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On the property of solutions to a system of equations modelling thermal explosion in combustible dusty gas containing fuel droplets with Arrhenius Power-law Model

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Abstract

This paper is essentially devoted to the property of solutions to a system of ordinary differential equations modelling thermal explosion in combustible dusty gas containing fuel droplet with generalised temperature dependent rate of reaction governed by Arrhenius power-law model. Theorems are stated and proofs provided on the qualitative properties of new system equations governing the physical model. New closed-form solutions are obtained based on quadratic approximations to the Arrhenius terms under realistic conditions. The results show that the delay before ignition depend significantly on interphase heat exchange parameter α_s and energy needed to transfer heat from gas phase to solid phase parameter μ. It is intended to describe the numerical analysis of the new problem in a later paper.

Nomenclature
English symbols

A	pre-exponential factor (1/s)
B	dimensionless parameter expressing ratio of latent heat of evaporation and specific combustion energy
C_f	molar concentration combustible gaseous mixture ($kmol/m^3$)
C_{pg}	specific heat capacity of the gas phase at constant pressure ($JK^{-1}kg^{-1}$),
C_{ps}	specific heat capacity of the solid phase at constant pressure ($JK^{-1}kg^{-1}$),,
E	activation energy ($J/kmol$)
L	latent heat of evaporation (J/kg)
m_f	molar mass (kg/$kmol$)
n	numerical exponent
n_d	number of droplets per unit volume (m^{-3}),
n_s	number of solid particles per unit volume (m^{-3})
Q	specific combustion energy ((J/kg)
R_d	droplet radius (m)
R_s	solid particle radius (m)
R_u	universal gas constant ($Jkmol^{-1}K^{-1}$)
t	time (s)
T_{g0}	combustible gas initial temperature (K)
T_{s0}	dusty particle initial temperature (K)
T_g	combustible gas temperature (K)
T_s	dusty particle temperature (K)

Greek Symbols

ρ	density of the combustible gaseous mixture (kg/m^3)
□	thermal conductivity ($Wm^{-1}K^{-1}$)
φ	volumetric phase content (dimensionless)
σ	Stefan-Boltzmann's constant
β	dimensionless activation energy

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τ dimensionless time

∂ dimensionless parameter expressing reciprocal of the characteristic time for adiabatic temperature rise

α_1 dimensionless parameter expressing heat loss via convection from gas phase#

α_2 dimensionless parameter expressing heat loss via radiation from gas phase

α_3 dimensionless parameter expressing interphase heat exchange between gas and solid particle

μ dimensionless parameter expressing energy needed to transfer heat from gas phase to solid phase

μ^* dimensionless parameter expressing energy needed to evaporate all fuel droplets

ψ dimensionless parameter expressing energy needed to consume all gas concentration

Subscripts

0 initial

g gas mixture,

f combustible gas component of the mixture (fuel);

d liquid droplets;

p constant pressure,

s solid particles

ig ignition

1.0 Introduction

The concept of thermal explosion in combustible dusty gas containing fuel droplets is of great importance in safety aspect of nuclear facilities, furnaces, gas turbines and internal combustion engines, coal mine etc. The essence of the inert solid particles in the combustible gas is to delay the ignition or explosion which could cause catastrophe. The phase of research on combustion and explosion that began at the end of the nineteenth century and continues to this day is associated with the invention of the internal combustion engine, with the development of explosive technology and of internal ballistics for artillery, and, in recent decade, with the extensive introduction of jet and diesel engines. In many respects, these have stimulated the rapid development of combustion science [36]. The procedure for thermal explosion in gases, which contain fuel droplets, has been of much interest. After Semenov [32] developed the basic theory of phenomenon of thermal explosion, models that are more complicated have been suggested in [27] and [34].

The focus of this research work is therefore on the long standing problem of thermal explosion and ignition in a combustible gas containing fuel droplets and its numerous applications to furnaces, gas turbines and internal combustion engines [23, 27, and 33, 34, 35]. Over recent years, the theoretical analysis of this problem has been performed mainly by the use of modern computers. In this regard, there exist many computational packages that have proved useful. These packages have been developed to take into account heat and mass transfer and combustion processes in the mixture of gas and fuel droplets in a self-consistent manner [8 and 28-30]. This approach, however, is not particularly helpful in aiding and understanding the relative contribution of various processes. An alternative approach to the problem is to analyse the equations in some limiting cases. This cannot replace computational methods but can complement them. One of these analyses is based on the geometrical asymptotic method of integral manifolds [13-22]. Klammer et al. [25] investigated ignition, combustion and detonation processes in dusty gases with combustion reaction. The dusty gas was considered as a two-continuum medium taking into account transport effects in the phases and non-equilibrium chemical reactions. Two-dimensional problems of ignition and detonation were developed in a plane gallery caused by a supersonic inflow stream and heating of the closed end of the gallery are studied with the analytical method of catastrophe and two finite-difference numerical methods. Krainov and Shaurman [26] studied the limits of flame propagation in a gas with suspended inert particles in the presence of external heat removal. The mathematical model used was based on an unsteady heat-diffusion two-temperature model of gas combustion in the presence of inert particles. The problem was solved by a numerical method. A parametric analysis was performed, and critical values of the parameter that characterizes external heat removal were obtained. Dispersed-phase parameters were determined for which the two-temperature nature of the medium was insignificant. For this case, an analytic estimate for the critical parameters of flame quenching was obtained. At the moment of flame quenching, the normal flame-propagation velocity in a dusty, gas decreases by a factor of $\sqrt{\in}$ compared with the flame velocity in the dusty gas under adiabatic conditions. Ben-Dor and Igra [9] considered the relaxation zone behind normal shock waves in a reacting dusty gas. It was assumed that the gas is monatomic. The conservation equations for a suspension composed of an ionized gas and small solid dust particles were formulated and then solved numerically. The solution revealed that the presence of the dust has a significant effect on the postshock flow field. Because of the dust, the relaxation zone was longer than in the pure plasma case; the equilibrium values for the suspension pressure and density was higher than in the dust-free case, whereas the values obtained for the temperature, degree of ionization, and velocity was lower. The numerical solution was executed for shock Mach numbers ranging from 10 to 17. It was found that the thermal relaxation length for the plasma decreases rapidly with increasing shock Mach number, whereas the thermal relaxation length for the suspension increases slightly with increasing M. Gol'dshtein et al. [13] studied criteria for thermal explosion with reactant consumption in a dusty gas. The dynamical regimes of the system were classified as slow regimes, thermal explosion with delay and thermal explosion (without delay). The critical transition conditions for the different dynamical regimes were analysed. They emphasized that the critical conditions for transition between slow regimes and explosion with delay was a thermal explosion limit. The Thermal explosion limit was described in the phase space by a so-called duck-trajectory. El-sayed [11] investigated the critical conditions of the adiabatic explosion problem of a gas–solid (dusty gas) mixture. The definitions used for the homogeneous gas to determine criticality were used for the gas–solid mixture. The analysis revealed that the classical definition of the critical point can be adopted and modified to determine the critical condition in $\gamma-\eta$ and $\gamma-\theta$ domains. It was also found that using the definition of criticality as an inflection point in the critical trajectory in the $\theta-\eta$ plane gives the same results as given by the classical definition of criticality. It was interesting to see that the critical and maximum points in the gas temperature–concentration domain could coincide. It was found that the presence of a solid produces more than one critical temperature. The limiting cases of the problem were also offered. The numerical solution showed that the supercritical trajectory shows a thermal runaway for the gas over the solid at the end of reaction.

