Journal of the Nigerian Association of Mathematical Physics
Volume **14 (May, 2009), pp 181 – 188**
© J. of NAMP

Well test analysis of horizontal wells in a two-layered reservoir system: Mathematical derivation

E. S. Adewole
Petroleum Engineering Department,
University of Benin, Benin City, Nigeria

Abstract

In this paper, a well test analysis procedure is discussed for a two-layered oil reservoir drained from each layer by a horizontal well. Reservoir mathematical model are derived for each layer so that analysis can be done strictly for each layered reservoir. Procedures for obtaining all the directional permeabilities, wellbore skin, degree of crossflow and individual layers average pressures are discussed for a pressure drawdown test procedure

Nomenclature

$p_D = \frac{kh\Delta p}{141.2q\mu B}$; $t_D = \frac{0.001056\ kt}{\mu\phi c_t L^2}$; $i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}}$,
i = positions along x or y or z axes, *ft*;
$h_D = 1/L_D$;
Δ = drop;
p = pressure, *psi;*
k = permeability, *md;*
h = pay thickness, *ft*;
t = time, hours;
q = flow rate, STB/Day;
μ = oil viscosity, *cp*;
B = oil formation volume factor, bbl/STB;
c_t = total fluid compressibility, *1/psi*;
L = well length, *ft;*
erf = error function;
τ = dimensionless dummy time variable $t_D p'_D$ dimensionless pressure derivative.

Subscripts

x, y, z = *x, y*, or *z*, directions; *D* = dimensionless; *w* = wellbore; *e* = external

1.0 Introduction

Horizontal well test and analysis procedures have been adequately reported in the literature especially in the late 1990's [1, 2, 3,4]. At first, test analyses procedures were modeled in the form of the conventional test analyses procedures such as the (Horner's) buildup and pressure drawdown plots. Later, dimensionless pressure derivative plots were introduced [5, 8], to solve test analyses shortcomings associated with the conventional test analyses methods.

e-mail: steveadewole@yahoo.co.uk
Telephone: 08039237561

The major aims of all transient test analyses are to evaluate wellbore condition, reservoir heterogeneity and reservoir boundary types. This information helps in planning well work over or repair, well location strategy for optimum recovery, production or injection profiling and determination of source of available reservoir energy (boundary types).

Layered reservoirs present peculiar problems. Apart from possessing more than one permeability distribution, the nature of their interfaces have to be established to determine a more technically suitable completion pattern for both economic benefits and high productivity. The interface may be permeable or not permeable. Permeable interfaces allow crossflow while no crossflow occurs if the interfaces are not permeable. Well completion strategies in layered reservoirs with crossflow and no crossflow interfaces have been discussed for vertical wells [9, 10]. Not so much similar efforts have been made for horizontal wells, but many authors have discussed analytical methods of delineating flow boundaries [1, 2, 3, 8, 11, 12, 13]. These methods can be used to determine both lateral and vertical extents of the reservoir from a horizontal well test. The remaining aspect yet to be discussed in as much detail remains the degree of crossflow determination. The degree of cossflow determines whether to crossflow the individual layers or commingle all the layers.

In this paper, a two-layered oil reservoir drained collectively and individually, with a horizontal well will be modeled mathematically and a detailed test analysis procedure based on the model will be discussed. Major information that will be derived from the well test analysis are (1) directional permeabilities, k_x , k_y, and k_z, (2) wellbore damage skin factor, s. (3) average reservoir pressure, p_e, (4) minimum wellbore pressure, p_{wm}, for oil production from only one layer, and (5) degree of crossflow, β. Both layer dimensionless pressure distribution and their derivatives will be derived and utilized.

2.0 Reservoir and Mathematical model descriptions

An anisotropic two-layered reservoir system is assumed. As shown below in Figure 2.1, each layer is penetrated with a horizontal well.

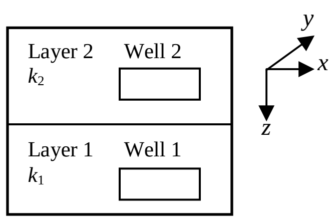

Figure 2.1: Layered Reservoir Model

For a permeable interface let the propensity of interface fluid crossflow be given by the index β, where $\xi \le \beta \le \infty$, practically, and ξ is a small positive value, greater than zero. As will be clear later, for an isotropic reservoir layers if $\beta = 1$, then the interface will experience no crossflow even though it may be permeable except created artificially under large pressure drop. Lower values of β mean that the interface will experience limited crossflow, while larger values mean that the interface will experience sufficient crossflow. [14] describes the mathematical development of the pressure distribution for the reservoir system shown in Figure 2.1 above. The layers dimensionless pressure distribution expressions according to the references are as follows for laterally infinite layers:

