Journal of the Nigerian Association of Mathematical Physics
Volume **14 (May, 2009), pp 151 – 158**
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Energy generation in a plant due to variable sunlight intensity

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Abstract

A mathematical model for energy generation in the cells of the leafs of a plant was designed. These modeled equations were solved assuming that the sunlight intensity is not constant. Our present result when compared with that of Mbah and Ezeorah* [1] *showed that this result as presented is more realistic. It is also shown that variation of the constant terms affect the level of carbohydrate produced. The effect of the sunlight intensity is shown where excessively high sunlight is shown to produce less carbohydrate which we interpreted to mean the deactivation of the enzymatic actions by this high level of sunlight intensity. Particularly, we showed the effect of the term b_2 which is likened to the case of diabetes in man.

1.0 Introduction

This study became necessary to us after having seen and studied the energy generation in the human cells, Mbah et al [1]. We understand that the plants and animals undergo the same process in the energy generation in the cells for the maintenance of life and activities of the cell. While the animal cells and in particular, the human cells do not produce food substances but rather depended on the food produced by plants, plants on its own produces its food in the presence of light energy from the sun, water, carbon dioxide from the air and the green colouring matter of the leaf called ***chlorophyll*** by a process known as the ***photosynthesis*****.** One then wishes to know how the cells of the leaf of a plant produce this and what quantity it produces per day that are actually stored as a source of energy in other parts of the plant for the animal world to use in generating its own energy for life activities and sustenance.

From our knowledge of what a plant is like, we know that a plant (tree) is made up of the leafs, the trunk (stem, braches) and the roots. We equally know that a leaf is made up of the cells and according to Weisz [2] and Guttmann and Hopkins [3]; a green leaf contains a ***chloroplast*** which contains the green colouring matter called the ***chlorophyll***. When we talk about energy obtained by man from plant, we mean the energy stored in the stem, trunks and or roots in the form of carbohydrates, proteins, vitamins etc as well as those stored in the leafs, Weisz [2], in addition to the energy subsequently generated by the leafs in the presence of light energy from the sun.

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The variation in the amount of energy supplied by a plant is as a result of the variation in the energy generation by the cells of the leaf which in most cases is a function of the sunlight energy released and used during the photolytic process. In our earlier work, we considered this sunlight energy release as constant. Here, we are to assume that the sunlight energy, k, is not constant all-through but varies with time and denoted as $k = \lambda + \varepsilon t$. This is the basis for our present study of the energy generated for storage in the roots, stem and branches of a plant by the cells of the leafs of a plant.

Thus, the determination of the storable energies in other parts of the plant such as the roots, stem and branches is a matter of determining the net energy manufactured or trapped daily from the sun by the leafs via the cells. Hence, if we can correctly get the net energy that is produced by a single cell which is meant for storage outside this cell, then we can get the entire quantity for the plant by simply summing up over the entire cells in the leaf and later over the entire leafs of the plant. Hence, we need to talk about photosynthesis which involves the green pigments of the leaf (Chlorophyll) located in the chloroplast of the cells in the leaf, the molecule of water (H_2O), the CO_2 in the air and finally the light energy. Detailed information about photosynthesis can be found in any good biology textbook although we used Weisz [3] and Guttmann and Hopkins [4]. At the end of the photosynthetic process, the end-products are Adenosine Triphosphate (ATP) and carbohydrate when chlorophyll **a** is used. There are other forms of chlorophyll other than this and they are used for the production of other food needs of the cells. The light energy is trapped in the ATP formation and this ATP is embedded in the formed carbohydrate. Hence, storing manufactured carbohydrate in the stem and roots is the same as storing the energy trapped from the sun light energy. Thus, let us determine the amount of energy that is trapped daily by a given cell of the leaf. It is known that a given cell of a leaf contains about 1 – 80 chloroplasts which are the sites for photosynthesis. From Weisz [3] and Guttmann and Hopkins [4], a given cell produces more energy (food) than it requires immediately. Some of these food or energy produced are used by the cell itself for its maintenance of life and functioning of the cell. Some others are then stored in the cell itself while the bulk of the produced food or energy are sent away from the cells to the stem and roots for storage. At this place, they are kept away from circulation so that they will only be available when required. In the absence of sunlight, some of this stored food/energy is converted back to carbohydrate to help supply energy to the cells in the stem, roots and even the cells. It may be necessary to state that in the cells conversion of the carbohydrates produced to Starch and other polysaccharides, certain enzymes are involved. In fact, system of enzymes is involved in this process, Weisz [3]. We shall assume that the quantity of system of enzymes involved in this process of conversion is determined by the quantity of energy generated in excess of the immediate cell requirement and thus for storage. We also know that little bit of the enzymes are destroyed in the process of this action, not necessarily by the carbohydrates, but by the system itself. Hence, we develop the model equation for the energy generation for storage as follows.