In this paper, attempt has been made to extend the problem of thermal explosion in a combustible gas containing fuel droplets with addition of inert solid particles and to generalised the problem based on temperature dependence of the reaction rate (i.e. Arrhenius Power-law model equation [1, 24]) given as $K(T_g) = A\left(T_g / T_{g0}\right)^n \exp\left(-E / R_u T_g\right)$ while taking into account convective and radiative heat losses, and temperature dependence of density and thermal conductivity of the gas. Therefore, a generalised physical model for thermal explosion in combustible dusty gas mixture containing fuel droplets is developed in the present paper. The main interest is focused on property of solutions to the new system of coupled non-linear ordinary differential equations governing the physical model. Moreover, new closed-form solutions are obtained based on quadratic approximations to the Arrhenius terms under realistic conditions. The results show that the delay before ignition depend significantly on interphase heat exchange parameter α_s and energy needed to transfer heat from gas phase to solid phase parameter μ .

2.0 Mathematical model

In this study, we consider the problem of thermal explosion in combustible gas containing fuel droplets with addition of solid particles. Dusty gas is a combustible gas with addition of solid particles. It is assumed that the solid phase is inert, mono-size and uniformly heated and the dusty gas is optically thick. The reaction is fast and highly exothermic [13, 22]. The rate of reaction is based on generalized temperature dependent Arrhenius equation and temperature dependent thermo-physical properties of the gas are taking into account. The fuel droplet's surface temperature is assumed to be constant. Following [1], the system of governing equations has the form:

$$C_{pg}\varphi_g\rho_{g0}T_{go}T_g^{-1}\frac{dT_g}{dt} = Q_f m_f \varphi_g c_f A_0 T_{go}^{-n}T_g^n \exp\left(\frac{-E}{R_uT_g}\right) - 4\pi R_d n_d \lambda_{g0}\sqrt{\frac{T_g}{T_{g0}}}\left(T_g - T_{g0}\right)$$

$$-4\pi R_d^2 n_d \sigma(T_g^4 - T_{g0}^4) - 4\pi R_s n_s \lambda_s\left(T_g - T_s\right) \quad (2.1)$$

$$\rho_s C_{ps}\frac{dT_s}{dt} = 4\pi R_s n_s \lambda_s (T_g - T_s) \quad (2.2)$$

$$\frac{dR_d}{dt} = -\frac{1}{L\rho_d}\frac{\lambda_{g0}}{R_d}\sqrt{\frac{T_g}{T_{g0}}}\left(T_g - T_{g0}\right) - \frac{1}{L\rho_d}\sigma(T_g^4 - T_{g0}^4) \quad (2.3)$$

$$\varphi_g\frac{dc_f}{dt} = -\varphi_g c_f A_0 T_{g0}^{-n}T_g^n \exp\left(\frac{-E}{R_uT_g}\right) + \frac{4\pi R_d n_d \lambda_{g0}}{Lm_f}\sqrt{\frac{T_g}{T_{g0}}}\left(T_g - T_{g0}\right) + \frac{4\pi R_d^2 n_d \sigma}{Lm_f}(T_g^4 - T_{g0}^4) \quad (2.4)$$

with the following initial conditions:

$$T_g(0) = T_{g0}, T_s(0) = T_{s0},\ R_d(0) = R_{d0}, c_f(0) = c_{f0}. \quad (2.5)$$

2.1 Non-dimensional analysis

In this study, we introduce the following dimensionless variables:

$$,\theta_s = \frac{\left(T_g - T_s\right)E}{R_uT_{s0}^2}\ r = \frac{R_d}{R_{d0}}, \quad \eta = \frac{c_f}{c_{f0}}\ ,\tau = \frac{t}{t_*}. \quad (2.6)$$

We assume that at initial stage the temperature of gas and solid particles are the same i.e. $T_{go} = T_{s0}$. Therefore, using (2.6) in (2.1)-(2.5), the dimensionless system of governing equations has the following form:

$$\left(1+\beta\theta_g\right)^{-1}\frac{d\theta_g}{d\tau} = \delta\eta\left(1+\beta\theta_g\right)^n \exp\left(\frac{\theta_g}{1+\beta\,\theta_g}\right) - r\left\{\alpha_1\theta_g\sqrt{(1+\beta\theta_g)}\right.$$

$$\left. + \alpha_2 r\left[(1+\beta\theta_g)^4 - 1\right]\right\} - \alpha_3\left(\theta_g - \theta_s\right) \quad (2.7)$$

$$\frac{d\theta_s}{d\tau} = \alpha_3\mu\left(\theta_g - \theta_s\right) \quad (2.8)$$

$$\frac{dr}{d\tau} = -\frac{\mu^*}{r}\left\{\alpha_1\theta_g\sqrt{(1+\beta\theta_g)} + \alpha_2 r\left[\left(1+\beta\theta_g\right)^4 - 1\right]\right\} \quad (2.9)$$

$$\frac{d\eta}{d\tau} = -\alpha_4\delta\,\eta\left(1+\beta\theta_g\right)^n \exp\left(\frac{\theta_g}{1+\beta\,\theta_g}\right) + \psi\, r\left\{\alpha_1\theta_g\sqrt{(1+\beta\theta_g)} + \alpha_2 r\left[\left(1+\beta\theta_g\right)^4 - 1\right]\right\} \quad (2.10)$$

with the initial conditions:

$$\theta_g(0) = 0,\ \theta_s(0) = 0, r(0) = 1,\ \eta(0) = 1. \quad (2.11)$$

In equations (2.7) – (2.11) the following dimensionless parameters have been introduced:

$$\left.\begin{aligned}
&\beta = \frac{R_uT_{g0}}{E},\ B = \frac{L}{Q_f},\ \delta = \frac{EQ_f m_f c_{f0}At_*}{\rho_{g0}C_{pg}R_uT_{g0}^2}\exp\left(-\frac{E}{R_uT_{g0}}\right),\ \alpha_1 = \frac{4\pi R_{d0}n_d\lambda_{g0}t_*}{\rho_{g0}C_{pg}\varphi_g},\\
&\alpha_2 = \frac{4\pi R_{d0}^2 n_d\sigma T_{g0}^2 Et_*}{\rho_{g0}C_{pg}\varphi_g R_u},\ \alpha_3 = \frac{4\pi R_s n_s\lambda_s t_*}{\rho_{g0}C_{pg}\varphi_g},\ \alpha_4 = B\psi,\ \mu^* = \frac{\rho_{g0}C_{pg}\varphi_g R_uT_{g0}^2}{4\pi R_{d0}^3 n_d\rho_d EL},\\
&\mu = \frac{\rho_{g0}C_{pg}\varphi_g}{\rho_sC_{ps}},\ \psi = \frac{\rho_{g0}C_{pg}\varphi_g R_uT_{g0}}{Em_f c_{f0}L}.
\end{aligned}\right\} \quad (2.12)$$

Some special cases of the system of coupled non-linear ordinary differential equations (2.7) -(2.10) with initial conditions (2.11) and related problems have been studied for $n \neq 0$ and $\alpha_3 = 0$ (see for example [1], [3], [4] and the references therein). In the events that $n = 0$ and parameters α_1, α_2 and α_3 are varied, analyses have been performed both analytically and numerically in ([6, 12-22, 32, 36] and references sited therein).

