5].

3.2 Communication model

A transmission from node x_i is successfully received by node x_j if the following conditions are satisfied:

(i) Node x_j is within the transmission range of node x_i, i.e. $|x_i - x_j| \le r$, where $|x_i$-$x_j|$ represents the distance of node x_i and node x_j in the plane.

(ii) For every other node x_k that is simultaneously transmitting over the same channel, we have, $|x_k - x_j| \ge (1+\Delta)|x_i - x_j|$.

This condition guarantees a guard zone around the receiving node to prevent a neigbouring node from transmitting on the same channel at the same time. The radius of the guard zone is a product of $(1+\Delta)$ and the distance between the sender and the receiver. The parameter Δ defines the size of the guard zone and we require $\Delta > 0$.

3.3 Routing strategy

As stated earlier, a hybrid network consists of two transmission modes: the ad-hoc mode and the infrastructure mode. In the ad-hoc mode, data are forwarded from the source to destination in a multi-hop fashion without using any infrastructure. In the infrastructure mode, data are forwarded through the infrastructure. It can be shown that in terms of throughput capacity, it is optimal to enter and exit the infrastructure only once. Also, it is optimal for a node to communicate with the nearest base station in order to reach the infrastructure. Let us denote the base station nearest to node, x_i, as $B(x_i)$. By infrastructure mode we imply that data are first transmitted from the source, x_i, to $B(x_s)$ over the wireless channel; the base station then transmits the data through wired infrastructure $B(x_d)$, which finally transmits the data to the destination (x_d)

Here we consider a routing strategy in which if the destination is located in the same cell as the source node, data are forwarded in the ad-hoc mode. Otherwise, data are forwarded in the infrastructure mode. Since the destination for a source node is randomly chosen in the unit area disk, the probability that a node commits to inter-cell communication is $1 - 1/m$. We can generalize the routing strategy to represent a family of routing strategies by relaxing the condition that the ad-hoc mode is chosen to send data. Instead of requiring that the destination be located in the same cell as the source, a node uses the ad-hoc mode to send data as long as the destination is located within k nearest neighbouring cells from the source node, where $k \ge 0$. This defines the range within which the ad-hoc mode transmissions should be used. We call this family of routing strategies the k-nearest cell routing strategies.

3.4 Throughput capacity computation model (TCCM)

In this paper, we adopt the asymptotic notations defined in Cover and Thomas [7] Consider an arbitrary cell, k, let Y_i denote a random factor that represents whether node $i(1 \le i \le n)$ and its destination are located in cell k. The random variables are defined as follows:

$$Y_i = \begin{cases} 1; \textit{Both node i and its destination are in cell k} \\ 0; \textit{Otherwise} \end{cases} \tag{3.1}$$

The hybrid network considered in this paper has m cells/nodes and the corresponding destinations are randomly and independently distributed in the unit area disk. The probability that a node, i, is located in cell k is $\frac{1}{m}$ and the probability that the destination of node i is located in cell k is also $\frac{1}{m}$. Therefore,

$E\,|\,Y_i\,| = \dfrac{1}{m^2}$. We then define a random variable $N_k = \sum_{i=1}^{n} Y_i$ to represent the number of source and destination pairs communicating using the ad-hoc mode within cell *k*. Since $\{Y_i\}_1^n$ is an *i.i.d* sequence of random variables with $E\,|\,Y_i\,| = \dfrac{1}{m^2}$, by Strong Law of Large Numbers and with probability, *i*, we have:

$$\frac{N_k}{n} = \frac{1}{n}\sum_{i=1}^{n} Y_i \rightarrow \frac{1}{m^2} \quad as\ n \rightarrow \infty \tag{3.2}$$

Given, $m = O(\sqrt{n})$, we have $\lim_{n\to\infty} \dfrac{n}{m^2} \rightarrow \infty$, and thus $\lim_{n\to\infty} N_k \rightarrow \infty$. Accordingly, for random ad-hoc network of N_k nodes and a common transmission rate of W_1, the per node capacity is $\Theta\left(\dfrac{W_1}{\sqrt{N_k \log N_k}}\right)$, as N_k approaches infinity. Therefore the capacity of cell k contributed by ad-hoc transmission is: $T_a(N_k) = \Theta\left(\sqrt{\dfrac{N_k}{\log N_k}}W\right)$.