(1) Upper Layer (Layer 2)

$$p_{D2} = \frac{E_2\sqrt{\pi\beta}}{2}\sqrt{\frac{k}{k_y}}\int_0^{t_D}\left[erf\ \frac{\sqrt{k/k_x}+x_D}{2\sqrt{\beta\tau}} + erf\ \frac{\sqrt{k/k_x}-x_D}{2\sqrt{\beta\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\beta\tau}}{\sqrt{\tau}} \tag{2.1}$$

$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n-1)^2\pi^2\tau}{4h_D^2}\sin\frac{(2n-1)\pi z_D}{2h_D}\sin\frac{(2n-1)\pi z_{wD}}{2h_D}d\tau$$

where $$\beta = (\phi_1 c_{t1}\mu_1 L_1^2 k_2)/((\phi_2 c_{t2}\mu_2 L_2^2 k_1) \tag{2.2}$$

(2) Lower Layer (Layer 1)

$$p_{D1} = \frac{E_1\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_0^{t_D}\left[erf\ \frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}} + erf\ \frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}} \tag{2.3}$$

$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau$$

In both Equations 2.1 and 2.3, the effects of the individual layers boundaries along the $-x$ and $-y$ axes have not been felt and are considered infinite. Only the interface effects (along the top or bottom $-z$ axis of the layers) are assumed now felt. When all the boundaries are felt, the following equations are then used in place of Equations 2.1 and 2.3, for central well location along the x-axis, according to [15]:

$$p_{D2} = \frac{4\pi E_2\beta}{y_{eD}x_{eD}}\int_0^{t_D}\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2\tau\beta}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}})\right]$$

$$(\times\left[1+2\sum_{m=1}^{\infty}\exp(-\frac{m^2\pi^2\tau\beta}{y_{eD}^2})\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}})\right] \tag{2.4}$$

$$\times\sum_{l=1}^{\infty}\exp(-\frac{(2l-1)^2\pi^2\tau\beta}{4h_D^2})\sin\frac{(2l-1)\pi z_{wD}}{2h_D}\sin\frac{(2l-1)\pi z_D}{2h_D}d\tau$$

$$p_{D1} = \frac{4\pi E_1}{y_{eD}x_{eD}}\int_0^{t_D}\left[1+\frac{2x_{eD}}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp(-\frac{n^2\pi^2\tau}{x_{eD}^2})\sin\frac{n\pi}{x_{eD}}\cos\frac{n\pi x_{wD}}{x_{eD}}\cos\frac{n\pi x_D}{x_{eD}})\right]$$

$$\times\left[1+2\sum_{m=1}^{\infty}\exp(-\frac{m^2\pi^2\tau}{y_{eD}^2})\cos\frac{m\pi y_{wD}}{y_{eD}}\cos\frac{m\pi y_D}{y_{eD}})\right] \tag{2.5}$$

$$\times\sum_{l=1}^{\infty}\exp(-\frac{(2l+1)^2\pi^2\tau}{4h_D^2})\cos\frac{(2l+1)\pi z_{wD}}{2h_D}\cos\frac{(2l+1)\pi z_D}{2h_D}d\tau$$

In other words, Equations 2.4 and 2.5 prevail long after the prevalence of 2.1 and 2.3 if (1) the interface experiences no crossflow or limited crossflow, (2) large layers k_h/k_v ratios and (3) low layers wells withdrawal or injection rates. Otherwise, production of fluid from or injection into one of the two layers would produce the effects of one enlarged reservoir in which case all the parameters would now be the average or equivalent sum of the parameters of all the layers. Furthermore, the flow across the interface would now be dependent upon the degree of communication, β, between the layers. By equating dimensionless pressures and flow velocities at the interface, the expression for E can be derived as follows if Equations 2.1 and 2.3 are used [14, 15]:

$$E=\frac{2}{\sqrt{\pi}}\left[\frac{\int_{z_{wD1}}^{h_{D1}} I_{11}dz_D+\int_{h_{D1}}^{h_D} I_{22}dz_D}{\int_{z_{wD1}}^{h_{D1}} I_{11}^2dz_D+\int_{h_{d1}}^{h_D} I_{22}^2dz_D}\right] \tag{2.6}$$

where

$$I_{11}=\sqrt{\frac{k}{k_y}}\int_0^{t_D}\left[erf\,\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}}+erf\,\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}}$$
$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi(z_D=h_{D1})}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau \tag{2.7}$$