2.0 The Model

In this model, it is known that in the presence of the enzymes denoted by X, the carbohydrate or energy produced in excess will be stored in the stem, branches and roots. We shall sum up all the stored energy whether in the cell or other parts and represent it by E. Using these therefore and our study of the work of Mbah [5] and Mbah [6] on the glucose/Insulin levels in the human blood, we have the present model equations as:

$$\frac{dE}{dt} = a_i Q e^{-(\lambda+\varepsilon t)(t-\bar{t})} - a_2 EX - a_3 E \quad (2.1)$$

$$\frac{dX}{dt} = b_1 E - b_2 X \quad (2.2)$$

where a_1, a_2, a_3, b_1, b_2 are constants to be determined ($\lambda+\varepsilon t$) is the factor associated with the sunlight energy and ε is relatively small E is the energy generated by the cell, X is the system of enzymes involved in the conversion process of the carbohydrate to starch and other polysaccharides, t is the time while $\bar{t}$ =1/2 day which is 12:00 hours. From this model, it is assumed that the cell production of food/energy is at its peak at 12.00 noon on a 24 hours count for a day. In this present work, we shall assume that k is not constant throughout the day. Thus we shall solve while assuming that enough carbohydrate is already stored in the cell such that in further production of carbohydrates by the cells, none of it is further stored in the leaf. If that is the case, we shall write $a_2 EX$, as $a_2 E_o X$ so that the equations become:

$$\frac{dE}{dt} = a_i Q e^{-(\lambda+\varepsilon t)(t-\bar{t})} - a_2 E_0 X - a_3 E \quad (2.3)$$

$$\frac{dX}{dt} = b_1 E - b_2 X \quad (2.4)$$

Equation (2.4) says that the presence of the carbohydrates in the cell illicit the action of the enzymes release and some how, the quantity of the enzymes involved in the conversion process is a function of the quantity of the carbohydrate produced and the quantity of the enzymes either utilized by the system or catalyzed along the line of the conversion. We know that in every plant cell, some enzymes exists whose function is related to the type of cell in question. So, in the cells of the leaf, we have these catalyzing enzymes although, more is recruited to the particular position in the cell once much more carbohydrate is produced by the cell. In the absence of these enzymes, it is noticed that only very little quantity of the carbohydrate will be stored and the bulk of it will be dissipated as heat energy during transpiration just like the excretion of sugar in the urine of a diabetic patient due to lack of insulin in the blood of the patient, Mbah [6]. We can see that equation (2.3) is now linear unlike equation (2.1) above.

Based on all these, we shall solve these two equations (2.3) and (2.4) simultaneously by solving for the complementary and particular solutions. The complementary parts of equations (2.3) and (2.4) are:

$$\frac{dE}{dt} = -a_2 E_0 X - a_3 E \quad (2.5)$$

$$\frac{dX}{dt} = b_1 E - b_2 X \quad (2.6)$$

whose complementary solutions are:

$$\begin{pmatrix} E_c \\ X_c \end{pmatrix} = \begin{pmatrix} \xi \\ \beta \end{pmatrix} e^{\gamma t} \quad (2.7)$$

From equations (2.5) and (2.6), we have the matrix coefficient of the complementary equation as:

$$T = \begin{pmatrix} -a_3 & -a_2 E_0 \\ a_3 & -a_2 \end{pmatrix}$$

whose eigen-vector is given as:

$$\begin{pmatrix} -a_3 - \gamma & -a_2 E_0 \\ a_3 & -a_2 - \gamma \end{pmatrix}$$

which is evaluated to get $\gamma^2 + (a_3 + b_2)\gamma + a_3b_2 + b_1a_2E_0 = 0$

$$\frac{-(a_3+b_2) \pm \left\{(a_3+b_2)^2 - 4(a_3b_2 + b_1a_2E_0)\right\}^{\frac{1}{2}}}{2} \qquad (2.8)$$