Now, let $c_2 = \liminf_{n\to\infty} \dfrac{T_a(N_k)}{\sqrt{\dfrac{N_k}{\log N_k}}W_1}$ and $c_3 = \limsup_{n\to\infty} \dfrac{T_a(N_k)}{\sqrt{\dfrac{N_k}{\log N_k}}W_1}$. From equation (3.2), we have $\lim_{n\to\infty} \dfrac{\sqrt{\dfrac{N_k}{\log N_k}}}{\sqrt{\dfrac{N_k}{\dfrac{n/m^2}{\log(n/m^2)}}}} = 1$. Therefore,

$$\liminf_{n\to\infty} \frac{T_a}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}W_1} = \liminf_{n\to\infty} \frac{T_a(N_k)}{\sqrt{\dfrac{N_k}{\log N_k}}W_1} \frac{\sqrt{\dfrac{N_k}{\log N_k}}}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}} = c_2 \text{ and}$$

$$\limsup_{n\to\infty} \frac{T_a N_k}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}W_1} = \limsup_{n\to\infty} \frac{T_a(N_k)}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}W_1} \frac{\sqrt{\dfrac{N_k}{\log N_k}}}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}} = c_3$$

The term $\lim_{n\to\infty} \dfrac{T_a(N_k)}{\sqrt{\dfrac{n/m^2}{\log(n/m^2)}}W_1}$ is upper and lower bound by two constraints. Therefore, the per cell throughput capacity contributed by the ad-hoc mode communications is:

$$T_a = \Theta\left(\sqrt{\frac{n/m^2}{\log n/m^2}}W_1\right) \tag{3.3}$$

We now calculate the capacity contributed by the infrastructure mode communications. Since all the infrastructure mode traffic has to go through the base station and the base station can only receive data at the rate of W_2 bits/sec, at any given time, T_i, the received data is upper bound by W_2. For the lower bound, if each node in the infrastructure mode employs a transmission range of l (the side length of each cell), there is a schedule for each node to transmit to the base station in a round robin fashion, yielding a throughput of W_2. Therefore

$$T_i = \Theta(W_2) \qquad (3.4)$$

Since there is no interference between the ad-hoc mode and the infrastructure mode (Oliver et. al., 1991) [7], the aggregate throughput capacity is $T_a + T_i$, i.e.

$$T(n,m) = \Theta(mT_a + mT_i) = \Theta\left(\sqrt{\frac{n}{\log(n/m^2)}}W_1 + mW_2\right) \qquad (3.5)$$

The maximum throughput capacity is maximized if $W_2/W \to 0$ and the convenient corresponding throughput capacity becomes:

$$T(n,m) = \Theta\left(\sqrt{\frac{n}{\log\frac{n}{m^2}}}W\right) \qquad (3.6)$$

From equations (3.3) and (3.6) we then establish relationships among the number of node, bandwidth, base station infrastructure and throughput capacity. These relationships are discussed in the next section.

4.0 Presentation and discussion of simulated results

Figures 4.1 and 4.2 shows the per-cell capacity and the aggregate throughput capacity of the hybrid networks respectively as function nodes. It can be observed from these figures that the capacity of the network increases with the nodes as the base station increases. This is due to the fact that as the base station increases; more links are formed resulting in increase in the density nodes and network throughput capacity. Indeed, it is obvious that the base station infrastructure can highly impact the network throughput and this enhances the overall performance of the networks. However, it should be noted here that if *m* grows asymptotically slower than, $\sqrt{n}$, the maximum capacity decreases. In this case, the benefit of adding base station infrastructure is insignificant. Therefore, in a case where base stations can be added at a speed asymptotically faster than $\sqrt{n}$, the maximum capacity of the network will increase with the number of base stations. Also, if the number of base stations scales slower than some thresholds, the throughput capacity is dominated by the contribution of ad-hoc mode transmission.