and

$$I_{22}=\sqrt{\beta}\sqrt{\frac{k}{k_y}}\int_0^{t_D}\left[erf\,\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\beta\tau}}+erf\,\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\beta\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\beta\tau}}{\sqrt{\tau}}$$
$$\times\sum_{n=1}^{\infty}\frac{(2n-1)\pi}{2}\exp(-\frac{(2n-1)^2\pi^2\beta\tau}{4h_D^2})\cos\frac{(2n-1)\pi(z_D=h_{D1})}{2h_D}\sin\frac{(2n-1)\pi z_{wD}}{2h_D}d\tau \tag{2.8}$$

2.1 **Well test analysis Mathematical model**

For application in well test analysis, assuming Well 1, the solution to Equation 2.3 is written as

$$p_{D1}=-\frac{h_D}{4}\sqrt{\frac{kk}{k_yk_z}}Ei(-\frac{r_D^2}{4t_D})+\frac{E\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_{t_{De}}^{t_D}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}}+erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}}$$
$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau. \tag{2.9}$$

Equation 2.9 is obtained by superposing the infinite-acting (radial) flow period and the early linear flow period. During the infinite-acting flow, E = 0 since no external physical boundary has been encountered. According to [5, 6, 7] the pressure derivative for Equation 2.9 is

$$t_Dp_D'=-\frac{h_D}{4}\sqrt{\frac{kk}{k_yk_z}}e^{-r_D^2/4t_D}+E\frac{\sqrt{\pi t_D}}{2}\sqrt{\frac{k}{k_y}}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}}+erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right]e^{-(y_D-y_{wD})^2/4\tau}$$
$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D} \tag{2.10}$$

Note that Equations 2.1, 2.3, 2.4, 2.5, 2.9 and 2.10 do not depend on the ratio $\sqrt{k/k_y}$ as a result of multiplication by E.

2.2 Well test analysis procedure

For various of h_D, r_D, $\sqrt{k/k_x}$, y_D and y_{wD} (= r_{wD}), type curves can be produced for p_{D1} versus t_D and $t_D p'_D$ versus t_D, respectively, on log-log axes. A plot of $t_D p_D$ against $e^{-r_D^2/4t_D}$ and a plot of p_D against $Ei(-r^2_D/4t_D)$ on linear axes yield the same slope of ($-0.25h_D\sqrt{kk/k_y k_z}$). However, in a well test only pressures, p, and flow times, t, are read directly; other parameters to be obtained from the well test are calculated. If the values of h_D and $\sqrt{k/k_x}$ are obtained from type-curve matching, then with the slope obtained from the plots above there are now three equations and three unknowns to be solved for. These unknowns are k_x, k_y, and k_z and the three equations are

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_z}} \tag{2.11}$$

$$(-0.25h_D\sqrt{kk/k_y k_z} = A) \tag{2.12}$$

and

$$\sqrt{k/k_x} = B \tag{2.13}$$

Solving the three equations simultaneously, we have

$$k_x = \frac{8A^{3/2}}{h_D^{3/2}B^{3/2}},\ k_y = \frac{8A^{1/2}}{h_D^{5/2}B^{1/2L}L^2},\ \text{and}\ k_z = \frac{8A^{3/2}}{h_D^{1/2}B^{-5/2}L^2}, \tag{2.14}$$

2.3 Behaviour of *E*

At early dimensionless times, p_D is not affected by E, since no boundary is felt yet. For all well parameters, values of E remain constant beyond the early flow period [14, 16]. Negligible values of E are obtained for large values of h_D, showing that the factor E becomes negligible as the layers thickness becomes larger. In other words, the layers p_D values still behave as if the pay thickness are infinite. Note that for reservoirs without crossflow, E = 1 for all expressions to be valid. During the period beyond radial flow, i.e., when E is constant, it therefore means that

$$E = constant \tag{2.15}$$

Hence

$$E - constant = 0 \tag{2.16}$$

where *constant* is the constant value obtained beyond the early radial flow period. Equation 2.15 is valid only at $t_D \geq t_{De}$ and t_{De} is the least value of t_D that satisfies Equation 2.15. It may also be the dimensionless time when the interface is felt. Thus, for any set of wellbore and reservoir layers parameters a particular value of β that satisfies Equation 2.16 can be estimated numerically using the Newton-Raphson scheme as follows: Assume that β = 0, the worst scenario. Obtain

$$E' = \frac{\partial E}{\partial \beta} \tag{2.17}$$

Then

$$\beta_{i+1} = \beta_i - \frac{E(\beta = 0)}{E'(\beta = 0)} \tag{2.18}$$

If $|(\beta_{i+1} - \beta_i)| \approx$ epsilon, then β_i is the degree of crossflow across the interface.