3.0 Properties of Solutions

The aim of this section is to formulate some basic theorems relating to the properties of solutions of initial value problem (2.7)–(2.11) and establish the proofs respectively.

***Theorem* 3.1 (**Existence and Uniqueness Theorem**)**

If $|\tau| \le c$, $0 \le \theta_g \le c_1$, $0 \le \theta_s \le c_2$, $|r-1| \le c_3$, $|\eta - 1| \le c_4$, $0 \le \alpha_1 \le d_1$, $0 \le \alpha_2 \le d_2$, $0 \le \alpha_3 \le d_3$, $0 \le \beta << 1$, $0 \le B << 1$, $\delta > 0$, $\alpha_4 \ge 0$, $\mu > 0$, $\mu^* > 0$ and $\psi^* > 0$, then for $c > 0$ and $n \in \{-2, 0, 0.5\}$, there exists a unique solution of the system of coupled non-linear differential equations (2.7) – (2.10) which satisfies the initial conditions (2.11) .

Proof

Let

$$\left.\begin{aligned} \theta_g' &= g_1(\tau, \theta_g, \theta_s, r, \eta) \\ \theta_s' &= g_2(\tau, \theta_g, \theta_s, r, \eta), \\ r' &= g_3(\tau, \theta_g, \theta_s, r, \eta), \\ \eta' &= g_4(\tau, \theta_g, \theta_s, r, \eta) \end{aligned}\right\} \tag{3.1}$$

such that

$$\left.\begin{aligned} g_1(\tau, \theta_g, \theta_s, r, \eta) &= (1+\beta\theta_g)\{H_P(\theta_g, \eta) - H_L(\theta_g, r) - H_E(\theta_g, \theta_s)\} \\ g_2(\tau, \theta_g, \theta_s, r, \eta) &= \mu H_E(\theta_g, \theta_s) \\ g_3(\tau, \theta_g, \theta_s, r, \eta) &= -\frac{\mu^*}{r^2} H_L(\theta_g, r) \\ g_4(\tau, \theta_g, \theta_s, r, \eta) &= -\alpha_4 H_P(\theta_g, \eta) + \psi^* H_L(\theta_g, r) \end{aligned}\right\} \tag{3.2}$$

where

$$H_P(\theta_g, \eta) = \delta\eta(1+\beta\theta_g)^n \exp\left(\frac{\theta_g}{1+\beta\theta_g}\right), \tag{3.3}$$

$$H_L(\theta_g, r) = r\left\{\alpha_1\theta_g\sqrt{(1+\beta\theta_g)} + \alpha_2 r\left[(1+\beta\theta_g)^4 - 1\right]\right\}, \tag{3.4}$$

$$H_E(\theta_g, \theta_s) = \alpha_3(\theta_g - \theta_s). \tag{3.5}$$

Now, we need to show that $g_i(\tau, \theta_g, \theta_s, r, \eta)$ for each $i = 1,2,3,4$ is Lipschitz continuous. Differentiating $g_i(\tau, \theta_g, \theta_s r, \eta)$ for each $i = 1,2,3,4$ with respect to θ_g, θ_s, r and η respectively. Hence, we have

$$\left.\begin{aligned} &\left|\frac{\partial g_1}{\partial \theta_g}\right| \le \beta\{H_P^{\max} + H_L^{\max} + H_E^{\max}\} + \left(\frac{1+n\beta(1+\beta c_1)}{(1+\beta c_1)}\right) H_P^{\max} + \frac{(1+c_3)}{2} J^{\max} + (1+\beta c_1) H_E^{\max} \\ &\left|\frac{\partial g_1}{\partial \theta_s}\right| \le d_3, \quad \left|\frac{\partial g_1}{\partial r}\right| \le \frac{(1+\beta c_1)}{\gamma(1+c_3)}\{H_L^{\max} + d_2(1+c_3)^2[(1+\beta c_1)^4 - 1]\}, \quad \left|\frac{\partial g_1}{\partial \eta}\right| \le \frac{H_P^{\max}}{(1+c_4)} \end{aligned}\right\}, \tag{3.6}$$

$$\left|\frac{\partial g_2}{\partial \theta_g}\right| \le d_3\mu, \left|\frac{\partial g_2}{\partial \theta_s}\right| \le d_3\mu, \left|\frac{\partial g_2}{\partial r}\right| = 0, \left|\frac{\partial g_2}{\partial \eta}\right| = 0, \tag{3.7}$$

$$\left.\begin{aligned} &\left|\frac{\partial g_3}{\partial \theta_g}\right| \le \frac{\mu^*}{2(1+\beta c_1)(1+c_3)} J^{\max}, \quad \left|\frac{\partial g_3}{\partial \theta_s}\right| = 0 \\ &\left|\frac{\partial g_3}{\partial r}\right| \le \frac{\mu^*}{(1+c_3)^5}\left\{H_L^{\max} + d_2(1+c_3)^2\left[(1+\beta c_1)^4 - 1\right]\right\}, \quad \left|\frac{\partial g_3}{\partial \eta}\right| = 0 \end{aligned}\right\}, \tag{3.8}$$

$$\left.\begin{aligned} &\left|\frac{\partial g_4}{\partial \theta_g}\right| \le \frac{B\psi^*}{(1+\beta c_1)}\left\{\frac{1+n\beta(1+\beta c_1)}{(1+\beta c_1)} H_P^{\max} + \frac{(1+c_3)}{2} J^{\max}\right\}, \quad \left|\frac{\partial g_4}{\partial \theta_s}\right| = 0, \\ &\left|\frac{\partial g_4}{\partial r}\right| \le \frac{\psi^*}{(1+c_3)}\left\{H_L^{\max} + d_2(1+c_3)^2\left[(1+\beta c_1)^4 - 1\right]\right\}, \quad \left|\frac{\partial g_4}{\partial \eta}\right| \le \frac{B\psi^*}{(1+c_4)} H_P^{\max} \end{aligned}\right\}. \tag{3.9}$$

where

$$\left.\begin{aligned} H_P^{\max} &= \delta(1+c_4)(1+\beta c_1)^n \exp\left(\frac{c_1}{1+\beta c_1}\right), \\ H_L^{\max} &= (1+c_3)\left\{d_1 c_1\sqrt{1+\beta c_1} + d_2(1+c_3)\left[(1+\beta c_1)^4 - 1\right]\right\}, \\ H_E^{\max} &= d_3\left|c_1 - c_2\right|, \\ J^{\max} &= \left\{d_1(2+3\beta c_1)\sqrt{1+\beta c_1} + 8\beta d_2(1+c_3)(1+\beta c_1)^4\right\} \end{aligned}\right\}. \tag{3.10}$$