For the particular solution, this exists only for the equation (2.3) and since the equations are solved simultaneously, we have the solution as:

$$\begin{pmatrix} E_p \\ X_p \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.9)$$

If we differentiate this equation (2.9) with respect to t and then substitute into equations (2.3) and (2.4), we then have:

$$-A(\lambda + 2\varepsilon t - \varepsilon\bar{t})e^{-(\lambda+\varepsilon t)(t-\bar{t})} = a_1Qe^{-(\lambda+\varepsilon t)(t-\bar{t})} - a_2E_0Be^{-(\lambda+\varepsilon t)(t-\bar{t})} - a_3Ae^{-(\lambda+\varepsilon t)(t-\bar{t})}$$

$$-B(\lambda + 2\varepsilon t - \varepsilon\bar{t})e^{-(\lambda+\varepsilon t)(t-\bar{t})} = b_1Ae^{-(\lambda+\varepsilon t)(t-\bar{t})} - b_2Be^{-(\lambda+\varepsilon t)(t-\bar{t})}$$

These will simplify to:

$$-A(\lambda + 2\varepsilon t - \varepsilon\bar{t}) = a_1Q - a_2E_0B - a_3A,\ \left[a_3 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right] + a_2E_0B = a_1Q$$

$$\Rightarrow -B(\lambda + 2\varepsilon t - \varepsilon\bar{t}) = b_1A - b_2B,\ \left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right]B - b_1A = 0$$

Thus, we have to solve for A and B that will satisfy these equations so that we obtain

$$A = \frac{a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right]}{z} \text{ and } B = \frac{a_1b_1Q}{z} \qquad (2.10)$$

$$z = \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)^2 - (b_2 + a_3)\left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right) + a_3b_2 + a_2b_1E_0 \qquad (2.11)$$

Thus, from equation (2.9), we have:

$$\begin{pmatrix} E_p \\ X_p \end{pmatrix} = \frac{1}{z}\begin{pmatrix} a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right] \\ a_1b_1Q \end{pmatrix} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.12)$$

From equations (2.7) and (2.12), we have the general solution to equations (2.3) and (2.4) as

$$\begin{pmatrix} E \\ X \end{pmatrix} = \begin{pmatrix} E_c \\ X_c \end{pmatrix} + \begin{pmatrix} E_p \\ X_p \end{pmatrix}$$

so that more explicitly, we have

$$\mathrm{E}(t) = \xi e^{\gamma t} + \frac{a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right]}{z} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.13)$$

$$X(t) = \beta e^{\gamma t} + \frac{a_1b_1Q}{z} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.14)$$

From these two equations, (2.13) and (2.14), we need to find the form or expressions for ξ and β. Thus, using equation (2.7), we have:

$$\begin{pmatrix} E_c \\ X_c \end{pmatrix} = \begin{pmatrix} \xi \\ \beta \end{pmatrix} e^{\gamma t} = \begin{pmatrix} \xi_1 e^{\gamma_1 t} + \xi_2 e^{\gamma_2 t} \\ \beta_1 e^{\gamma_1 t} + \beta_1 e^{\gamma_2 t} \end{pmatrix}$$

This means that

$$E_c = \xi_1 e^{\gamma_1 t} + \xi_2 e^{\gamma_2 t} \qquad (2.15)$$

$$X_c = \beta_1 e^{\gamma_1 t} + \beta_2 e^{\gamma_2 t} \qquad (2.16)$$

If we differentiate these and the substitute into equations (2.5) and (2.6), we then get

$$\gamma_1\xi_1 e^{\gamma_1 t} + \gamma_2\xi_2 e^{\gamma_2 t} = -a_2E_0X_c - a_3E_c \qquad (*)$$

$$\gamma_1\beta_1 e^{\gamma_1 t} + \gamma_2\beta_2 e^{\gamma_2 t} = b_1E_c - b_2X_c \qquad (**)$$