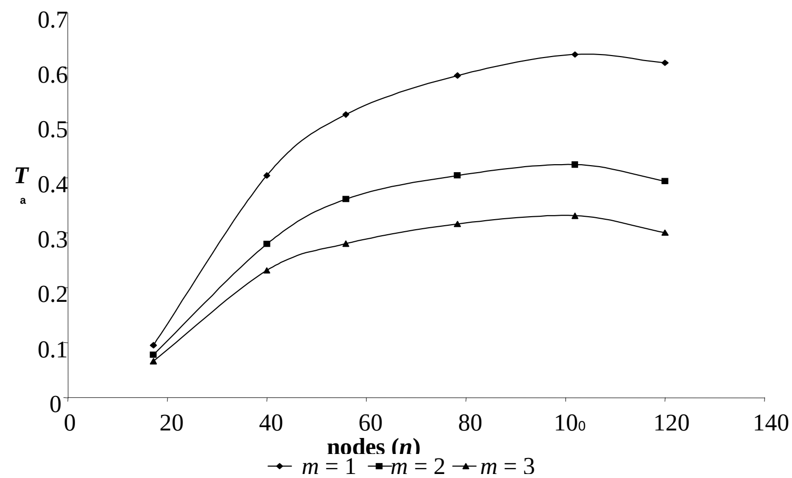

Figure 4.1: A graph of per-cell throughput capacity vs. number of nodes

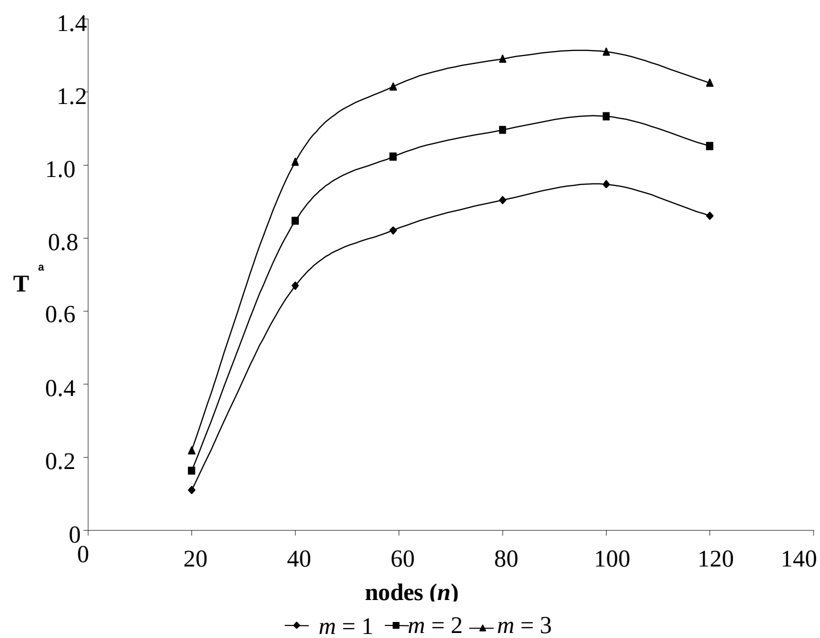

Figure 4.2: A Graph of aggregate throughput capacity vs. number of nodes

The benefit of adding base stations is minimal. If the number of base stations scale faster than the threshold, the capacity contributed by the infrastructure dominates the overall network throughput capacity. In this case, the maximum throughput capacity scales linearly with the number of base stations, providing an effective improvement over pure ad-hoc networks.

5.0 Conclusion

This paper has examined the throughput capacity of hybrid wireless networks with the aid of infrastructure communications. The hybrid network considered in this paper consists of an ad hoc network and sparse network of base stations. The base stations are connected by a wired network and placed in the ad hoc network in a regular pattern, which allows data to be forwarded either in a multi-hop fashion (in the case of ad hoc networks) or through the infrastructure (in the case of cellular networks).

A computational model for obtaining the throughput capacity for the hybrid wireless network have been analytically presented and simulated. The goal is to investigate the benefits of the infrastructure to the throughput capacity. As observed from our results, the throughput capacity increases linearly with the base station infrastructure provided the number of nodes *n*, does not grow asymptotically slower than $\sqrt{n}$. The results show a remarkable improvement in performance, with the help of the base station infrastructure using a dynamic routing strategy. Our results is in contrast with the results obtained in Gupta and Kumar [1], where the capacity of a random network does not scale well with the number of nodes *n*, in the system. Hence, to achieve significant capacity gain for hybrid networks, investment in the wired infrastructure should be high enough.

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