Then there exists a number (Lipschitz constant) $K,\ 0 \le K < \infty,$ such that

$$K = \max_i\left\{\left|\frac{\partial g_i}{\partial \theta_g}\right|, \left|\frac{\partial g_i}{\partial \theta_s}\right|, \left|\frac{\partial g_i}{\partial r}\right|, \left|\frac{\partial g_i}{\partial \eta}\right|\right\},\ i = 1,2,3,4. \tag{3.11}$$

Consequently, $\frac{\partial g_i}{\partial \theta_g}, \frac{\partial g_i}{\partial \theta_s}, \frac{\partial g_i}{\partial r}, \frac{\partial g_i}{\partial \eta}$ for each $i = 1,2,3,4.$ is bounded and $g_i(\tau, \theta_g, \theta_s, r, \eta)$ for each $i = 1,2,3,4.$ is Lipschitz continuous. Hence, there exists a unique solution of the system of coupled non-linear ordinary differential equations (2.7)–(2.10) which satisfies the initial conditions (2.11). ∎

***Theorem* 3.2**

Let $\delta > (\alpha_1 + \alpha_3),\ \delta > 0,\ \alpha_3 > 0$. Then θ_g is an increasing function of τ.

Proof

Suppose $\beta \to 0$, $r = 1$ and $\eta = 1$, the system of equations (2.7)-(2.10) reduces to

$$\frac{d\theta_g}{d\tau} = \delta\exp(\theta_g) - \alpha_1\theta_g - \alpha_3(\theta_g - \theta_s) \tag{3.12}$$

$$\frac{d\theta_s}{d\tau} = \alpha_3\mu(\theta_g - \theta_s), \quad \theta_g \ge \theta_s \tag{3.13}$$

with the initial conditions $\theta_g(0) = 0,\ \theta_s(0) = 0$.

We shall prove the result by Picard iteration. Thus, we

$$\left.\begin{aligned}&\theta_{g0}(\tau)=0,\\&\theta_{s0}(\tau)=0,\\&\theta_{gk}(\tau)=\theta_{g0}(\tau)+\int_0^\tau\left[\delta\exp(\theta_{gk-1}(s))-\alpha_1\theta_{gk-1}(s)-\alpha_3\left(\theta_{gk-1}(s)-\theta_{sk-1}(s)\right)\right]ds,\\&\theta_{sk}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^\tau\left(\theta_{gk-1}(s)-\theta_{sk-1}(s)\right)ds.\end{aligned}\right\}\quad(3.14)$$

For $k=1$, we have

$$\left.\begin{aligned}&\theta_{g1}(\tau)=\theta_{g0}(\tau)+\int_0^\tau\left[\delta\exp(\theta_{g0}(s))-\alpha_1\theta_{g0}(s)-\alpha_3\left(\theta_{g0}(s)-\theta_{s0}(s)\right)\right]ds\\&\theta_{s1}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^\tau\left(\theta_{g0}(s)-\theta_{s0}(s)\right)ds.\end{aligned}\right\}\quad(3.15)$$

Thus, we obtain

$$\theta_{g1}(\tau)=\int_0^\tau\delta\,ds=\delta\tau,\ \theta_{s1}(\tau)=0.\quad(3.16)$$

We assume it is true for $k=n$, that is

$$\left.\begin{aligned}&\theta_{gn}(\tau)=\theta_{g0}(\tau)+\int_0^\tau\left[\delta\exp(\theta_{gn-1}(s))-\alpha_1\theta_{gn-1}(s)-\alpha_3\left(\theta_{gn-1}(s)-\theta_{sn-1}(s)\right)\right]ds\\&\theta_{sn}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^\tau\left(\theta_{gn-1}(s)-\theta_{sn-1}(s)\right)ds.\end{aligned}\right\}\quad(3.17)$$

For $k=n+1$, we get

$$\left.\begin{aligned}&\theta_{gn+1}(\tau)=\theta_{g0}(\tau)+\int_0^\tau\left[\delta\exp(\theta_{gn}(s))-\alpha_1\theta_{gn}(s)-\alpha_3\left(\theta_{gn}(s)-\theta_{sn}(s)\right)\right]ds\\&\theta_{sn+1}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^\tau\left(\theta_{gn}(s)-\theta_{sn}(s)\right)ds.\end{aligned}\right\}\quad(3.18)$$

Therefore, we find that

$$\left.\begin{aligned}&\theta_{gn+1}(\tau)-\theta_{gn}(\tau)=\int_0^\tau\Big\{\delta\left[\exp(\theta_{gn}(s))-\exp(\theta_{gn-1}(s))\right]-\alpha_1\left(\theta_{gn}(s)-\theta_{gn-1}(s)\right)\\&\qquad\qquad-\alpha_3\left[\left(\theta_{gn}(s)-\theta_{sn}(s)\right)-\left(\theta_{gn-1}(s)-\theta_{sn-1}(s)\right)\right]\Big\}ds\\&=\int_0^\tau\Big\{\delta\left[\exp(\theta_{gn}(s))-\exp(\theta_{gn-1}(s))\right]-(\alpha_1+\alpha_3)\left(\theta_{gn}(s)-\theta_{gn-1}(s)\right)\\&\quad+\alpha_3\left(\theta_{sn}(s)-\theta_{sn-1}(s)\right)\Big\}ds\\&\theta_{sn+1}(\tau)-\theta_{sn}(\tau)=\alpha_3\mu\int_0^\tau\left\{\left(\theta_{gn}(s)-\theta_{sn}(s)\right)-\left(\theta_{gn-1}(s)-\theta_{sn-1}(s)\right)\right\}ds.\end{aligned}\right\}\quad(3.19)$$

By Maclaurin series expansion of $\exp(\theta_{gn})$ and $\exp(\theta_{gn-1})$, we obtain

$$
\left.\begin{aligned}
\theta_{gn+1}(\tau)-\theta_{gn}(\tau) &= \int_0^{\tau}\Bigg\{\delta\Bigg[\Bigg(1+\theta_{gn}+\frac{\theta_{gn}^2}{2!}+\frac{\theta_{gn}^3}{3!}+\cdots\Bigg)-\Bigg(1+\theta_{gn-1}+\frac{\theta_{gn-1}^2}{2!}+\frac{\theta_{gn-1}^3}{3!}+\cdots\Bigg)\Bigg]\\
&\quad -(\alpha_1+\alpha_3)\big(\theta_{gn}(s)-\theta_{gn-1}(s)\big)+\alpha_3\big(\theta_{sn}(s)-\theta_{sn-1}(s)\big)\Bigg\}ds\\
\theta_{gn+1}(\tau)-\theta_{gn}(\tau) &= \int_0^{\tau}\Bigg\{\delta\big(\theta_{gn}(s)-\theta_{gn-1}(s)\big)+\delta\Bigg[\frac{\big(\theta_{gn}^2-\theta_{gn-1}^2\big)}{2!}+\frac{\big(\theta_{gn}^3-\theta_{gn-1}^3\big)}{3!}+\cdots\Bigg]\\
&\quad -(\alpha_1+\alpha_3)\big(\theta_{gn}(s)-\theta_{gn-1}(s)\big)+\alpha_3\big(\theta_{sn}(s)-\theta_{sn-1}(s)\big)\Bigg\}ds.
\end{aligned}\right\}\quad (3.20)
$$