3.0 Individual reservoir layers average pressure

The general relationship for the flowing well pressure and reservoir layers pressure is given as

$$p(x, y, z, t) = p_e - \frac{141.2q\mu B}{kh}(p_D + s) \tag{3.1}$$

where s is the wellbore skin and p_D is given by Equation 3.1, assuming Well 1 is the monitoring well. From Equation 3.1, it is expected that the growth of p_D, occasioned by the exponential integral is interrupted the moment the interface is felt and beyond. If the dimensionless time at which the constant value is attained is t_{Dss}, then

$$p_{D1} = -\frac{h_D}{4}\sqrt{\frac{kk}{k_y k_z}} Ei(-\frac{r_D^2}{4t_D}) + \frac{E\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_{t_{De}}^{t_{Dss}}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}} + erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right]\frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}} \tag{3.2}$$

$$\times\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau$$

At $t_D > t_{Dss}$ flow is now propagated beyond the interface in the x-y plane for highly compressible oil. The rapidity at which this is attained depends on the degree of crossflow of the interface. Note that in Equation 3.2, p_{D1} does not depend on dimensionless time.

Using Equation 3.2 in Equation 3.1 and for a constant drawdown or injection rate, a plot of $p(x,y,z,t)$ recorded at the surface against p_D yields a straight line with intercept p_e on the vertical axis. The intercept should be obtained by extrapolating the portion of the curve that is parallel to the p_D axis; that is, the minimum wellbore pressure, p_{wm}, required for fluid production or injection from the bottom layer only and at the prevailing rate. The slope of the curve, m = $141.2q\mu B/kL$, can be used to calculate k, kL, k/μ, etc.

3.1 Estimation of Skin Factor, *s*

To permit estimation of the skin factor, s, Equation 3.2 is written to include the skin factor and oil field units as follows:

$$p = p_e - \frac{162.6q\mu B}{kL}\left\{\begin{bmatrix}\frac{h_D}{4}\sqrt{\frac{kk}{k_yk_z}}\left(\log t + \log\frac{k}{\phi\mu c_t L^2} - 2.625\right) + \frac{E\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_{t_{De}}^{t_D}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}} + erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right] \\ \frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau + 0.87s\end{bmatrix}\right\} \tag{3.3}$$

After flowing the well for $1hr$, Equation 3.3 is now given as:

$$p_{1hr} = p_e - \frac{1626q\mu B}{kL}\left\{\begin{bmatrix}\frac{h_D}{4}\sqrt{\frac{kk}{k_yk_z}}\left(\log\frac{k}{\phi\mu c_t L^2} - 2.625\right) + \frac{E\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_{t_{D\text{int}}}^{t_{D1hr}}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}} + erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right] \\ \frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau + 0.87s\end{bmatrix}\right\}. \tag{3.4}$$

Solving,

$$s = 1.151\left\{\frac{(p_e - p_{1hr})}{m} - \left(\begin{bmatrix}\frac{h_D}{4}\sqrt{\frac{kk}{k_yk_z}}\left(\log\frac{k}{\phi\mu c_t L^2} - 2.625\right) + \frac{E\sqrt{\pi}}{2}\sqrt{\frac{k}{k_y}}\int_{t_{D\text{int}}}^{t_{D1hr}}\left[erf\frac{\sqrt{k/k_x}+x_D}{2\sqrt{\tau}} + erf\frac{\sqrt{k/k_x}-x_D}{2\sqrt{\tau}}\right] \\ \frac{e^{-(y_D-y_{wD})^2/4\tau}}{\sqrt{\tau}}\sum_{n=1}^{\infty}\exp(-\frac{(2n+1)^2\pi^2\tau}{4h_D^2}\cos\frac{(2n+1)\pi z_D}{h_D}\cos\frac{(2n+1)\pi z_{wD}}{h_D}d\tau + 0.87s\end{bmatrix}\right)\right\} \tag{3.5}$$

where $m = 162.6q\mu B/kL$. Note that in Equation 3.5, s does not depend on dimensionless time since it is evaluated within constant dimensionless time interval t_{De} and t_{D1hr}. The p_{1hr} must be obtained from the straight line on a plot of p_{wf} against log t.

All infinite summations are performed at the wellbore, i.e., at $n = 1$. The integrals can be evaluated numerically.