Using equation (*) and substituting appropriately for X_c and E_c using equations (2.15) and (2.16), we get:

$$\xi_1\gamma_1 e^{\gamma_1 t} + \xi_2\gamma_2 e^{\gamma_2 t} = -a_2E_0\left(\beta_1 e^{\gamma_1 t} + \beta_2 e^{\gamma_2 t}\right) - a_3\left(\xi_1 e^{\gamma_1 t} + \xi_2 e^{\gamma_2 t}\right)$$

$$\Rightarrow (\gamma_1 + a_3)\xi_1 e^{\gamma_1 t} + (\gamma_2 + a_3)\xi_2 e^{\gamma_2 t} = -a_2E_0\beta_1 e^{\gamma_1 t} - a_2E_0\beta_2 e^{\gamma_2 t}$$

Equating terms of corresponding coefficients, we have:

$$\beta_1 = \frac{\gamma_1 + a_3}{-a_2E_0}\xi_1 \text{ and } \beta_2 = \frac{\gamma_2 + a_3}{-a_2E_0}\xi_2 \qquad (2.17)$$

Substituting these values appropriately into equations (2.13) and (2.14) gives:

$$E(t) = \xi_1 e^{\gamma_1 t} + \xi_2 e^{\gamma_2 t} + \frac{a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right]}{z} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.18)$$

$$X(t) = -\frac{\gamma_1 + a_3}{a_2E_0}\xi_1 e^{\gamma_1 t} - \frac{\gamma_2 + a_3}{a_2E_0}\xi_2 e^{\gamma_2 t} + \frac{a_1b_1Q}{z} e^{-(\lambda+\varepsilon t)(t-\bar{t})} \qquad (2.19)$$

From these two equations, we need to find the values of ξ_1 and ξ_2 at a given time t and let this time t be $t = t^*$ which will be any comfortable time of interest such as the starting point t =7.00 hours. Thus, solving equations (2.18) and (2.19) simultaneously, we obtain :

$$\xi_1 = \left(\frac{a_2E_0}{\gamma_2 - \gamma_1}\right) e^{-\gamma_1 t^*}\left\{\left(\frac{\gamma_2 + a_3}{a_2E_0}\right)E^* + X^* - \frac{a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right](\gamma_2 + a_3)}{za_2E_0} e^{-(\lambda+\varepsilon t)(t-\bar{t})} - \frac{a_1b_1Q}{z}\right\} \qquad (2.20)$$

and

$$\xi_2 = \left(\frac{a_2E_0}{\gamma_1 - \gamma_2}\right) e^{-\gamma_2 t^*}\left\{\left(\frac{\gamma_1 + a_3}{a_2E_0}\right)E^* + X^* - \frac{a_1Q\left[b_2 - \left(\lambda + 2\varepsilon t - \varepsilon\bar{t}\right)\right](\gamma_1 + a_3)}{za_2E_0} e^{-(\lambda+\varepsilon t)(t-\bar{t})} - \frac{a_1b_1Q}{z}\right\} \qquad (2.21)$$

To obtain these expressions in equations (2.20) and (2.21), we had to solve at $t = \bar{t}$ which can be any time like the peak of the time of production of the carbohydrate/energy by the plant taking that we have normal sunlight weather in which the peak of the sunlight radiation of energy is at $t = \bar{t}$ which is at 12:00 noon on a 24 hours count for a day. Thus, if we substitute these two expressions into equation (2.18), we get the complete expression for the energy generated by a cell of a given leaf when there is proper sunlight energy radiation. This energy is that which is to be stored at other parts of the plant outside the producing cell. Similarly, substituting equations (2.20) and (2.21) into equation (2.19), we get the quantity of the system of enzymes that took part in the conversion process, who survived being degraded by the process, for every amount of the carbohydrate or energy manufactured or produced.

3.0 Analysis

To get a meaningful discussion of the models, graphs need to be produced using the solutions of the modeled equations. To do this, we used the MATLAB software to write the programme and generate the required graphs. Given that the values used for the constants in the equations are: a_1 = 0.028, a_2 = 0.004, a_3 = 0.03, b_1 = 0.45, b_2 = 0.33, λ = 0.69, ε = 0.004, $E_0 = E^*$ = 20units, $X_o = X^*$ = 1.5, $\bar{t}$ = 12.00 and t^* = 7.00, we then obtain the graph shown as Figure 3.1 below:

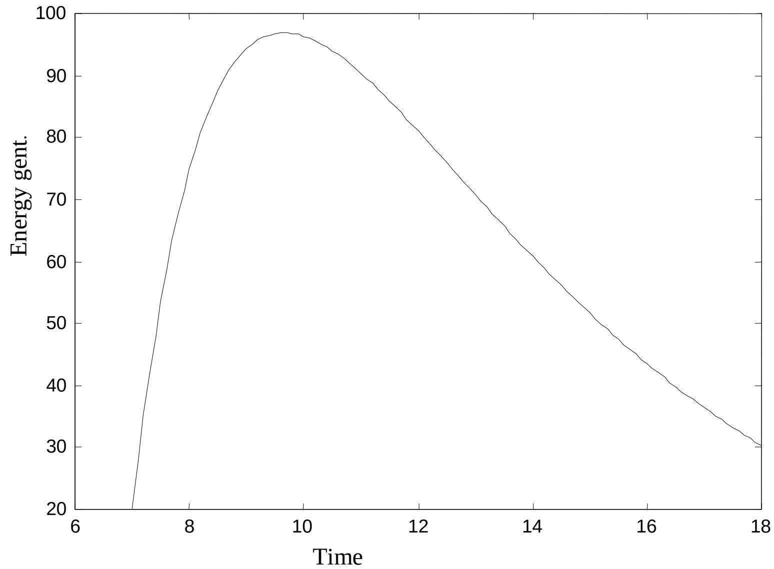

Figure 3.1: Energy generation over a 12 hour period

Even when we take the same values for the constants and then λ=0.494 with ε = 0.004, we saw that the result obtained graphically was far from this above showing that these values are not realistic if the sunlight energy used in the photosynthetic process was considered variant.

From the models and the subsequent graph, we saw that increasing the value of b_2 decreases the rate of clearance of the carbohydrate by conversion to starch by the enzymes thereby leaving much of it unconverted by the 18.00 hours. A change in the value of b_2 from 0.33 to 0.4 increases the quantity of the unconverted carbohydrate from 05 units to 11 units. This can be shown as Figure 3.2.

This result is expected and thus validated because increasing the value of b_2 implies less enzymes that are involved in the conversion which also implies higher carbohydrate presence in the cell.

Similarly, varying the values of a_2 and a_3 affects the level of carbohydrate left unconverted in the cell. This prac4ically explains that happens in reality and this shows t(at the existence of a particular tree is partly dependent on how fast it 0ses the produced carbohydrate .

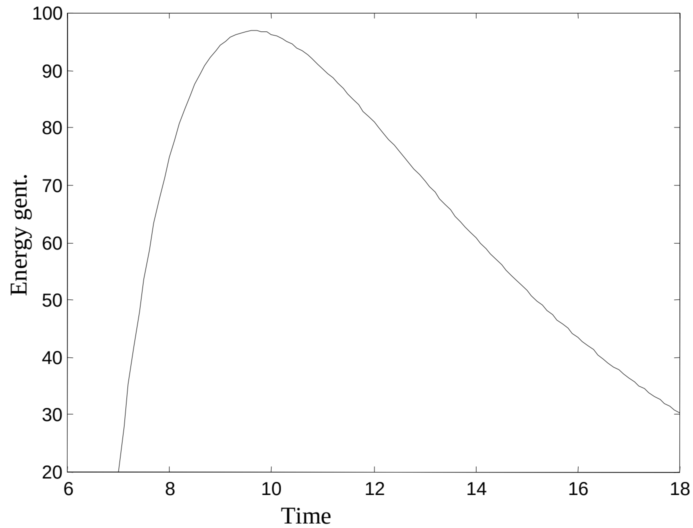

Table 3.2: Variation in the clearance rate (b_2)

Also, if much of this carbohydrate is not stored may be due to high value of b_2 or very small value of a_2 , then much of this produced carbohydrate will be wasted through transpiration . This resembles the case of a diabetic patient, Mbah [7], who abruptly gets, can because much of the glucose meant for storage are urinated out thereby leading to no reservation of glucose for the body cells' usage fat times of needed. This as we saw in Mbah [7], makes the cells to now utilize other food sources such as stored fat, vitamins etc to generate the required energy in the cell. A simple example is the percentage variation of the value of a_2 from 0.004 to 0.001 which changes the quantity of the unconverted carbohydrate from 100 units to 120 units. Thus the role played by the term a_2 is very important in the reservation or conservation of energy by the plants for the animal world. The change in the a_2 value is shown as Figure 3.3 below:

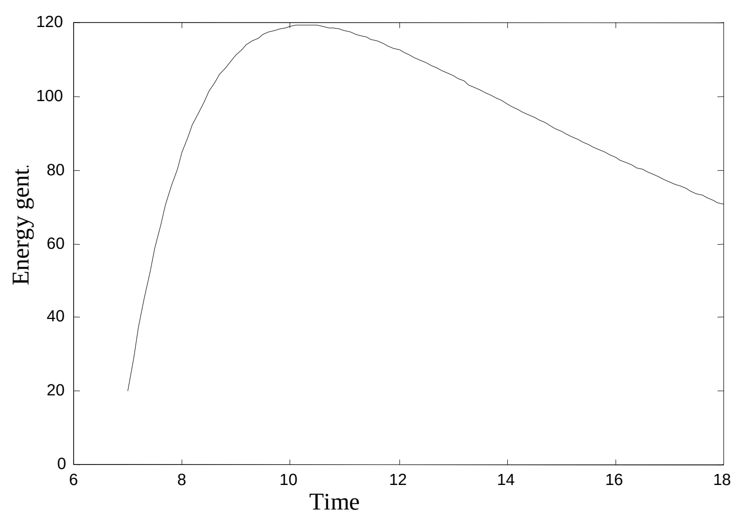

Figure 3.3: Effect of a_2 in the storage of energy by the plant

We can also see here that the quantity of the carbohydrate not cleared by the end of the day is still very high showing that a lot of it will be dissipated as heat during transpiration which serves no purpose both to the plant and or the animals.

From our earlier work, Mbah and Ezeorah [1], we assumed $k = 0.494$ and the other constants as stated and we obtained good result though with high level of unconverted carbohydrate which can be appropriately explained. However, using the same values for the constants but for the variable k, that is $k = \lambda + \varepsilon t$, we got a higher level of unconverted carbohydrate at all times. To be able to get the same quantity of the produced carbohydrate when k was assumed constant by using k as varied above, we saw that the constants, a_i, and b_j, $i = 1,2,3$ and $j = 1,2$, all took smaller values. This probably implies that assumption of constant value for k (sun light energy level) is an oversimplification. From practical point of view, we know that sunlight intensity is never the same from morning till night. This then means that the impact of this on the cells of the leaves during photolysis can never also be the same. A lot can still be done even for this our present assumption because the intensity does not increase or decrease indefinitely. It is pyramidal in nature and we shall handle this case in our further work although this appears more complex and more difficult to handle.

4.0 Conclusion

From this result, we can see that when the sunlight intensity varies, the model presents a more realistic situation compared to what obtained in our previous work where we assumed that the sun intensity was constant. Equally, we here showed that variation in the value of a_2 affects the level of the energy stored as food. In one of our works, we had modeled the energy generation in human system which motivated the present study. Therefore, with this model, one can completely have a good view of how energy is generated in the plant and subsequently in the human system as a result of the availability of this energy in the plant.

References

[1] Mbah, G.C.E. AND Ezeorah, J. N. (2009): Energy generation in a plant by the cells of the leafs of the plant. (Accepted for publication by the ICASTOR journal, India March, 2009

[2] Mbah, G.C.E., Oyesanya, M.O. and Ejikeme, C. L. (2007): Mathematical model on the energy generation in human cell. Part of M. Sc. Thesis, University of Nigeria, Nsukka. (Communicated)

[3] Weisz, P. B: The Science of Biology. New York. 1980

[4] Guttman, B.S and Hopkins III , J. W : Understanding Biology. Harcourt Brace Jovanovich, Inc. New York. 1983

[5] Mbah, G.C.E. (1998): Mathematical Modelling and control of blood glucose/Insulin concentrations in an insulin-dependent diabetic subject. A Ph. D. thesis, University of Nigeria, Nsukka

[6] Mbah, G.C.E. (2001): An analytical method of solution of the generalized mathematical model used for the study of insulin-dependent diabetes Mellitus. Jour. Nig. Mathl. Soc., Vol. 20, pp 65-76

[7] Mbah, G.C.E. (2002): Mathematical modeling approach to the treatment of insulin-dependent diabetes Disease. Global Journal of Pure and Applied Sciences, Vol. 8, No. 3 pp. 339-345.