Since $\delta > (\alpha_1+\alpha_3)$, $\delta > 0$, $\alpha_3 > 0$, $\mu > 0$ and $\theta_g \geq \theta_s$, we have

$$
\left.\begin{aligned}
\theta_{gn+1}(\tau)-\theta_{gn}(\tau) &> \int_0^{\tau}\Bigg\{\delta\Bigg[\frac{\big(\theta_{gn}^2-\theta_{gn-1}^2\big)}{2!}+\frac{\big(\theta_{gn}^3-\theta_{gn-1}^3\big)}{3!}+\cdots\Bigg]+\alpha_3\big(\theta_{sn}(s)-\theta_{sn-1}(s)\big)\Bigg\}ds\\
\theta_{sn+1}(\tau)-\theta_{sn}(\tau) &= \alpha_3\mu\int_0^{\tau}\Big\{\big(\theta_{gn}(s)-\theta_{sn}(s)\big)-\big(\theta_{gn-1}(s)-\theta_{sn-1}(s)\big)\Big\}ds \geq 0.
\end{aligned}\right\}\quad (3.21)
$$

Thus $$\theta_{gn+1}(\tau)-\theta_{gn}(\tau) > 0\,. \quad (3.22)$$

Hence $\theta_g(\tau)$ is an increasing function of τ. This completes the proof. ∎

***Theorem* 3.3**

Let $(\alpha_1+\alpha_3) < \delta,\ \delta > 1,\ \alpha_3 > 0$. Then $\theta_s \geq 0$.

Proof

We shall prove the result by Picard iteration.

$$
\left.\begin{aligned}
&\theta_{g0}(\tau)=0,\\
&\theta_{s0}(\tau)=0,\\
&\theta_{gk}(\tau)=\theta_{g0}(\tau)+\int_0^{\tau}\Big[\delta\exp(\theta_{gk-1}(s))-\alpha_1\theta_{gk-1}(s)-\alpha_3\big(\theta_{gk-1}(s)-\theta_{sk-1}(s)\big)\Big]ds,\\
&\theta_{sk}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^{\tau}\big(\theta_{gk-1}(s)-\theta_{sk-1}(s)\big)ds.
\end{aligned}\right\}\quad (3.23)
$$

For $k=1$, we have

$$
\left.\begin{aligned}
&\theta_{g1}(\tau)=\theta_{g0}(\tau)+\int_0^{\tau}\Big[\delta\exp(\theta_{g0}(s))-\alpha_1\theta_{g0}(s)-\alpha_3\big(\theta_{g0}(s)-\theta_{s0}(s)\big)\Big]ds\\
&\theta_{s1}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^{\tau}\big(\theta_{g0}(s)-\theta_{s0}(s)\big)ds.
\end{aligned}\right\}\quad (3.24)
$$

Thus, we obtain

$$\theta_{g1}(\tau)=\int_0^{\tau}\delta\,ds=\delta\tau\,,\ \theta_{s1}(\tau)=0\,. \quad (3.25)$$

For $k=2$, we have

$$
\left.\begin{aligned}
&\theta_{g2}(\tau)=\theta_{g0}(\tau)+\int_0^{\tau}\Big[\delta\exp(\theta_{g1}(s))-\alpha_1\theta_{g1}(s)-\alpha_3\big(\theta_{g1}(s)-\theta_{s1}(s)\big)\Big]ds\\
&\theta_{s2}(\tau)=\theta_{s0}(\tau)+\alpha_3\mu\int_0^{\tau}\big(\theta_{g1}(s)-\theta_{s1}(s)\big)ds.
\end{aligned}\right\}\quad (3.26)
$$

Thus, we obtain

$$\left.\begin{aligned}\theta_{g2}(\tau) &= \int_0^\tau \delta[\exp(\delta s) - (\alpha_1 + \alpha_3)s]ds \\ \theta_{s2}(\tau) &= \alpha_3\mu\int_0^\tau \delta s\, ds.\end{aligned}\right\} \tag{3.27}$$

So that

$$\left.\begin{aligned}\theta_{g2}(\tau) &= \exp(\delta\tau) - \frac{\delta}{2}(\alpha_1 + \alpha_3)\tau^2 - 1 > 0 \\ \theta_{s2}(\tau) &= \frac{\alpha_3\mu\delta}{2}\tau^2 > 0.\end{aligned}\right\} \tag{3.28}$$

We assume that $\theta_{sn}(\tau) > \theta_{sn-1}$. Hence, by Picard iteration we have

$$\theta_{sn+1}(\tau) = \theta_{s0}(\tau) + \alpha_3\mu\int_0^\tau \left(\theta_{gn}(s) - \theta_{sn}(s)\right)ds. \tag{3.29}$$

But $\theta_{gn}(\tau) - \theta_{sn}(\tau) \geq 0$, $\alpha_3 > 0$ and $\mu > 0$. Therefore, $\theta_{sn+1}(\tau) \geq 0$. This completes the proof. ∎

***Theorem* 3.4**

Let $\delta > (\alpha_1 + \alpha_3)$, $\delta > 0$, $\alpha_3 > 0$. Then $\theta_g \to \infty$ in finite time i.e. blows up.

Proof

Suppose $\beta \to 0$, $r = 1$ and $\eta = 1$ in equation (2.7), we have

$$\frac{d\theta_g}{d\tau} = \delta\exp(\theta_g) - (\alpha_1 + \alpha_3)\theta_g + \alpha_3\theta_s. \tag{3.30}$$

By Maclaurin series expansion equation (3.30) becomes

$$\frac{d\theta_g}{d\tau} = \delta\left(1 + \theta_g + \frac{\theta_g^2}{2!} + \frac{\theta_g^3}{3!} + \cdots\cdots\right) - (\alpha_1 + \alpha_3)\theta_g + \alpha_3\theta_s. \tag{3.31}$$

This implies that

$$\frac{d\theta_g}{d\tau} \geq \frac{\delta}{2!}\theta_g^2, \text{ since } \delta > (\alpha_1 + \alpha_3),\ \delta > 0 \text{ and } \alpha_3 > 0. \tag{3.32}$$

Now, let

$$\frac{d\theta_g}{d\tau} = \frac{\delta}{2!}\theta_g^2. \tag{3.33}$$

By separation of variable, we have

$$\frac{d\theta_g}{\theta_g^2} = \frac{\delta}{2!}d\tau. \tag{3.34}$$

Integrating, we get

$$\theta_g = \frac{2}{C - \delta\tau},\ C \text{ is constant of integration.} \tag{3.35}$$