4.0 Conclusion

Horizontal well analysis has been discussed for a layered reservoir with crossflow interface. For a pressure draw down test analysis procedure, the following conclusions can be deduced:

1. Type curve matching is required to obtain h_D and $\sqrt{k/k_x}$. Even though the degree of crossflow, β, can be obtained from type curve matching, it can also be obtained from Newton-Raphson scheme.
2. A plot of subsurface pressures versus computed dimensionless pressures is made to read the layers average pressures. This is against conventional approaches for obtaining average reservoir pressures in the literature.
3. The plot above can also be used to estimate the minimum bottomhole pressure to guarantee oil production from only one layer under a specific rate regime.
4. All directional permeabilities are inversely proportional to dimensionless thickness in varying degrees.
5. The slopes of dimensionless pressure against $Ei(-r_D^2/4t_D)$ and dimensionless pressure derivative against $\exp(-r_D^2/4t_D)$ are the same.

References

[1] Ozkan, E., Raghavan, R. and Joshi, S.D.: "Horizontal Well Pressure Analysis," *SPEFE* (Dec. 1989) 567 – 575.

[2] Odeh, S.A. and Babu, D.K.: "Transient Flow Behaviour of Horizontal Wells: Pressure Drawdown and Buildup Analysis," *SPEFE* (March 1990)7 –15, Trans., AIME, 289.

[3] Ozkan, E. and Raghavan, R.: "Performance of Horizontal Wells Subject to Bottom Water Drive," SPERE (Aug. 1990) 375 – 383, *Trans*., 289.

[4] Daviau, F., Mouronval, G., Bourdarot, G. and Curutchet, P.: "Pressure Analysis for Horizontal Wells," *SPEFE* (Dec. 1988) 716 – 724.

[5] Bourdet,D., Whittle,T.M., Douglas, A.A., and Pirard, Y-M.: "A New Set of Type Curves Simplifies Well Test Analysis," *World Oil*, (May 1983), 95 – 106.

[6] Bourdet, D., Ayoub, J.A., and Pirard, Y-M.: "Use of the Pressure Derivative in Well Test Interpretation," *SPEFE*, (June 1989), 293 – 302.

[7] Tiab, D. and Kumar, A.: "Application of the p_D' Function to Interference Analysis," *J.P.T.* (Aug. 1980), 1465 – 70.

[8] Ovwigho, K. and Adewole, E.S.: "Pressure Derivative Studies of a Laterally Infinite Reservoir with a Horizontal Well"; paper accepted for publication in the *Advanced Material Research*.

[9] Russell, D.G. and Prats, M.: "Performance of Layered Reservoirs with Crossflow- Single – Compressible – Fluid Case," *SPEJ*, March 1962, vol. 225, p. 53 – 67.

[10] Pendergrass, J.D. and Berry, V.J. Jr.: "Pressure Transient Performance of a Multilayered Reservoir with Crossflow," SPE *Trans*., AIME (Dec. 1962) vol. 225, 347.

[11] Kuchuck, F.J., Goode, P.A., Brice, B.W., Sherrard, D.W. and Thambynayagam, R.K.M.: "Pressure Analysis for Horizontal Wells," *JPT* (Aug. 1990)974.

[12] Goode, P.A. and Thambynayagam, R.K.M.: "Pressure Drawdown and Buildup Analysis for Horizontal Wells in Anisotropic Media," *SPEF*E (Dec.1987)683-697, *Trans.*, AIME 283.

[13] Clonts, M.D. and Ramey, H.J.Jr.: "Pressure-Transient Analysis for Wells with Horizontal Drainholes," paper SPE 15116 presented at the 1986 SPE California Regional Meeting, Oakland, April 2- 4.

[14] Adewole, E.S. (2004): "The Use of Pressure Functions to Model Pressure Distribution in a Layered Reservoir with Lateral Wells, Ph.D. Thesis, University. of Benin, Nigeria.

[15] Adewole, E.S. (2006): "The Use of Source and Green's Functions to Model Pressure Distribution in a Bounded Layered Reservoir with Lateral Wells; Part I: Mathematical Derivation," *Abacus: J. of the Mathematical Ass. of Nigeria*, vol. 33, Number 2B, Mathematical Series, Sept., p. 436-450.

[16] Adewole, E.S.: "Weighing Factor for Instantaneous Source Functions of a Permeable Interface", paper accepted for publication in the *J. of Nigerian Ass. of Mathematical Physics*, 2009.

[17] Earlougher, R.C. Jr.: Advances in Well Test Analysis, SPE, New York, 1977, Dallas, p.5.