By Picard iteration, we have $\theta_{g1}(\tau) = \delta\tau$. Start $\tau = \tau_*$, we have $\theta_{g1}(\tau) = \delta\tau_*$. Then $\tau = \tau_*$, $\theta_g(\tau) \geq \delta\tau_*$, $\theta_g(\tau_*) = \dfrac{2}{C - \delta\tau_*} = \delta\tau_*$. Thus, we obtain

$$\left.\begin{aligned} C &= \delta\tau_* + \frac{2}{\delta\tau_*}, \\ \theta_g &= \frac{2}{\delta\tau_* + (2/\delta\tau_*) - \delta\tau} \ . \end{aligned}\right\} \tag{3.36}$$

Hence, $\theta_g \to \infty$ when $\tau = \tau_* + \dfrac{2}{\delta^2\tau_*}$. This completes the proof. ∎

***Theorem* 3.5**

Let $\alpha_1 > 0,\ \mu^* > 0$. Then $r \to 0$ in finite time i.e. quenches.

Proof

Suppose $\beta \to 0$ in equation (2.9), we have

$$r\frac{dr}{d\tau} = -\alpha_1\mu^*\theta_g \ . \tag{3.37}$$

Since $\theta_g = \dfrac{2}{C - \delta\tau}$, we get $rdr = \dfrac{-2\alpha_1\mu^*}{C - \delta\tau}d\tau$. Integrating, we find that

$$\frac{r^2}{2} = \frac{2\alpha_1\mu^*}{\delta}\ln(C - \delta\tau) + D \tag{3.38}$$

where D is constant of integration. When $\tau = \tau_*$ and $r = r_*$, we get $D = \dfrac{r_*^2}{2} - \dfrac{2\alpha_1\mu^*}{\delta}\ln(C - \delta\tau_*)$.

Now, substituting D into (3.38), we obtain

$$\frac{r^2}{2} = \frac{r_*^2}{2} - \frac{2\alpha_1\mu^*}{\delta}\ln\left(\frac{C - \delta\tau_*}{C - \delta\tau}\right). \tag{3.39}$$

When $C - \delta\tau = 0$, we obtain

$$\frac{r^2}{2} = \frac{r_*^2}{2} - \infty \ . \tag{3.40}$$

Hence, $\dfrac{r^2}{2} = -\infty$. This means that r becomes zero before $C - \delta\tau = 0$. This completes the proof. ∎

***Theorem* 3.6**

Let $\delta > 0,\ B << 1,\ \psi > 0$. Then $\eta \to 0$ in finite time i.e. quenches.

Proof

Suppose $\beta \to 0$, in equation (2.10) $r = 1$, we have

$$\frac{d\eta}{d\tau} = -B\psi\delta\eta\exp(\theta_g) + \alpha_1\psi^*\theta_g \ . \tag{3.41}$$

By Maclaurin series expansion, we have

$$\left.\begin{aligned} \frac{d\eta}{d\tau} &= -B\psi\delta\eta\left(1 + \theta_g + \frac{\theta_g^2}{2!} + \frac{\theta_g^3}{3!} + \cdots\cdots\right) + \alpha_1\psi\theta_g \\ &= -B\psi\delta\eta - B\psi\delta\eta\theta_g - \frac{B\psi\delta\eta\theta_g^2}{2!} - \cdots\cdots + \alpha_1\psi\theta_g \ . \end{aligned}\right\} \tag{3.42}$$

That is $$\frac{d\eta}{d\tau} \le -B\psi\delta\eta\left(1+\frac{\theta_g^2}{2!}\right), \text{ since } \alpha_1 \approx B\delta\eta . \qquad (3.43)$$

Now let $$\frac{d\eta}{d\tau} = -B\psi\delta\eta\left(1+\frac{\theta_g^2}{2!}\right). \qquad (3.44)$$

Substituting $\theta_g = \dfrac{2}{C-\delta\tau}$ into (3.44) and separating the variable, we have

$$\frac{d\eta}{\eta} = -B\psi\delta\left(1+\frac{2}{(C-\delta\tau)^2}\right)d\tau . \qquad (3.45)$$

Integrating equation (3.45) gives

$$\ln\eta = -B\psi\delta\left(\tau+\frac{2}{\delta(C-\delta\tau)}\right)+E, \qquad (3.46)$$

where E is constant of integration. Employing the initial condition $\eta = 1$ at $\tau = 0$, we obtain $E = \dfrac{2B\psi}{C}$. Thus, we obtain

$$\ln\eta = -B\psi\delta\left(\tau+\frac{2}{\delta(C-\delta\tau)}\right)+\frac{2B\psi}{C}. \qquad (3.47)$$

Taking the exponential of both sides yields

$$\eta = F\exp\left(-B\psi\delta\left(\tau+\frac{2}{\delta(C-\delta\tau)}\right)\right) \qquad (3.48)$$

where $F = \exp(2B\psi/C)$. Hence, $\eta \to 0$ as $C-\delta\tau \to 0$. This completes the proof. ∎

4.0 Adiabatic case for combustible dusty gas

The basic idea of the model in this section is the competition between the heat production (due to exothermic reaction and high activation energy) and heat exchange between the gas and solid particle temperatures. The presence of heat exchange makes the adiabatic case $(\alpha_1 = 0, \alpha_2 = 0)$ qualitatively informative. The proposed model is a two-temperature (gas-solid) system with reactant consumption. The system (2.7)-(2.10) is reduced to the following system:

$$\frac{d\theta_g}{d\tau} = \delta\eta\exp(\theta_g) - \alpha_3(\theta_g - \theta_s) \qquad (4.1)$$

$$\frac{d\theta_s}{d\tau} = \alpha_3\mu(\theta_g - \theta_s) \qquad (4.2)$$

$$\frac{d\eta}{d\tau} = -\alpha_4\delta\eta\exp(\theta_g). \qquad (4.3)$$

Multiplying both sides of equation (4.1) by α_4 gives

$$\alpha_4\frac{d\theta_g}{d\tau} = \alpha_4\delta\eta\exp(\theta_g) - \alpha_3\alpha_4(\theta_g - \theta_s). \qquad (4.4)$$

Addition of equations (4.3) and (4.4) yields

$$\alpha_4\frac{d\theta_g}{d\tau} + \frac{d\eta}{d\tau} = -\alpha_3\alpha_4(\theta_g - \theta_s). \qquad (4.5)$$

Using equation (4.2) in equation (4.5), we get

$$\alpha_4 \frac{d\theta_g}{d\tau} + \frac{d\eta}{d\tau} = -\frac{\alpha_4}{\mu}\frac{d\theta_s}{d\tau}. \tag{4.6}$$

Integrating equation (4.6) with respect to τ gives

$$\alpha_4 \theta_g + \eta = -\frac{\alpha_4}{\mu}\theta_s + G. \tag{4.7}$$

Using the initial conditions $\theta_g = 0,\ \theta_s = 0,\ \eta = 1$ at $\tau = 0$, we get $G = 1$. Then equation (4.7) becomes

$$\eta + \alpha_4 \theta_g + \frac{\alpha_4}{\mu}\theta_s = 1. \tag{4.8}$$

Substituting equation (4.8) into equation (4.1) yields

$$\frac{d\theta_g}{d\tau} = \delta\eta \exp(\theta_g) - \alpha_3(1+\mu)\theta_g + \frac{\alpha_3 \mu}{B\psi}(1-\eta), \text{ since } \alpha_4 = B\psi. \tag{4.9}$$

If we take the assumption that, in terms of the explosive behaviour, the consumption of reactant is insignificant, we assume $d\eta/d\tau = 0$ and put $\eta = 1$ in equations (4.8) and (4.9). Thus we have

$$\theta_s + \mu\theta_g = 0 \tag{4.10}$$

$$\frac{d\theta_g}{d\tau} = \delta \exp(\theta_g) - \alpha_3(1+\mu)\theta_g. \tag{4.11}$$

Since equation (4.11) does possess closed form solution, following [7, 10] we assume the quadratic approximations to the Arrhenius term, i.e.

$$\exp(\theta_g) \approx 1 + (e-2)\theta_g + \theta_g^2. \tag{4.12}$$

Thus, equation (4.11) becomes

$$\frac{d\theta_g}{d\tau} = \delta\theta_g^2 + \Omega\,\theta_g + \delta \tag{4.13}$$

where $\Omega = [\delta(e-2) - \alpha_3(1+\mu)]$. Separating the variable and integrating equation (4.13), we have

$$\int \frac{d\theta_g}{\delta\theta_g^2 + \Omega\,\theta_g + \delta} = \tau + H. \tag{4.14}$$

Thus, we get

$$\frac{2}{\sqrt{4\delta^2 - \Omega^2}} \tan^{-1}\left(\frac{2\delta\theta_g + \Omega}{\sqrt{4\delta^2 - \Omega^2}}\right) = \tau + H. \tag{4.15}$$

Using the initial condition $\theta_g = 0$, we obtain

$$H = \frac{2}{\sqrt{4\delta^2 - \Omega^2}} \tan^{-1}\left(\frac{\Omega}{\sqrt{4\delta^2 - \Omega^2}}\right). \tag{4.16}$$

Putting equation (4.16) into equation (4.15) gives

$$\frac{2}{\sqrt{4\delta^2 - \Omega^2}} \tan^{-1}\left(\frac{2\delta\theta_g + \Omega}{\sqrt{4\delta^2 - \Omega^2}}\right) = \tau + \frac{2}{\sqrt{4\delta^2 - \Omega^2}} \tan^{-1}\left(\frac{\Omega}{\sqrt{4\delta^2 - \Omega^2}}\right) \tag{4.17}$$

Simplifying equation (4.17), we get

$$\theta_g(\tau) = \frac{1}{2\delta}\left\{\left(\frac{(4\delta^2 - \Omega^2)\tan\left(\frac{1}{2}\sqrt{4\delta^2 - \Omega^2}\,\tau\right) + \Omega\sqrt{4\delta^2 - \Omega^2}}{\sqrt{4\delta^2 - \Omega^2} - \Omega\tan\left(\frac{1}{2}\sqrt{4\delta^2 - \Omega^2}\,\tau\right)}\right) - \Omega\right\}. \tag{4.18}$$

Hence, substituting equation (4.18) into equation (4.10) yields

$$\theta_s(\tau) = \frac{\mu}{2\delta}\left\{\Omega - \left(\frac{\left(4\delta^2 - \Omega^2\right)\tan\left(\frac{1}{2}\sqrt{4\delta^2 - \Omega^2}\,\tau\right) + \Omega\sqrt{4\delta^2 - \Omega^2}}{\sqrt{4\delta^2 - \Omega^2} - \Omega\tan\left(\frac{1}{2}\sqrt{4\delta^2 - \Omega^2}\,\tau\right)}\right)\right\} \tag{4.19}$$

Thermal runaway occurs when θ_g becomes infinite $(\theta_g \to \infty)$ at finite time. The finite time (i.e. ignition time) identified as the time of this runaway is obtained from equation (4.17) as

$$\tau_{ig} = \frac{1}{\sqrt{4\delta^2 - \Omega^2}}\left(\pi - 2\tan^{-1}\left(\frac{\Omega}{\sqrt{4\delta^2 - \Omega^2}}\right)\right), \tag{4.20}$$

and the results are valid for $\delta(e-4)/1+\mu < \alpha_3 < \delta e/1+\mu$.

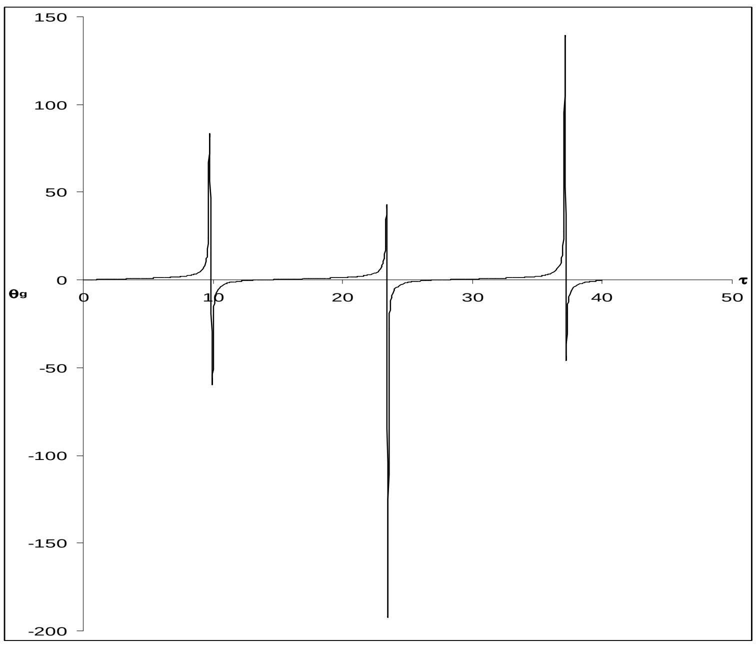

Figure 4.1: Plot of combustible gas mixture temperature profile with time for $\delta = 0.3$, $\mu = 2$ and $\alpha_3 = 0.2$.

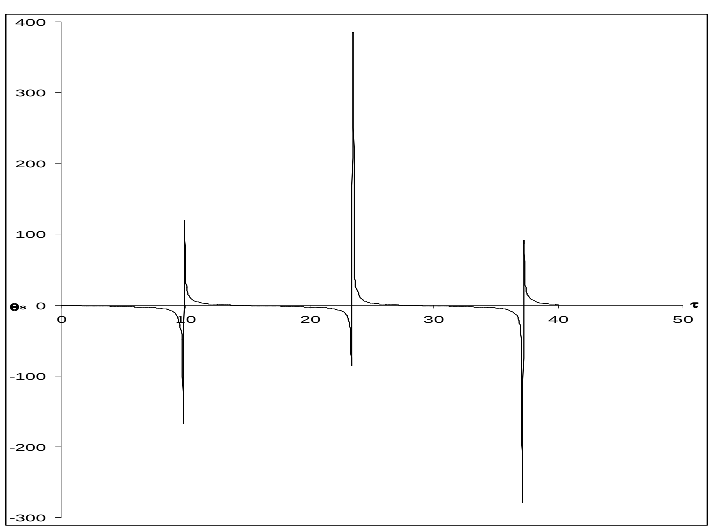

Figure 4.2: Plot of solid particle temperature profile with time for $\delta = 0.3,\ \mu = 2$ and $\alpha_3 = 0.2$.

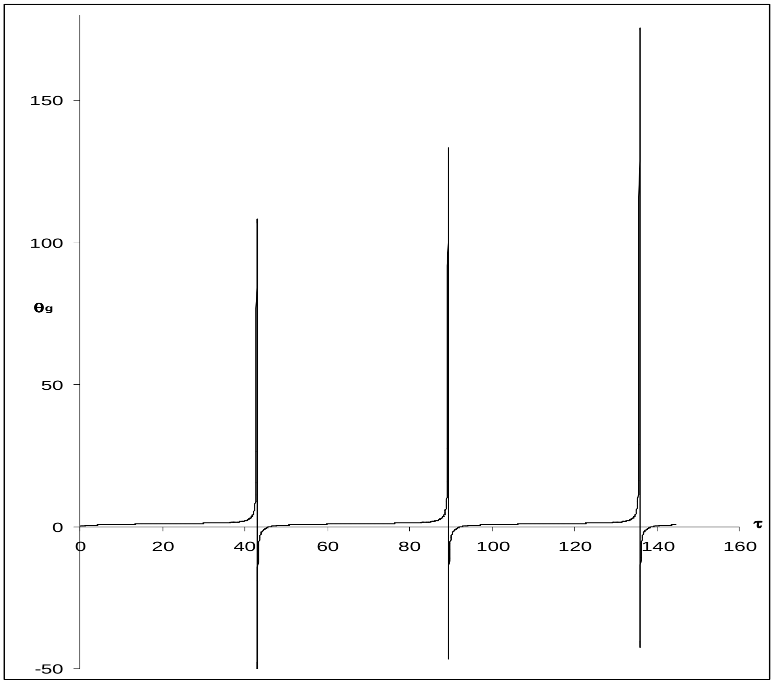

Figure 4.3: Plot of combustible gas mixture temperature profile with time for $\delta = 0.3,\ \mu = 1$ and $\alpha_3 = 0.4$.

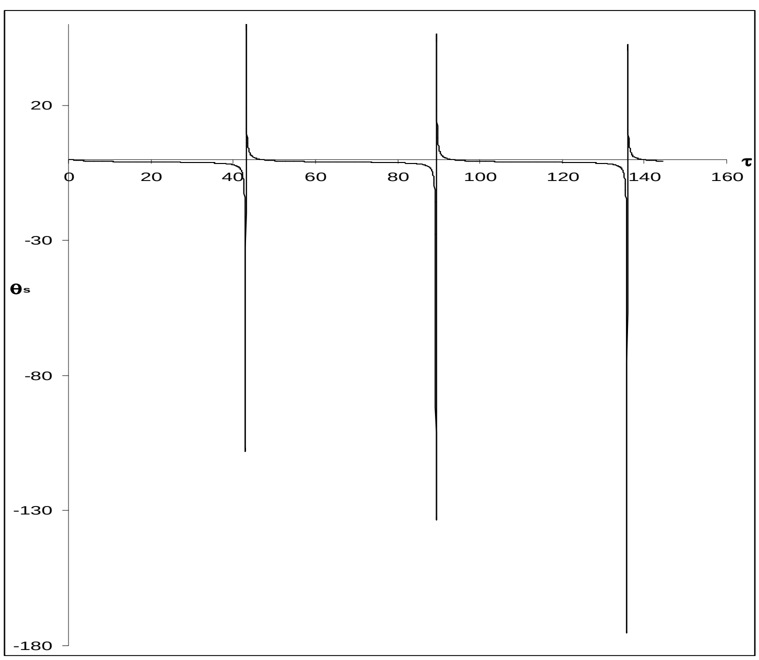

Figure 4.4: Plot of solid particle temperature profile with time for $\delta = 0.3,\ \mu = 1$ and $\alpha_3 = 0.4$.

5.0 Discussion of results

Properties of solutions to validate the mathematical model of physical problem under consideration are adequately delineated. The criteria to establish the proofs of the theorems for the new problem are clearly stated.

We display in Figures 4.1 to 4.2 the graphs of dimensionless combustible gas and solid particle temperatures against time when $\delta = 0.3,\ \mu = 2$ and $\alpha_3 = 0.2$. We notice that convectional thermal explosion occur at ignition time $\tau_{ig} = 9.8$ while the subsequent thermal explosions with freeze delay occur at different ignition times $\tau_{ig} = 23.4$ and $\tau_{ig} = 37.1$ respectively in gas phase whereas in solid phase it is vice versa.

Similarly, Figures 4.3 to 4.4 illustrate the graphs of gas and solid particle temperatures against time when $\delta = 0.3,\ \mu = 1$ and $\alpha_3 = 0.4$. It is also observed that convectional thermal explosion occur at ignition time $\tau_{ig} = 43.0$ while the subsequent thermal explosions with freeze delay occur at different ignition times $\tau_{ig} = 89.4$ and $\tau_{ig} = 135.8$ respectively in gas phase whereas in solid phase it is vice versa.

The analysis reveals that the ignition time exists when the interphase heat exchange parameter is within the range $\delta(e-4)/1+\mu < \alpha_3 < \delta e/1+\mu$.

6.0 Conclusion

The problem of thermal explosion in combustible dusty gas mixtures containing fuel droplets have been extended to permit a more general temperature dependent rate of reaction for most typical practical reactions under physically reasonable assumptions. The mathematical formulation involves a system of four highly non-linear ordinary differential equations. The properties of solutions, which validate system of governing equations representing the physical model, were analysed by formulating theorems and establishing the proofs respectively. In multiple phase processes, interphase heat exchange plays the role of heat losses in homogeneous combustible gas mixtures. Specifically, adiabatic models for the self-ignition provide conceptual information concerning the system parameters which influence the self-ignition and delay effects. The adiabatic approach offers the possibility of analytical investigations. Therefore closed form analytic solutions were obtained based on quadratic approximations to the Arrhenius term for combustible gas mixtures temperature, solid particle temperature and ignition time from the simplified governing equations. It was found that existence and type of delay before ignition depend significantly on interphase heat exchange parameter α_3 and energy needed to transfer heat from gas phase to solid phase parameter μ. However, there is a jump from conventional thermal explosion to thermal explosions with freeze delay. Likewise, the solid particles do experience jumps at different ignition times.

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